Proc. of 3rd KSV Conference
SNU, Seoul, Korea, Apr. 23, 2004

Fluids in Computer Animation

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최근 컴퓨터 애니메이션 분야에서는 계산 유체 역학 분야의 기법을 적용하여 다양한 유체 애니메이션 효과를 생성하려는 시도가 활발히 진행되고 있다. 본 강연에서는 연기와 가스, 그리고 물 등의 유체에 대하여 애니메이션 제작 분야에서 제안되고 적용되고 있는 유체 기법에 대하여 설명한 후, 본 서강대학교 컴퓨터 그래픽스 연구실에서 수행하고 있는 유체 애니메이션 관련 연구에 대하여 간략히 알아본다.

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2004. 4. 23

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Physically Based Fluids in Computer Animation

?

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} + \nu \nabla \cdot (\nabla \mathbf{u}) - \frac{1}{\rho}\nabla p + \mathbf{f}$$

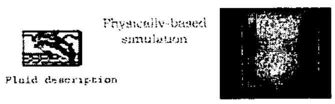
...

$$\phi_t + \mathbf{u} \cdot \nabla \phi = 0$$

$$\phi_t + sgn(\phi)(|P_{\nabla\psi} \cdot \nabla \phi| - 1) = 0$$

Two Major Tasks in Fluid Animation

- How are such natural phenomena as water, smoke, fire, flame, and explosion simulated numerically?



- How are the simulated results visualized realistically?

Important Issues in Fluid Animation

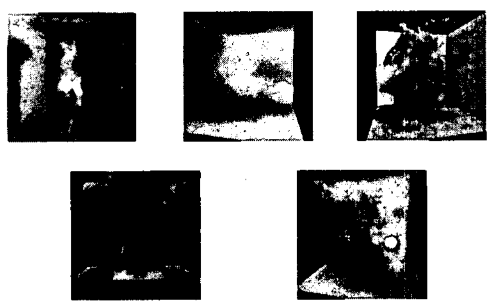
- Which one is more important in the physically based fluid animation?

  Preciseness ⟷ Cost & Complexity / Controllability

  - Preciseness and Realism
    - Physically-convincing motion must be generated.
  - Cost & Complexity
    - 5min x 60sec/min x 30fr/sec x 10min/fr = 90,000min = 62.5days
  - Controllability
    - Animation is all about control!
- Applicability to animation production is crucial.

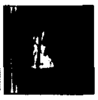
- Is it possible to have a fully-interactive real-time physically-based fluid animation?
  - The current programmable GPUs on PCs will be powerful and flexible enough to make this possible.

Physically Based Simulation of Smoke and Gas

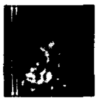


Recent Works on Gas and Smoke Simulations

- J. Stam, "Turbulent Wind Field for Gaseous Phenomena," *ACM SIGGRAPH '93*, pp. 369-376, 1993.



- J. Stam, and E. Fiume, "Depiction of Fire and Other Gaseous Phenomena Using Diffusion Process," *ACM SIGGRAPH '95*, pp. 129-136, 1995.



- N. Foster, and D. Metaxas, "Modeling the motion of a hot, turbulent gas," *ACM SIGGRAPH '97*, pp. 181-188, 1997.

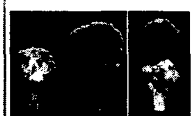
- J. Stam, "Stable Fluids," *ACM SIGGRAPH '99*, pp. 121-128, 1999.

- J. Stam, "Interacting with smoke and fire in real time," *CACM*, Vol. 43, pp. 76-83, 2000.

- R. Fedkiw, and J. Stam, and H. Jenson, "Visual Simulation of Smoke," *ACM SIGGRAPH '01*, pp. 15-22, 2001.

- J. Stam, "A simple fluid solver based on the FFT," Journal of Graphics Tools, Vol. 6, pp. 43-52, 2001.
- J. Stam, "Real-Time Fluid Dynamics for Games," Proceedings of Game Developer Conference, 2003.

