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FLUID MASS STREAMING IN A CHANNEL UNDER STANDING WALLS VIBRATIONS

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ABSTRACT

Peristaltic mass transfer of fluid in a channel with standing wall oscillations is analyzed. Averaged nonlinear Navier–Stokes equations of motion are examined for a wide range of Reynolds numbers and external pressure drops. Nonpropogating wall oscillations with relatively big amplitudes essentially increase the liquid flow. Most effective intensifying of mass transfer occurred for low Reynolds numbers.

KEYWORD : Streaming, Fluid pumping, Peristaltic motion

1. Introduction

Traveling wall waves in fluid filled channel normally induce fluid mass transport in the direction of wave propagation. This mechanism is very important for living organisms [1–4]. Peristaltic flows are being used in several modifications of pumps, including those using extreme modes of totally blocked "pieces" of liquid pumping with maximal mass transfer rate [5, 6]. Filtration velocity of liquid in two-phase porous media can be greatly intensified under the influence of some kinds of vibrations, it is a well known experimental effect, used in oil industry.

The vibrational Reynolds number $\mathrm{Re} = a^2\sigma/\nu$ is the main dimensionless parameter of the problem. Here σ is the characteristic frequency of the wall oscillations, ν is the kinematic viscosity of the fluid, and a is the characteristic channel width. These parameters determine the balance between inertial and viscous effects.

Nowadays, many investigations are devoted to the modeling of such processes. The basic physical principle of this effect seems to be clear: viscosity plays the dominant role in the formation of flow in the direction of waves propagation, i.e. along the channel walls and gives rise to a complex fluid flow with vortices, recurrent jets, where the main mechanism of the mass transport is the fluid drift.

In literature the mass transfer of fluid for different Reynolds numbers and pressure drops along the channel was investigated [7,8] . These works present results of analytical and numerical solutions of the Navier–Stokes equations for incompressible viscous fluids, two-dimensional fields of oscillating and drift velocities and mass transport characteristics. The possibilities for particles drift and recurrent flows were investigated, which are of interest for biomechanical and physiological applications. Investigations of complicated spiral trajectories of individual particles were carried out based on Lagrangian approach to fluid flows description. Investigations of particles' trajectories showed the possibility of fluid particles motion in the direction opposite to the main stream.

The two-phase model of the oscillatory fluid motion in a channel with elastic walls was suggested in [9]. It is basically different from the generally adopted one-way coupling models assuming unchangeable oscillation modes of the channel walls. The presence of the viscous fluid leads to essential decrease of bending oscillation velocity in the walls and damping of the waves.

The performed investigation demonstrates the efficiency of the mass transport at standing wave modes of wall oscillations, which are relatively rapidly damped along the tube. This statement changes the established opinion considering the traveling wave modes to be the basic mechanism for fluid pumping in a channel with vibrating walls. Weakly nonlinear solution suggested in [9] described fluid drift flow in the second order of approximation in the wave steepness, while strongly nonlinear modes were beyond the consideration.

The purpose of this paper is to develop a model of fluid mass transfer for a new method of peristaltic pumping with walls oscillating in a standing mode. The model will be based on a fully nonlinear statement of the problem. Nonlinear nonstationary Navier–Stokes equations are analyzed by integral method within the frame of a boundary layer type approximation. Such a model has no restrictions on the amplitudes of wall oscillations and allows one to go so far as to analyze different marginal modes of motion with almost complete blocking of the channel for different ranges of Re number.

Figure 1. Schema of the flat channel: L and a are the length and halfwidth of the channel, respectively; b is the wall oscillations amplitude; the area between the channel walls is filled with fluid.

2. Formulation of the problem. Integral method

Let us consider a planar channel of length L and width $2a$ (Fig. 1) filled with a viscous incompressible fluid. Orthogonal coordinate system is used, with X being the symmetry axis and Y being the perpendicular. If the plates are not moving, the fluid is either at rest, or its flow is determined by the influence of an external pressure gradient.

