Dynamic Analysis of Viscoelastic Composite Thin-Walled Blade Structures

Jae-Hyun Shin, Sungsoo Na, Chul Hue Park

점탄성-복합재 박판 블레이드 구조물의 진동 해석

신재현 · 나성수*· 박철휴**

Key Words : Viscoelastic material(점탄성 재료), GHM method(GHM 방법), Thin-Walled Blade(박판 블레이드)

Abstract

This paper concerns the analytical modeling and dynamic analysis of advanced cantilevered blade structure implemented by a dual approach based on structural tailoring and viscoelastic materials technology. Whereas structural tailoring uses the directionality properties of advanced composite materials, the passive materials technology exploits the damping capabilities of viscoelastic material(VEM) embedded into the host structure. The structure is modeled as a composite thin-walled beam incorporating a number of nonclassical features such as transverse shear, secondary warping, anisotropy of constituent materials, and rotary inertias. The case of VEM spreaded over the entire span of the structure is considered. The displayed numerical results provide a comprehensive picture of the synergisitic implications of the application of both techniques, namely, the tailoring and damping technology on vibration response of thin-walled beam structure exposed to external time-dependent excitations.

1. INTRODUCTION

The cantilevered composite thin-walled beam structure is the most important structure that can serve as a basic model for a number of constructions used in the aeronautical and space industries, such as airplane wings, helicopter blades, fan blades, robotic manipulator arms and space booms.

For such structures, the development and implementation of adequate methodologies aiming at controlling their free and forced vibration characteristics are likely to contribute to the improvement of their performance and avoidance of the occurrence of resonance and any dynamic instability.

One of the possible ways towards achieving such goals consists in the implementation of viscoelastic materials embedded into the host structure, which increase the energy dissipation due to the characteristics of VEM which minimize vibrations to improve resiliency. For instance, increasing the damping levels in turbo-fan blades is of current interest to both NASA and the Air Force as a means of making commercial and military turbo-fan engines more reliable. Increasing the damping capabilities in the beams or blades will improve the fatigue life and reduce aeroelastic instability matter. The appropriate viscoelastic FEM modeling of VEM will be called the GHM(Golla, Hughes, McTavish) and the system is analyzed in the time domain. Validation of the method is well defined in earlier monographs.[1]

Although of an evident importance, to the best of author's knowledge, no such studies including vibration and dynamic analysis of composite blade embedded VEM exposed to external loads can be found in the specialized literature. The early works on analyses of sandwich structures with a viscoelastic core were done by Kerwin[2] , Ross[3] et al., DiTaranto[4], and Mead and Markus[5]. They presented the fourth and sixth-order theories for beams and plates to predict damping and handled the cases with arbitrary boundary conditions. The governing equations of flexural vibration of symmetrical sandwich rectangular plate were presented by Mead.[6]. C. Park[7] derived a new technique to formulate the finite element model of a sandwich beam by using GHM. Composite laminated beams and plates with a viscoelastic core were also discussed. Cupial and Niziol [8] discussed the three layered composite plate

† Korea university, Mechanical engineering department
E-mail : icecom@korea.com TEL : (02)3290-3854

* Korea university, Mechanical engineering department
E-mail : nass@korea.ac.kr TEL : (02)3290-3370

** Pohang University of Science and Technology
E-mail : drparkch@postech.ac.kr

with a viscoelastic core layer and two laminated face layers by the first-order shear deformation theory.

2. FORMULATION OF THE COMPOSITE-VEM

2.1 Basic Assumption and Kinematics of the Modeling and Formulation

The model of the host structure considered encompasses a number of features such as: (a) transverse shear, (b) second warping, (c) anisotropy of constituent materials, (d) the beam cross-sections feature a symmetric biconvex profile.

The points of the beam cross-sections are identified by the global coordinates *x*, *y* and *z*, where *z* is the spanwise coordinate (see Fig. 1) and by a local one, *n*, *s*, and *z*, (see Fig. 2) where *n* and *s* denote the thicknesswise coordinate normal to the beam mid-surface and the tangential one along the contour line of the beam cross-section, respectively. Figure 1 shows the typical composite-VEM thin walled blade model that is considered in the present analysis.