- A. Treuille, A. McNamara, Z. Popovic, and J. Stam, "Keyframe Control of Smoke Animation," *ACM Transactions on Graphics (ACM SIGGRAPH '03)*, Vol. 22, No. 3, pp. 716-723, 2003.



- N. Rasmussen, D. Nguyen, W. Geiger, and R. Fedkiw, "Smoke simulation for large scale phenomena," *ACM Transactions on Graphics (ACM SIGGRAPH '03)*, Vol. 22, No. 3, pp. 703-707, 2003.

- X. Wei, Y. Zhao, Z. Fan, W. Li, S. Yoakum-Stover, and A. Kaufman "Blowing in the wind," *ACM Siggraph/Eurographics Symposium on Computer Animation '03*, pp. 75-85, 2003.

Basic Equations in [Stam99]

- The incompressible Navier-Stokes equations

For the velocity $\mathbf{u} = (u, v, w)$,

- Conservation of mass

$$\nabla \cdot \mathbf{u} = 0$$

- Conservation of momentum

$$\frac{\partial \mathbf{u}}{\partial t} = \underbrace{-(\mathbf{u} \cdot \nabla)\mathbf{u}}_{\text{advection}} + \underbrace{\nu \nabla \cdot (\nabla \mathbf{u})}_{\text{diffusion}} - \underbrace{\frac{1}{\rho}\nabla p}_{\text{pressure}} + \underbrace{\mathbf{f}}_{\text{external force}}$$

- Helmholtz-Hodge decomposition

  $\mathbf{w} = \mathbf{u} + \nabla q$, where $\nabla \cdot \mathbf{u} = 0$ and q is a scalar field.

  - "Any vector field is the sum of a mass conserving field and a gradient field."

- Projection operator $\mathbf{P}$

  $$\nabla \cdot \mathbf{w} = \nabla^2 q \longrightarrow \mathbf{u} = \mathbf{P}\mathbf{w} = \mathbf{w} - \nabla q$$

- The combined Navier-Stokes equations
  - Using the fact that $\mathbf{P}\mathbf{u} = \mathbf{u}$ and $\mathbf{P}\nabla p = 0$, the following equation is obtained:

  $$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{P}(-(\mathbf{u} \cdot \nabla)\mathbf{u} + \nu \nabla \cdot (\nabla \mathbf{u}) + \mathbf{f})$$

Updating the Velocity Field u

- The general procedure

$u(x, t)$ $\qquad \frac{\partial \mathbf{u}}{\partial t} = \mathbf{P}(-(\mathbf{u}\cdot\nabla)\mathbf{u} + \nu\nabla\cdot(\nabla\mathbf{u}) + \mathbf{f})$ $\qquad u(x, t + \Delta t)$

$w_0(x) \xrightarrow{\text{add force}} w_1(x) \xrightarrow{\text{advect}} w_2(x) \xrightarrow{\text{diffuse}} w_3(x) \xrightarrow{\text{project}} w_4(x)$

1. The *add force* step: **f**

$$\mathbf{w}_1(\mathbf{x}) = \mathbf{w}_0(\mathbf{x}) + \Delta t\,\mathbf{f}(\mathbf{x}, t)$$

2. The *advect* step: $-(\mathbf{w}_1 \cdot \nabla)\,\mathbf{w}_1$
   - Use the method of characteristics for the effect of advection: a semi-Lagrangian scheme

$$\mathbf{w}_2(\mathbf{x}) = \mathbf{w}_1(\mathbf{p}(\mathbf{x}, -\Delta t))$$

where $\mathbf{p}(\mathbf{x}, t)$ is the characteristics of the vector filed.

   - Implementation:
     - Build a particle tracer and linear (or cubic) interpolator.

3. The *diffuse* step: $\nu\nabla \cdot (\nabla \mathbf{w}_2)$
   - Use an implicit method for the effect of viscosity.

$$(\mathbf{I} - \nu\Delta t\nabla^2)\mathbf{w}_3(\mathbf{x}) = \mathbf{w}_2(\mathbf{x}),$$

where **I** is the identity operator.

   - Implementation:
     - Use the linear solver POIS3D form FISHPAK after discretization.