Displacements of both walls are symmetrical with respect to the axis $Y = 0$.

The fluid motion is modeled by an incompressible viscous laminar flow.

In this case, the mass balance equation is given by

$$U_X + V_Y = 0, \qquad (1)$$

where U and V are respectively the X and Y components of the fluid velocity vector.

The Navier–Stokes equations projected on the two coordinate axes are written as

$$\begin{aligned} U_T + UU_X + VU_Y &= -\frac{1}{\rho_f} P_X + \nu_f\left(U_{XX} + U_{YY}\right), \\ V_T + UV_X + VV_Y &= -\frac{1}{\rho_f} P_Y + \nu_f\left(V_{XX} + V_{YY}\right), \end{aligned} \qquad (2)$$

where ρ_f is the fluid density, ν_f is its kinematic viscosity, T is the time, and P is the pressure.

In order to formulate the problem completely, it is necessary to set up the conditions at the solid wall. The kinematic conditions are determined by the adhesion of the viscous fluid to the solid oscillating wall (no cavitation allowed),

$$H_T = V, \quad U = 0 \quad \text{at} \quad Y = H(X,T), \qquad (3)$$

H being the variable width.

Oscillations of the walls are assumed standing, harmonic in time with frequency σ uniform along the channel

$$H(X,T) = a + b\cos(\sigma T), \qquad (4)$$

b is the wall oscillation amplitude.

The symmetry of the problem allows one to seek for the fluid flow solution with the corresponding symmetry, i.e.

$$U_Y = V = 0 \quad \text{at} \quad Y = 0, \qquad (5)$$

and the last boundary condition is the pressure drop,

$$P(X = L/2, T) - P(X = -L/2, T) = \Delta P(T). \qquad (6)$$

Dimensionless variables are introduced as

$$x = \frac{X}{L}, \quad y = \frac{Y}{a}, \quad t = \sigma T, \quad u = \frac{U}{\sigma L},$$
$$v = \frac{V}{\sigma a}, \quad p = \frac{P}{\sigma^2 L^2 \rho_f}\mathrm{Re}, \quad h = \frac{H}{a}.$$

In this work we suppose that the ratio of the channel width to the length of channel is small: $a/L \ll 1$. This assumption is also valid for small characteristic gradients of the flow parameters along the longitudinal coordinate in comparison with the flow parameters changes across the channel (approximation similar to the boundary layer theory).

This approximation makes it possible to neglect the terms of $(a/L)^2$ in the dimensionless form of equations of motion (1)-(2). Thus, according to these assumptions, the governing system of equations and the boundary conditions are written as

$$u_x + v_y = 0, \qquad (7)$$

$$\mathrm{Re}\left(u_t + uu_x + vu_y\right) = -p_x + u_{yy}, \quad p_y = 0, \qquad (8)$$

$$u\big|_{u=h} = 0, \quad v\big|_{u=h} = h_t, \qquad (9)$$

$$h(t) = 1 + \varepsilon\cos t \qquad (10)$$

$$p\left(\tfrac{1}{2}, t\right) - p\left(-\tfrac{1}{2}, t\right) = \Delta p(t), \qquad (11)$$

$$u_y\big|_{y=0} = v\big|_{y=0} = 0. \qquad (12)$$

Due to the symmetry of the problem, only the upper half of the channel will be considered.

Integral method. Let us introduce one more physical variable, a normalized mass flux through the cross section of the tube,

$$Q(x,t) = \int_0^{h(t)} u(x,y,t)\,dy. \qquad (13)$$

Integrating the continuity equation (7) over cross section and using the latter of boundary conditions (9) gives

$$Q_x + h_t = 0. \qquad (14)$$

Taking into account the known function of the wall oscillations (10), finally one arrives at

$$Q(x,t) = Q_0(t) + \varepsilon x \sin t \qquad (15)$$

where $Q_0(t)$ is a free function of time to be determined.