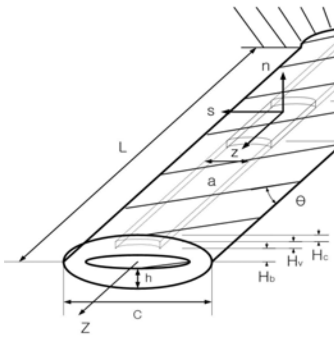

Fig. 1 Composite-VEM thin walled blade model

Following coordinates description, $\theta_x(z,t)$ and $\theta_y(z,t)$ denote the rotations about axes *x* and *y* respectively, while γ_{yz} and γ_{xz} denote the transverse shear in the planes *yz* and *xz* respectively and the primes denote derivatives with respect to the *z*-coordinate.

The primary warping function is expressed as [9]

$$F_w = \int_0^s [r_n(s) - \psi] ds \tag{1}$$

where the torsional function ψ and the quantity $r_n(s)$ are [9]

$$\psi = \frac{\oint_C \frac{r_n(s)}{h(s)} ds}{\int_C \frac{ds}{h(s)}} \tag{2}$$

and

$$r_n(s) = x(s)\frac{dy}{ds} - y(s)\frac{dx}{ds} \tag{3}$$

respectively. Fig. 2 displays the configuration of a cross-section of a thin walled beam structure and reveals the geometrical meaning of $a(s)$ and $r_n(s)$ as well.

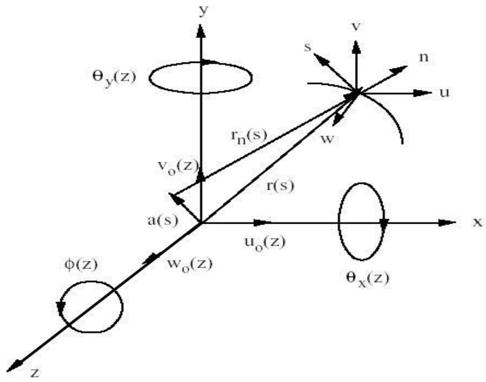

Fig 2. Displacement field for the thin walled beam

In accordance with the above assumptions and in order to reduce the 3-D problem to an equivalent 1-D, the components of the displacement vector are expressed as [9]

$$\begin{aligned} u(x,y,z,t) &= u_0 - y\phi(z,t) \\ v(x,y,z,t) &= v_0 + x\phi(z,t) \\ w_0(x,y,z,t) &= w_0(z,t) + \theta_x(z,t)[y(s) - n\frac{dx}{ds}] \\ &\quad + \theta_y(z,t)[x(s) + n\frac{dy}{ds}] - \phi'(z,t)[F_w(s) + na(s)] \end{aligned} \tag{4}$$

$$\begin{aligned} \theta_x(z,t) &= \gamma_{yz}(z,t) - v_0'(z,t) \\ \theta_y(z,t) &= \gamma_{xz}(z,t) - u_0'(z,t) \\ a(s) &= -y(s)\frac{dy}{ds} - x(s)\frac{dx}{ds} \\ \gamma_{v_{yz}} &= (\frac{H_{b^*} + 2H_v}{2H_v})\frac{\partial y}{\partial z} \end{aligned} \tag{5}$$

Eqs. (4) and (5) reveal that the kinematic variables, $v_0(z,t)$, $w_0(z,t)$, $\theta_x(z,t)$, $\theta_y(z,t)$ and $\phi(z,t)$ representing three translations in the *x*, *y*, *z* directions and three rotations about the *x*, *y*, *z* directions, respectively are used to define the displacement components, *u*, *v* and *w*.

Notice that the *z* - axis is located as to coincide with the locus of symmetrical points of the cross-section along the wing span.