4. The *project* step: $\mathbf{P}(\mathbf{w}_3)$
   - Apply the projection operator to make the velocity field divergent-free.

$$\nabla^2 q = \nabla\cdot\mathbf{w}_3 \longrightarrow \mathbf{w}_4 = \mathbf{w}_3 - \nabla q$$

   - Implementation:
     - Use the linear solver POIS3D form FISHPAK after discretization.

Moving Substances through the Fluid

- A non-reactive substance is advected by the fluid while diffusing at the same time.
- The following equation can be used to evolve density, temperature, texture coordinates, etc.

$$\frac{\partial a}{\partial t} = -\mathbf{u}\cdot\nabla a + \kappa_a\nabla^2 a - \alpha_a a + S_a$$

where κ_a is a diffusion constant, α_a is a dissipation rate, and S_a is a source term.

- Dissipation term

$$(1 + \Delta t\alpha_a)a(\mathbf{x}, t + \Delta t) = a(\mathbf{x}, t)$$

Summary of [Stam99]

- Based on the full Navier-Stokes equations
- Based on an 'unconditionally' stable computational model
  - Semi-Lagrangian integration scheme
- Easy to implement
- Appropriate for gas and smoke
- Suffer from too much 'numerical dissipation'
  - The flow tends to dampen too rapidly.
  - [FedkiwStamJensen01] attempted to solve this problem.

Basic Equations in [FedkiwStamJensen01]

- The incompressible Euler equations
  "Gases are modeled as inviscid, incompressible, constant density fluids."

$$\nabla \cdot \mathbf{u} = 0$$
$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \nabla p + \mathbf{f}$$

- The equations for the evolution of the temperature T and the smoke's density ρ

$$\frac{\partial T}{\partial t} = -(\mathbf{u} \cdot \nabla)T$$
$$\frac{\partial \rho}{\partial t} = -(\mathbf{u} \cdot \nabla)\rho$$

Updating the Velocity Field u

1. The *add force* step: **f**
   - Update the velocity field for the effect of forces.

$$\mathbf{f} = \mathbf{f}_{user} + \mathbf{f}_{buoy} + \mathbf{f}_{conf}$$

   - $\mathbf{f}_{user}$: user-defined force (for any purpose)
   - $\mathbf{f}_{buoy}$ gravity and buoyancy forces

$$\mathbf{f}_{buoy} = -\alpha\rho\mathbf{z} + \beta(T - T_{amb})\mathbf{z}, \text{ where } \mathbf{z} = (0,0,1)$$

   - $\mathbf{f}_{conf}$ vorticity confinement force
     - Use a vorticity confinement method by Steinhoff and Underhill.
     - Inject the energy lost due to numerical dissipation back into the fluid using a forcing term.
     - Reduce the numerical dissipation inherent in semi-Langrangian schemes.

$$\omega = \nabla \times \mathbf{u} \;\rightarrow\; \mathrm{N} = \frac{\eta}{|\eta|}\ (\eta = \nabla\,|\,\omega\,|) \;\rightarrow\; \mathbf{f}_{conf} = \epsilon h(N \times \omega)$$

   - Implementation: simple

2. The *advect* step: $-(\mathbf{u} \cdot \nabla)\mathbf{u}$
   - Use the method of characteristics for the effect of advection: a semi-Lagrangian scheme
   - Implementation:
     - Build a particle tracer and linear interpolator.
     - Same as [Stam99]

3. The *project* step: $P(\mathbf{w}_3)$
   - Apply the projection operator to make the velocity field divergent-free.
   - Same as [Stam99]

$$\nabla^2 p = \frac{1}{\Delta t}\nabla \cdot \mathbf{u}^*$$
$$\downarrow$$
$$\mathbf{u} = \mathbf{u}^* - \Delta t \nabla p$$

   - Implementation:
     - Impose free Neumann boundary conditions at the occupied voxels.
     - Use the conjugate gradient method with an incomplete Choleski pre-conditioner.