Momentum equation (8), averaged over a cross section of the channel, can be given as

$$\mathrm{Re}\left[Q_t + \left(\int_0^{h(t)} u^2 dy\right)_x\right] = -p_x h(x,t) + u_y\big|_{y=h(t)}. \qquad (16)$$

To model the longitudinal velocity $u(x, y, t)$, we assume the locally Poiseuille profile as

$$u(x,y,t) = \frac{3}{2}\frac{Q(x,t)}{h(t)^3}\left[h^2(t) - y^2\right]. \qquad (17)$$

This expression satisfied boundary conditions (9) and integral continuity equation (14).

Substitution of (17) into (16) yields

$$\mathrm{Re}\left[Q_t(x,t) + \frac{6}{5}\left(\frac{Q(x,t)^2}{h(t)}\right)_x\right] = -p_x h(t) - \frac{3Q(x,t)}{h^2(t)} \qquad (18)$$

Finally, to satisfy integral momentum equation (18) and the boundary condition for pressure (11) we still have two unknown functions: pressure $p(x, t)$ and time term in mass flux function $Q_0(t)$.

The problem could be solved in the following way. Integration of (18) over the channel length yields the ordinary differential equation to determine $Q_0(t)$,

$$\mathrm{Re}\left[Q_{0t}-\frac{12}{5}Q_0\frac{h_t}{h}+\frac{1}{2}\left(\frac{12}{5}h_t^2-h_{tt}\right)\right] +3\frac{Q_0}{h^2}-\frac{3h_t}{2h^2}=-\Delta p(t)h. \quad (19)$$

Pressure distributions along the tube could be gained from (18) after solving (19) for $Q_0(t)$.

3. Flow structure and mass transport of liquid in the channel

Vibrations of the walls are uniform along the tube (10), so they can only modify already existing liquid mass transport due to the difference of the external pressure.

Structure of the flow and mass transport in the presence of pressure drop and wall vibrations are determined by the following set of nondimensionless parameters: Re, ε, and Δp.

To make things simpler, let us assume the external pressure difference Δp constant (not time-dependent). Numerical solution of (19) and (18) with boundary conditions (11) for a wide range of governing parameters evidently shows that the maximal effect of the vibrations upon the flow characteristics could be achieved for low and moderate Reynolds numbers.

Let us consider low Rayleigh numbers: $\mathrm{Re} \ll 1$ first. Oscillating walls cause maximal increase of liquid mass transport in this case.

The streaming of liquid is characterized by the average of the mass flux at the exit over the period of oscillations,

$$Q_{\mathrm{av}} = \int_0^{2\pi} Q\left(\tfrac{1}{2},t\right) dt. \quad (20)$$

Mass transport of liquid along the channel is intensified on increasing the wall oscillations. The gain is given by

$$k = Q_{\mathrm{av}} / Q_{\mathrm{st}}. \quad (21)$$

The dependences of Q_{av} on the pressure drop Δp for all cases are almost linear. Thus, the mass transport gain k can be assumed as a pure function of the wall oscillation amplitude $k = k(\varepsilon)$. This function is shown at Fig. 2. Mass transport for large amplitudes of wall oscillations $\varepsilon \geq 0.9$ more than two times surpasses the stationary solution.

Figure 2. Dependence of the mass transport gain k on the wall oscillation amplitudes.

The dynamic of fluid flow through the channel is illustrated in Fig. 3 for wall oscillation amplitudes $\varepsilon \geq 0.6$ and $\mathrm{Re} = 0.1$. The function of liquid mass transport $Q(t)$ is shown in Fig. 3*b*. It has a highly nonstationary behavior with large oscillations being in the same phase, as wall oscillations shown in Fig. 3*a*.