The shear strain $\gamma_{v_{yz}}$ of VEM layer were derived form the kinematic relationships between the constrained layer and the base beam by Mead and Markus [10]

The transverse displacement is assumed to be the same for each layer, so that expressions for the kinetic and potential energy for the composite thin walled beam with embedded VEM treatment can be obtained. This assumption is valid as long as a thick constraining layer is attached to a thin viscoelastic layer [11].

.

2.2 Golla-Hughes-McTavish Damping Modeling

The Golla-Hughes-McTavish (GHM) modeling approach [12] provides an alternative method which includes viscoelastic damping effects without the restriction of steady-state motion by providing extra coordinates. GHM models hysteretic damping by adding additional "dissipation coordinates" to the system to achieve a linear nonhysteretic model providing the same damping properties. The dissipation coordinates are used with a standard finite element approach or, as in this work, with assumed modes method. The derivation of the GHM equations starts with the constitutive relation for a one dimensional stress-strain system using the theory of linear viscoelasticity [13].

$$\sigma(t) = G(t)\varepsilon(0) + \int_0^t G(t-\tau)\frac{d}{d\tau}\varepsilon(\tau)d\tau \qquad (6)$$

where σ is the stress. It is assumed that the strain, ε is zero for all time less than zero, and $G(t)$ is defined as a material relaxation function. The stress relaxation function represents the energy loss of the material, which is damping.

Linear second-order matrix form is maintained as well as symmetry and definiteness of the augmented system matrices. The time domain stress relaxation is modeled by a modulus function in the Laplace domain. This complex modulus can be written in Laplace domain from Eq. (6) as

$$\sigma(s) = sG(s)\varepsilon(s) \qquad (7)$$

$$G(s) = G^*(1+h(s)) = G^*\left(1+\sum_{n=1}^{k}\hat{\alpha}_n\frac{s^2+2\hat{\varsigma}_n\hat{\omega}_n s}{s^2+2\hat{\varsigma}_n\hat{\omega}_n s+\hat{\omega}_n^2}\right) \qquad (8)$$

where G^* is the equilibrium value of the modulus, i.e. the final value of the relaxation function $G(t)$, and s is the Laplace domain operator. The hatted terms are obtained from the curve fit to the complex modulus data for a particular VEM at a given temperature.

The equation of motion, which uses a complex shear modulus to describe damping, in the Laplace domain is

$$M_v s^2 x(s) + G^*\left(1+\sum_{n=1}^{k}\hat{\alpha}_n\frac{s^2+2\hat{\varsigma}_n\hat{\omega}_n s}{s^2+2\hat{\varsigma}_n\hat{\omega}_n s+\hat{\omega}_n^2}\right)K(s)x(s) = F(s) \qquad (9)$$

using the dissipation coordinates

$$z(s) = \frac{\hat{\omega}^2}{s^2+2\hat{\varsigma}\hat{\omega}s+\hat{\omega}^2}x(s) \qquad (10)$$

Such added degrees of freedom are also called internal variables. Eqs. (9) and (10) multiplied by G^*, and the following equation of motion can be assembled as

$$\begin{bmatrix} M & 0 \\ 0 & \frac{\hat{\alpha}G^*}{\hat{w}^2}K_v' \end{bmatrix}\begin{bmatrix} x(s) \\ z(s) \end{bmatrix}s^2 + \begin{bmatrix} 0 & 0 \\ 0 & \frac{2\hat{\alpha}\hat{\varsigma}G^*}{\hat{w}^2}K' \end{bmatrix}\begin{bmatrix} x(s) \\ z(s) \end{bmatrix}s + \begin{bmatrix} (1+\hat{\alpha})G^*K' & -\hat{\alpha}G^*K' \\ -\hat{\alpha}G^*K' & \hat{\alpha}G^*K' \end{bmatrix}\begin{bmatrix} x(s) \\ z(s) \end{bmatrix} = \begin{bmatrix} F(s) \\ 0 \end{bmatrix} \qquad (11)$$

where $z(s)$ is the vector of dissipation coordinates. This is the final form of the GHM model as described by McTavish and Hughes. [14]