Moving Substances through the Fluid

- Use the semi-Lagrangian scheme.

$$\frac{\partial T}{\partial t} = -(\mathbf{u} \cdot \nabla)T$$
$$\frac{\partial \rho}{\partial t} = -(\mathbf{u} \cdot \nabla)\rho$$

Updating the Velocity Field in [stam03]

- The general procedure

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{P}(-(\mathbf{u} \cdot \nabla)\mathbf{u} + \nu\nabla \cdot (\nabla \mathbf{u}) + \mathbf{f})$$

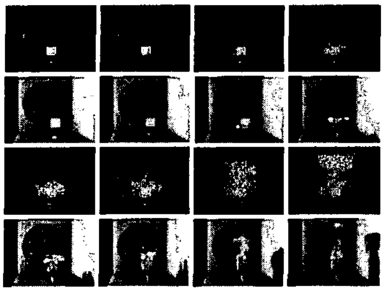
$\mathbf{u}(\mathbf{x}, t)$ … $\mathbf{u}(\mathbf{x}, t + \Delta t)$

$\mathbf{w}_0(\mathbf{x})$ *add force* $\mathbf{w}_1(\mathbf{x})$ *diffuse* $\mathbf{w}_2(\mathbf{x})$ *project* $\mathbf{w}_3(\mathbf{x})$ *advect* $\mathbf{w}_4(\mathbf{x})$ *project* $\mathbf{w}_5(\mathbf{x})$

- The basic integration method is same as in [stam99].
- The Gauss-Seidel method is used for solving the linear system instead of FFT.
  - 20 iterations are enough for the projection operator.

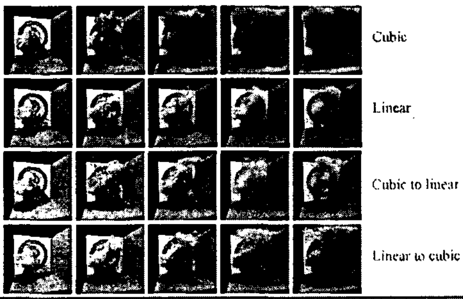
What are we doing with gas and smoke?

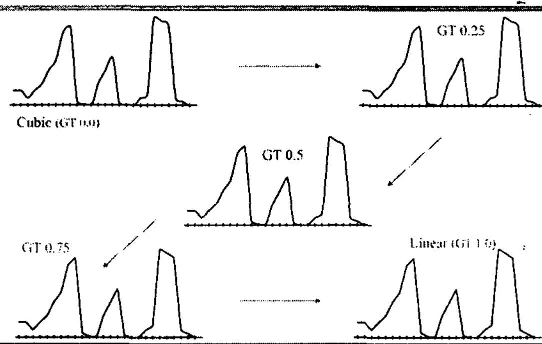
- Development of a stable simulation engine with intuitive user controls



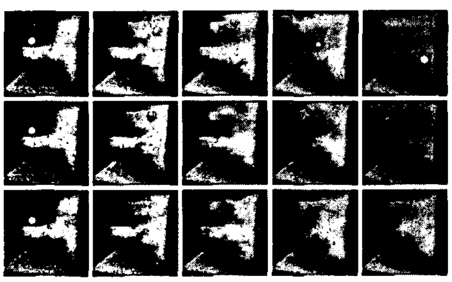
- Application to the animation production

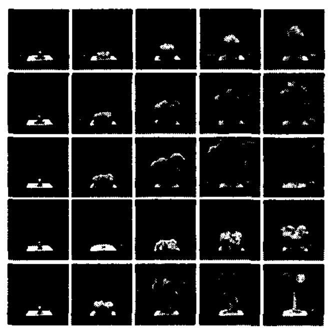
- Design of a controllable monotonic cubic interpolation filter