Distribution of longitudinal velocity along the channel is almost linear for all instants of time, and velocity gradient has two extrema during the period of oscillations. Time dependence of velocities along the axis of the tube is represented in Fig. 3*c* for three cross sections: (*1*) at the entrance, (*2*) at the exit, (*3*) in the middle of the channel. At the end of compression phase $h_t < 0$ the velocity at the exit of the channel has maximum, and at the entrance of tube—minimum. At the very beginning of the expansion of the channel ($h_t > 0$) velocity at the outlet is minimal and velocity at the inlet is maximal. Velocity in the middle cross section is almost constant in time and corresponds to a stationary solution with $\varepsilon = 0$.

Figure 3. Fluid mass transport dynamics $Q(\tfrac{1}{2}, t)$ (*b*) and velocity $u(t)$ *c* versus time in different cross sections of the tube:(*1*) $x = \tfrac{1}{2}$, (*2*) $x = -\tfrac{1}{2}$, and (*3*) $x = 0$; $\mathrm{Re} = 0.1$, $\Delta p = 10$, $\varepsilon = 0.6$.

Time variation of fluid flow parameters greatly increased for big amplitudes of wall oscillations. Dynamics of mass transport and velocity distributions are shown in Fig. 4. The mass transport oscillations are shown in Fig. 4*b*, wall oscillations are shown in Fig. 4*a*. The amplitudes of oscillations here are much bigger as compared to those shown in Fig. 3.

Figure 4. Fluid mass transport $Q(\tfrac{1}{2}, t)$ (*b*) and velocity $u(t)$ variations *c* versus time in different cross sections of the tube: (*1*) $x = \tfrac{1}{2}$, (*2*) $x = -\tfrac{1}{2}$, and (*3*) $x = 0$; $\mathrm{Re} = 0.1$, $\Delta p = 10$, $\varepsilon = 0.9$.

Another characteristic feature of this mode is the presence of two high velocity liquid jets going in opposite directions at the entrance and at the exit of the channel. At the end of contraction phase ($h_t > 0$) those jets go out of the channel; at the beginning of expansion of the channel ($h_t > 0$) the jets go inside the channel (see Fig. 4*c*). Jets of such type could play an important role in enhancing the mass transfer in porous media under the influence of vibrations.

Mass transport and flow dynamics for moderate Reynolds numbers $\mathrm{Re} \sim 1$ keep the same mean properties with slightly weaker characteristics. The dependence of the gain k on the wall oscillation amplitudes ε is shown in Fig. 5 for $\mathrm{Re} = 1$. Average fluid flow increased with enlarging of wall oscillations. Big amplitudes ($\varepsilon \geq 0.9$) can also generate periodically strong fluid jets outside the channel.

Figure 5. Mass transport gain for $\mathrm{Re} = 1$.

Next order of Reynolds parameter Re ~ 10 gives a quick decrease of mass transport characteristics: wall oscillations can give only about 20 % enhancement for big-amplitude vibrations.

Wall oscillations at high Reynolds numbers Re ~ 100 and more do not have any essential positive effect on the intensification of fluid mass transport through the channel. Moreover, gains for high Reynolds numbers could be less then unit.

4. Conclusion

The results of theoretical investigations show that standing transverse oscillations of channel walls can significantly increase the mass transport through the channel, as compared to an unperturbed flow. Investigations of the dependence of mass transport gain on the dimensionless governing parameters made it possible to distinguish conditions under which the effect of vibrations is maximum.

High velocity fluid jets induced periodically due to vibrations could play an important role in enhancing mass transfer in porous media under the influence of vibrations.

The found results can serve as an explanation to the fact that low frequency vibrations essentially increase the filtration rate in porous media and natural soils, in particular.