2.3 Hamilton's Principle and Assumed Modes Method

Thus far, displayed representations of displacement measures are used directly in Hamilton's variational equation,

$$\int_{t_1}^{t_2}(\delta T - \delta V + \delta W)dt = 0\,, \qquad (12)$$

$$\delta v_0 = \delta\theta_x = \delta\varphi = 0 \qquad \text{at } t = t_1, t_2$$

Herein T and V are the kinetic and strain energies, respectively, W is the work done by the external distributed loads, t_0 and t_1 are two arbitrary instants of the t, and δ is the variation operator. the kinetic energy T, and potential energy V for composite-VEM thin walled beam

The expressions for the virtual work done by externally applied forces are

$$\partial W_f = \int_0^L f(z,t)\delta v(z,t)dz \qquad (14)$$

The governing system of thin walled beams are expressed as

$$\delta v_0 : -a_{55}(v_0''+\theta_x') - a_{56}\varphi''' + b_1\ddot{v}_0 = f_y$$
$$\delta\theta_x : a_{55}(v_0'+\theta_x) + a_{56}\varphi'' - a_{33}\theta_x'' - a_{37}\varphi' + (b_4+b_{14})\ddot{\theta}_x = 0 \quad (15)$$
$$\delta\varphi : a_{56}(v_0'''+\theta_x'') + a_{66}\varphi'''' - a_{77}\varphi'' - a_{37}\theta_x' + (b_4+b_5)\ddot{\varphi} = 0$$

In addition, for composite-VEM beam the solution of Eqs. (9) must satisfy the geometric boundary conditions at $z = 0$

$$\varphi' = \varphi = v_0 = \theta_x = 0$$

and the natural boundary conditions at $z = L$,

$$\begin{aligned} &\delta\phi : -a_{66}\varphi''' + a_{77}\varphi' - a_{56}(v_0''+\theta_x') + a_{37}\theta_x' = 0, \\ &\delta\phi' : a_{56}(v_0'+\theta_x) + a_{66}\varphi'' = 0, \\ &\delta v_0 : a_{55}(v_0'+\theta_x) + a_{56}\varphi'' = 0, \\ &\delta\theta_x : a_{33}\theta_x' + a_{37}\phi' = 0. \end{aligned} \quad (16)$$

For computational reasons, it is necessary to discretize the boundary-values problem, which amounts to representing $v_0(z,t)$, $\theta_x(z,t)$ and $\phi(z,t)$ by means of series of space-dependent trial functions multiplied by time-dependent generalized coordinates as follows:

$$v_0(z,t) = V^T(z)q(t) \quad , \quad \theta_x(z,t) = \theta^T(z)q(t) \quad ,$$
$$\phi(z,t) = \varphi^T(z)q(t) \quad (17)$$

where

$$V(z) = [v_1, v_2, ..., v_n]^T \quad , \quad \theta(z) = [\theta_1, \theta_2, ..., \theta_n]^T \quad ,$$
$$\varphi(z) = [\varphi_1, \varphi_2, ..., \varphi_n]^T \quad (18)$$

are the vectors of trial functions, whereas

$$q^v(t) = [q_1^v, q_2^v, ..., q_n^v]^T \ , \quad q^\theta(t) = [q_1^\theta, q_2^\theta, ..., q_n^\theta]^T \ ,$$
$$q^\varphi(t) = [q_1^\varphi, q_2^\varphi, ..., q_n^\varphi]^T \quad (19)$$

are vectors of generalized coordinates and the superscript *T* denotes the transpose operation of a matrix.

These representations of displacement measures are used directly in Hamilton's variational equation.

Performing the integration with respect to the spanwise *z* coordinate and with respect to time, and keeping in mind Hamilton's condition, from Eq. (15) one obtains the discrete equation of motion:

$$[M_L]\ddot{x}(t) + [K_L + K_v']x(t) = F(t) \quad (20)$$

where $[M_L]$ and $[K_L + K_v']$ are the mass and stiffness matrices, respectively, of the structure considered.