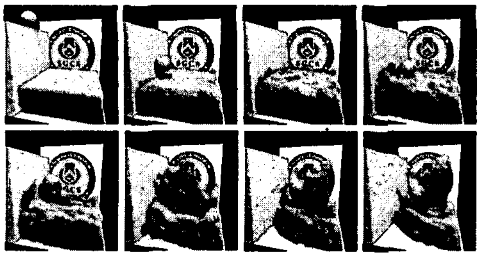


- Animation of chemically reactive fluids



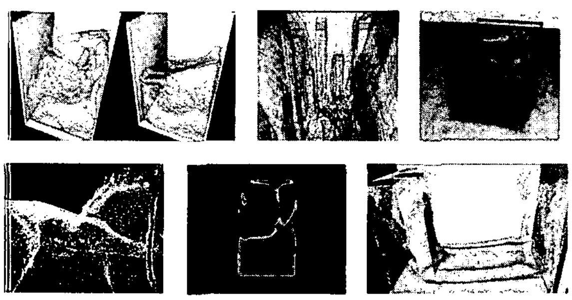


- Effective interaction of moving objects with fluids



- Adaptive and parallel computation of large scale simulations for HD-level animations
- Real-time simulation on programmable graphics processors

Physically Based Simulation of Water



Recent Works on Water Simulation

- N. Foster, and D. Metaxas, "Realistic animation of liquids," *Graphical Models and Image Processing* 58, pp. 471-483, 1996.

- N. Foster, and D. Metaxas, "Controlling fluid animation," *Computer Graphics International '97*, pp 178-188, 1997.

- N. Foster, and R. Fedkiw, "Practical animation of liquids," *ACM SIGGRAPH '01*, pp. 15-22, 2001.

- D. Enright, R. Fedkiw, J. Ferziger, and I. Mitchell, "A hybrid particle level set method for improved interface capturing," *J. Comp. Phys.* 183, pp. 83-116, 2002.

- D. Enright, S. Marschner, and R. Fedkiw, "Animation and rendering of complex water surfaces," *ACM Transactions on Graphics (ACM SIGGRAPH '02)*, Vol. 21, No. 3, pp. 736-744, 2002.

- C. Chiu, J. Chuang, C. Lin, and J. Yu, "Modeling Highly Deformable Fluid," http://cggmwww.csie.nctu.edu.tw/images/jwchiu/ModelingLiquid.pdf, 2002.

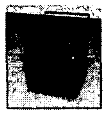
- R. Fedkiw, "Simulating Natural Phenomena for Computer Graphics." *Geometric Level Set Methods in Imaging, Vision and Graphics*, edited by S. Osher and N. Paragios, pp. 461-479, Springer Verlag, New York, 2003.

Cell Properties

- LIQUID
  - This cell is on that is fully filled with liquid. It is used for the calculation of pressure.
- EMPTY
  - This cell is one in which no liquid exists at all.
- SURFACE
  - This cell is one that exists between the LIQUID and the EMPTY cell.
- BOUNDARY
  - This cell is one that is contained in a solid object. It can be wall, obstacle, and other object that water can't flow in. Velocity of this cell is not governed by Navier-Stokes equation.

Basic Algorithm in [FosterFedkiw01]

```
For each time step {
    Update the velocity field by solving the Navier-
    Stokes equations.
    Update the position of the liquid volume using the
    new velocity field.
}
```



$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} + \nu \nabla \cdot (\nabla \mathbf{u}) - \frac{1}{\rho}\nabla p + \mathbf{f}$$

- How do we track the dynamically evolving surface of a volume of liquid?
  - Use a hybrid surface model based on particles and level set
  - Both the Lagrangian and the Eulerian approach

Representing the Water Surface

- Particle versus level set for dynamically evolving surfaces
  - Particle: a Lagrangian approach
    - Easy to move around in the velocity field.
    - Tricky to handle topological changes.
    - Hard to extract a smooth polygonal representation of the liquid surface.

$$\frac{dx_p}{dt} = \mathbf{u}_x$$

  - Level set: an Eulerian approach

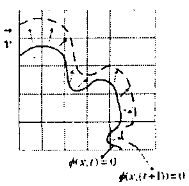
$$\phi_t + \mathbf{u} \cdot \nabla\phi = 0$$

    - Easily produce smooth surfaces.
    - Relatively easy to handle topological changes.
    - Suffer from severe volume loss in under-resolved regions.