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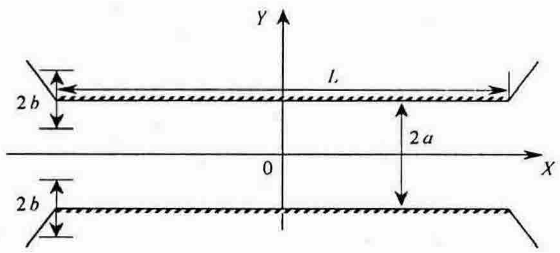

Figure 1. Schema of the flat channel: L and a are the length and halfwidth of the channel, respectively; b is the wall oscillations amplitude; the area between the channel walls is filled with fluid.

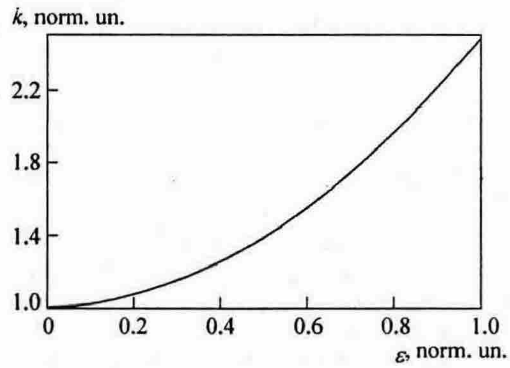

Figure 2. Dependence of the mass transport gain *k* on the wall oscillation amplitudes.

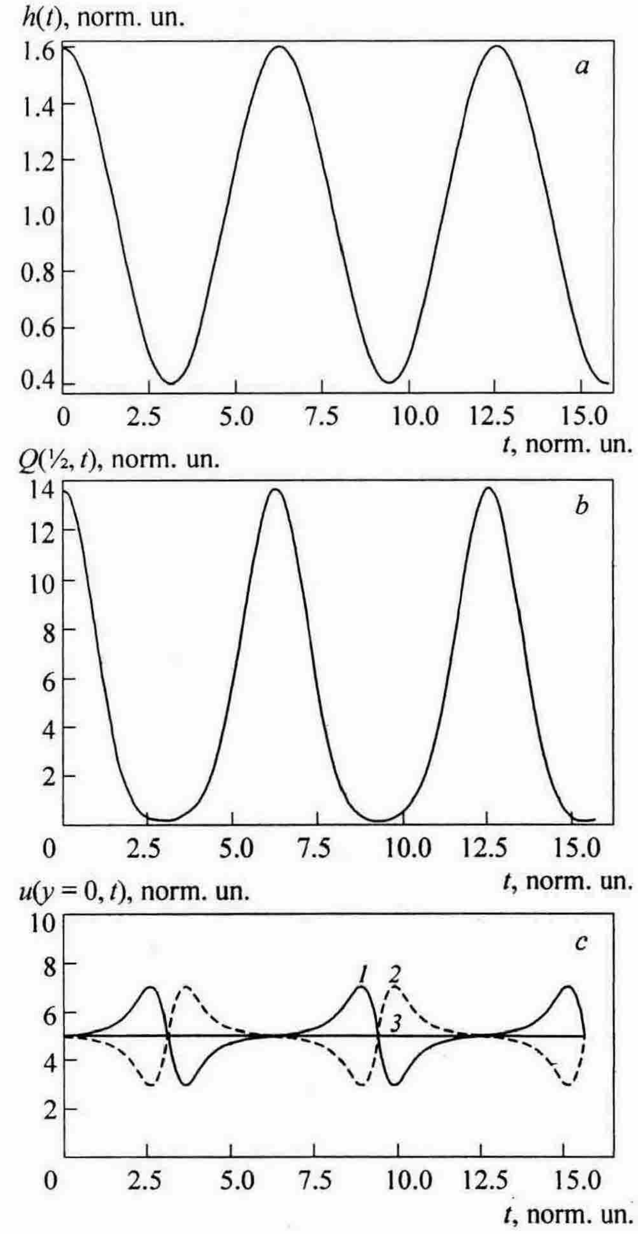

Figure 3. Fluid mass transport dynamics $Q(½, t)$ (b) and velocity $u(t)$ c versus time in different cross sections of the tube:(1) $x = ½$, (2) $x = -½$, and (3) $x = 0$; Re = 0.1, $\Delta p = 10$, $\varepsilon = 0.6$.