This case corresponds to shearable structure featuring warping and manufactures of an anisotropic composite material.

$$F = \int_0^L f_y v_i dz \quad (21)$$

The entire motion equation incorporating added damping matrix by GHM method is expressed as

$$[M_G]\ddot{x}(t) + [D_G]\dot{x}(t) + [K_G]x(t) = F(t) \quad (22)$$

where

$$[M_G] = \begin{bmatrix} M_L & 0 \\ 0 & \frac{\hat{\alpha}G^*}{\hat{w}^2}K_v' \end{bmatrix}, \ [\mathrm{D_G}] = \begin{bmatrix} 0 & 0 \\ 0 & \frac{2\hat{\alpha}\hat{\varsigma}G^*}{\hat{w}^2}K_v' \end{bmatrix},$$
$$[\mathrm{K_G}] = \begin{bmatrix} K_L + (1+\hat{\alpha})G^*K_v' & -\hat{\alpha}G^*K_v' \\ -\hat{\alpha}G^*K_v' & \hat{\alpha}G^*K_v' \end{bmatrix}$$

By the defining of the state vector as $x(t) = [q^T(t)\ z^T(t), \dot{q}^T(t)\ \dot{z}^T(t)]^T$ can be cast in state space form

$$\dot{x}(t) = Ax(t) + BF(t) \quad (23)$$

where

$$A = \begin{bmatrix} 0 & I \\ -M_G^{-1}K_G & -M_G^{-1}D_G \end{bmatrix} \ B = \begin{bmatrix} 0 \\ M_G^{-1} \end{bmatrix} \quad (24)$$

3. NUMERICAL SIMULATIONS

The considered geometrical and physical characteristics of the blade structure and VEM are respresented by Table 1 and Table 2, respectively.

Table 1. Composite properties (Ref.9)

Properties	Value	Properties	Value
L	2.032 m	E_T	5.17e9 N/m
H_b	4.953e-3 m	E_L	20.68e10 N/m
$\rho_b = \rho_c$	1528.15 kg/m^3	G_{TT}	3.1e9 N/m
ν	0.25	G_{LL}	2.55e9 N/m

Table 2 Viscoelastic material properties (Isotropy) - 3M ISD 112 (Ref.15)

Properties	Value	Properties	Value
L	2.03 m	E_v	1.4e6N/m
$H_v = H_c$	2.54.e-4 m 1.27 e-4 m	G_v	5e5N/m
ρ_v	1250 Kg/m^3	ν	0.4

Table 3. GHM parameters (Ref.15)

Properties	Value
a	6
ξ	4
$\hat{\omega}$	10000 rad/s

The GHM parameters used in the present study are shown in Table 3. The result of numerical simulation using one mini oscillator GHM method is presented in Fig. 3-7.

Fig. 3-6 show the passive impulse time response for tip displacement in the transverse direction of the composite-VEM thin walled blade model for difference ply angles, taper ratios and VEM thicknesses. The system model is sandwich beam type that has the same height in bottom and top composite beam and full embedded in it with VEM. Fig. 7 displays the effect of embedded VEM thickness to the dynamic response of the structure. The same trend is reflected in the ref.(18). Fig. 7 also shows the frequency response of tip displacement of the composite-VEM thin walled blade model for difference VEM thickness.

The effect of VEM can be found simply in the result of Figs. 3-7.

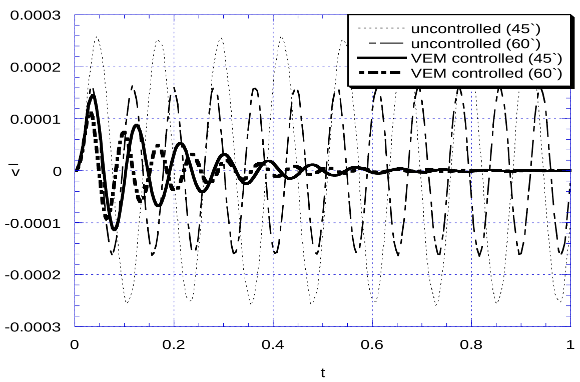

Fig.3 The nondimensional uncontrolled/controlled tip displacement of the beam subjected to impulse input for two ply angles