Level Set Method

- Traces the interface of moving surfaces using a signed distance function.
- Signed distance function $\phi = \phi(x, y, z, t)$
  - The shortest distance from a point to the interface
  - The sign indicates whether a point is inside, on, or outside of the surfaces.
- The evolution of the interface is described by the following simple convection equation:

$$\phi_t + \mathbf{u} \cdot \nabla\phi = 0$$



- How do we define $\mathbf{u}$ in order to create desired visual effects?
- How do we discretize the equation temporarily and spatially in a stable manner?
  - It is convenient to make the signed distance function to the interface so that $|\nabla\phi| = 1$.
    - Reinitialize the function near the interface.

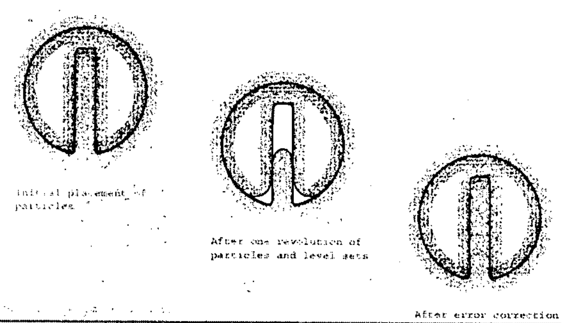
$$\phi_\tau + \frac{\phi}{\sqrt{\phi^2 + |\nabla\phi|^2 \Delta x}}(|\nabla\phi| - 1) = 0$$

where τ is the fictitious time and ϕ_τ is amount to adjust value ϕ.
  - Use a fifth-order accurate Hamilton-Jacobi WENO method to approximate the spatial derivatives.
  - Use a third-order accurate TVD Runge-Kutta method for the temporal integration.

Particle Level Set Method [Enright et al.02]

- A hybrid front tracking technique to model the surface interface that uses marker particles combined with a dynamic implicit surface level set.
- Based on the hybrid method proposed in [FosterFedkiw01].
- The particles are used to correct errors in moving the level set surfaces.

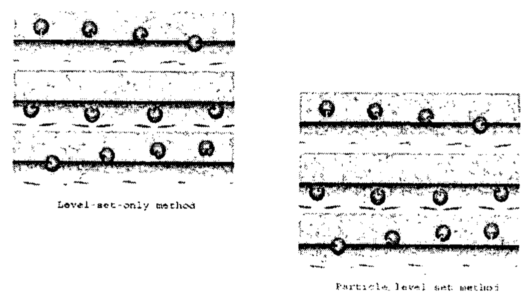
Example of Particle Level Set Method



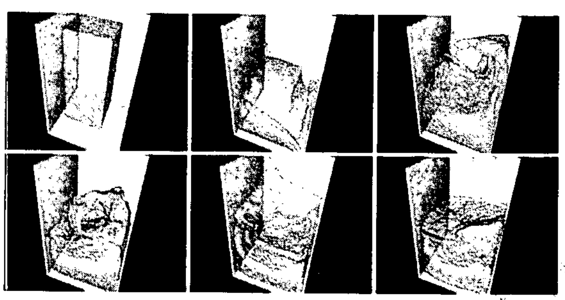
Algorithm of Particle Level Set Method

```
Place both positive and negative particles near the
   interface.
For each time step {
     1. Update the velocity field by solving the Navier-
        Stokes equations.
     2. Evolve both the particles and the level set
        functions forward in time.
     3. Correct errors in the level set function using
        particles.
     4. Apply re-initialization, correcting errors in the
        level set function using particles.
     5. Adjust the particle radii.
     6. Perform particle reseeding, if necessary.
}
```