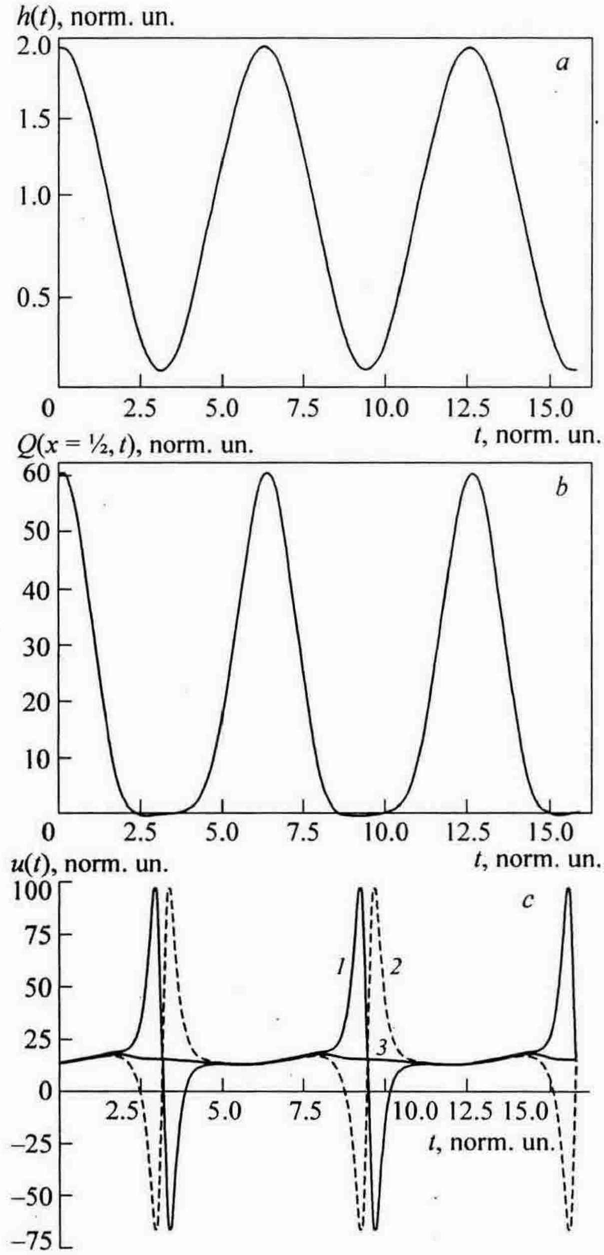

Figure 4. Fluid mass transport $Q(½, t)$ (b) and velocity $u(t)$ variations c versus time in different cross sections of the tube: (1) $x = ½$, (2) $x = -½$, and (3) $x = 0$; Re = 0.1, $\Delta p = 10$, $\varepsilon = 0.9$.

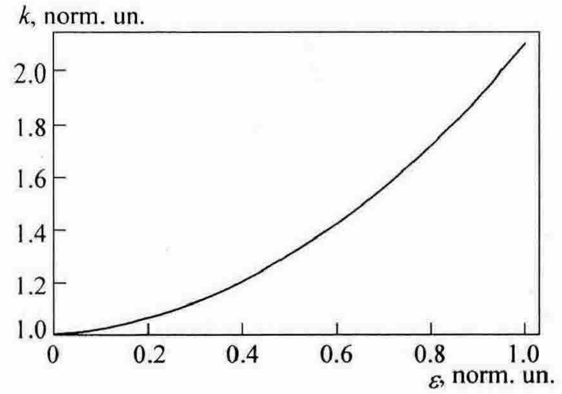

Figure 5. Mass transport gain for Re = 1.