(ply angle = 45, 60, taper ratio=1.5, H_v =2.54e-4)

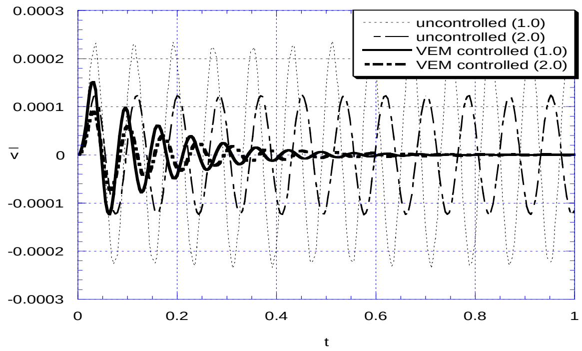

Fig.4 The nondimensional uncontrolled/controlled tip displacement of the beam subjected to impulse input for two taper ratios

(ply angle = 60, taper ratio=1.0, 2.0, H_v =2.54e-4)

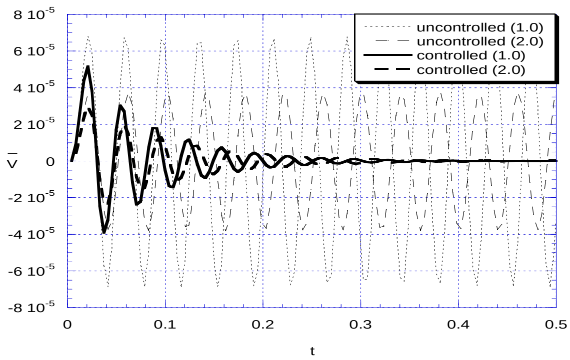

Fig.5 The uncontrolled/controlled tip displacement of the beam subjected to impulse input for two taper ratios

(ply angle = 70, taper ratio=1.0, 2.0, H_v =1.27e-4)

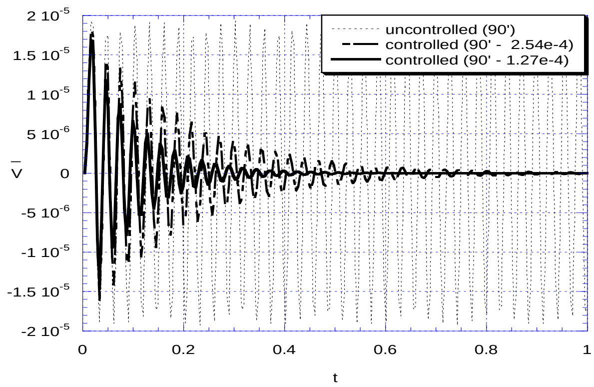

Fig.6 The nondimensional uncontrolled/controlled tip displacement of the beam subjected to impulse input for two VEM thicknesses

(ply angle = 90, taper ratio=1.5, H_v =2.54e-4, 1.27e-4)

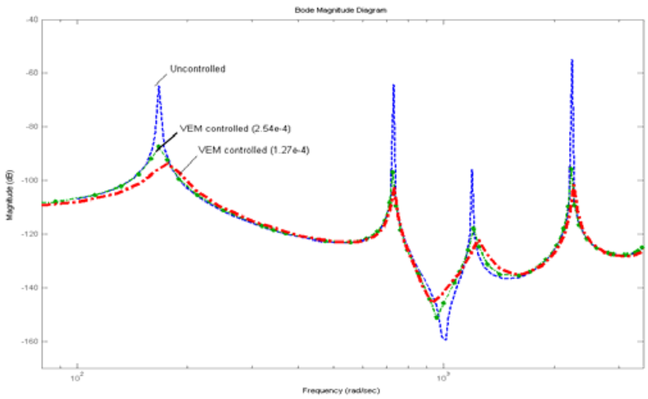

Fig.7 The uncontrolled/controlled of frequency response of the tip displaced of the beam
(ply angle = 80, taper ratio=1.5, H_v =2.54e-4, 1.27e-4)