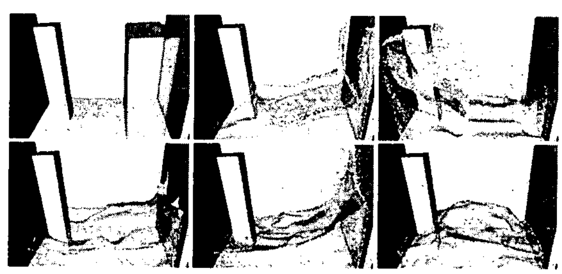

Comparison of the Old and the New Schemes



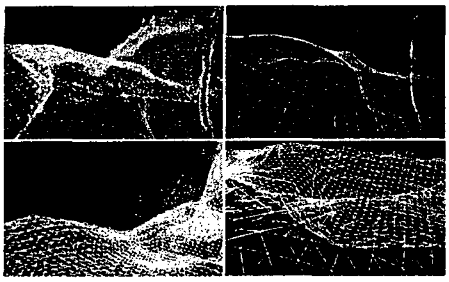
Implementation Results



Particle-only method without smoothing



Particle level set method with smoothing



Polygonized surface with particles

Slices showing positive and negative particles

What are we doing with water and level set?

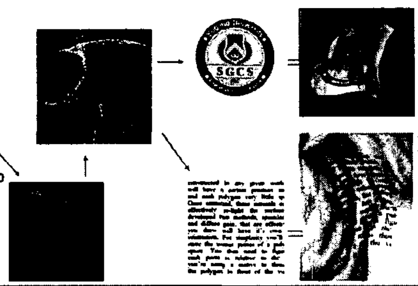
- Development of a stable simulation engine with intuitive user controls

- Local parameterization of polygonal objects using projection level set

$$\phi_t + C|P_{\nabla\psi}\nabla\phi| = 0$$

$$\phi_t + sgn(\phi)(|P_{\nabla\psi}\cdot\nabla\phi| - 1) = 0$$

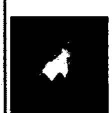
$$sgn(\phi) = \frac{\phi}{\sqrt{\phi^2+|P_{\nabla\psi}\nabla\phi|^2\Delta x}}$$



Physically Based Simulation of Fire, Flame, and Explosion

Recent Works on Fire and Flame Simulation

- P. Beaudoin, S. Paquet, and P. Poulin, "Realistic and controllable fire simulation," *Graphics Interface 2001*, pp. 159 - 166, 2001.



- X. Wei, W. Li, K. Mueller, and A. Kaufman, "Simulating fire with texture splats", *IEEE Visualization 2002*, pp. 227-234, 2002.

- D. Nguyen, R. Fedkiw, and H. Jensen, "Physically based modeling and animation of fire," *ACM Transactions on Graphics (ACM SIGGRAPH '02)*, Vol. 21, No. 3, pp. 721-728, 2002.

- A. Lamorlette, and N. Foster, "Structural modeling of flames for a production environment," *ACM Transactions on Graphics (ACM SIGGRAPH '02)*, Vol. 21, No. 3, pp. 729-735, 2002.

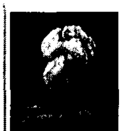
- S. Hasinoff, and K. Kutulakos, "Photo-consistent 3D fire by flame-sheet decomposition", *IEEE International Conf. on Computer Vision*, pp. 1184-1191, 2003.

Recent Works on Explosion Simulation

- M. Neff, and E. Fiume, "A visual model for blast waves and fracture," *Graphics Interface '99*, pp. 193-202, 1999.
- O. Mazarak, C. Martins, and J. Amanatides, "Animating exploding objects," *Graphics Interface '99*, pp. 211-218, 1999.

- G. Yngve, J. O'Brien, and J. Hodgins, "Animating explosions," *ACM SIGGRAPH '00*, pp. 29-36, 2000.

- B. Bashforth, and Y.-H. Yang, "Physics-based explosion modeling," *Graphical Models*, Vol. 63, No. 1, pp. 21-44, 2001.



- B. Feldman, J. O'Brien, and O. Arikan, "Animating suspended particle explosions," *ACM Transactions on Graphics (ACM SIGGRAPH '03)*, Vol. 22, No. 3, pp. 708-715, 2003.

- N. Rasmussen, D. Nguyen, W. Geiger, and R. Fedkiw, "Smoke simulation for large scale phenomena," *ACM Transactions on Graphics (ACM SIGGRAPH '03)*, Vol. 22, No. 3, pp. 703-707, 2003.