4. CONCLUSIONS

The structure formulation is based on the Extended Galerkin Method. Frequency dependent damping is modeled using the GHM method. Furthermore, GHM is able to account for increased damping associated with various VEM thickness. The displayed numerical results provide a comprehensive picture of result of implementing VEM embedded over the entire span of composite blade to show reductions of vibration. Further study will include the case of a patch of VEM incorporating its optimized placement exposed to various types of external loads. Although of an evident importance, to the best of author's knowledge, no such studies including vibration and dynamic analysis of composite blade embedded VEM exposed to external loads can be found in the specialized literature.

5. ACKNOWLEDGEMENT

Sungsoo Na acknowledges the support of the work by the Basic Research Program of the Korea Science and Engineering Foundation, Grant No. R01-2002-000-00129-0.

REFERENCE

(1) M. J. Lam, D.J. Inman, W.R. Saunders., 1997 "Vibration Control through Passive Constrained Layer Damping and Active Control," Journal of Intelligent Material Systems and Structures, Vol.8, pp.663-677
(2) Kerwin EM, 1959 "Damping of flexural waves by a constrained viscoelastic layer." Journal of the Acoustical Society of America Vol. 31:952–62.
(3) Ross D, Ungar EE, Kerwin EM., 1959 "Flexural vibrations by means of viscoelastic laminate." ASME Structure Damping :48–87.
(4) DiTaranto RA., 1965 "Theory of vibratory bending for elastic and viscoelastic layered finite-length beams." ASME Journal of Applied Mechanics ; Vol. 32:881–6.
(5) Mead DJ, Markus S., 1969 "The forced vibrations of a three-layer damped sandwich beam with arbitrary boundary conditions" .AIAA Journal Vol.10:163–75.
(6) Mead DJ. 1972 "The damping properties of elastically supported sandwich plates." Journal of Sound and Vibration; Vol. 24:275–95.
(7) C. Park, W. Cheol, B Yang, 1998 "Vibration analysis of three layer sandwich beam", Journal of ksnve Vol.8 pp157~170
(8) Cupial P, Niziol J., 1995 "Vibration and damping analysis of a three layered composite plate with a viscoelastic mid-layer." Journal of Sound and Vibration ;183:99–114.
(9) S. Na , Librescu, 2000 "Optimal vibration Control of Thin-Walled Anisotropic Cantilevers Exposed to Blast Loadings" Journal of Guidance, Control and Dynamics Vol,23, No3.
(10) Mead DJ, Markus S., 1969 "The force vibration of a three layer damped sandwich beam with arbitray boundary conditions" Journal of sound and vibration Vol. 10 pp 163-175
(11) J. B. Kosmatka, O. Mehmed, 2002 "Development of an Integral Damping Treatment for NASA's Next Generation Hollow Fan Blades," Smart Structures and Materials; Damping and Isolation, Vol. 4697, pp.15-24
(12) McTavish, D. J., and Hughes, P. C., 1993 "Modeling of Linear Viscoelastic Space Structures," Journal of Vibration and Acoustics, 115, 103-110.
(13) Shames, I. H., and Cozzarelli, F. A. 1992 "Elastic and Inelastic Stress Analysis" Prentice-Hall, Englewood Cliffs, 1st edition
(14) Ravi, S. S. A., Kundra, T. K., and Nakra, B. C. 1994 "Eigenvalue Reanalysis of Sandwich Beams with Viscoelastic Core Using Perturbation Method," International Journal of Analytical Experimental Modal Analysis, Vol. 9(3), 203-217.
(15) Y. M. Shi, Z. F. LI, H. X. Hux and Z. F. Fu, 2001 "The Modeling And Vibration Control Of Beams With Active Constrain Layer Damping" Journal of Sound and vibration Vol. 245(5) pp785-800
(16) C.T. Sun and T.P. Lu 1995 "Vibration Damping of Structural Elements" prentice hall