# Energy Bounding Algorithm for Stable Haptic Interaction

Jong-Phil Kim*, and Jeha Ryu**
* Department of Mechatronics, Kwang-Ju Institute of Science and Technology , Gwangju, Korea
(Tel : +82-62-970-2425; E-mail: lowtar@kjist.ac.kr)
** Department of Mechatronics, Kwang-Ju Institute of Science and Technology , Gwangju, Korea
(Tel : +82-62-970-2389; E-mail: ryu@kjist.ac.kr )

**Abstract**: This paper introduces a novel control algorithm, energy bounding algorithm, for stable haptic interaction. The energy bounding algorithm restricts energy generated by zero-order hold within consumable energy by physical damping that is energy consumption element in the haptic interface. The passivity condition can always be guaranteed by the energy bounding algorithm. The virtual coupling algorithm restricts the actuator force with respect to the penetration depth and restricts generated energy. In contrast, energy bounding algorithm restricts the change of actuator force with respect to time and restricts generated energy by zero-order hold. Therefore, much stiffer contact simulation can be implemented by the energy bounding algorithm. Moreover, the energy bounding algorithm doesn't is not computationally intensive and the implementation of it is very simple.

**Keywords:** haptic interaction, stability, passivity condition, sampled-data system

## 1. INTRODUCTION

Virtual reality technology has been rapidly developed with rapid development of semi-conductor technology and popularization of high performance personal computers together. Virtual reality or virtual environment is a wide-field presentation of computer-generated, multi-sensory information. Human operators are able to navigate and explore in real time in this synthetic world. Even though visual information through visual feedback interface such as CRT, HMD, and CAVE is the most important factor in VR technology, haptic information that can give force and tactile sense through haptic interfaces and help human operator interact with virtual environment more impressively and strenuously.

Stability is a key design element in haptic systems. In contrast to ordinary robotic manipulators, haptic interface cooperates with human operator and therefore unstable behavior of haptic interface may harm human operator. In order to protect human operator and haptic device from physical damage and in order to diminish the deterioration of haptic sense caused by unnecessary oscillation of haptic interface, it is indispensable to build stable haptic interaction. Construction of a stable haptic system, however, is reported as a very complicated problem comparing with that of common robotic system since a haptic system is composed of human operator, haptic interface, and virtual environment. Dynamics of human operator contains many uncertainties and is very difficult to model precisely. A virtual environment is constructed according to development purpose and it usually becomes highly nonlinear system. Haptic interface is a robotic manipulator which interfaces with a discrete virtual environment and continuous human operator.

For stable haptic interaction, Colgate and Schenkel [3] proposed the use of a virtual coupling which is derived through the passivity theorem. Virtual coupling can restrict impedance required by the virtual environment to such a level that haptic interfaces can be operated within stability-guaranteed impedance range (Z-width [1]) independent from the virtual environment. Thus, it enables to decouple design procedure of stable controller of haptic interface from that of virtual environment. Since a virtual coupling is designed based on the energy consumption damping elements such as friction, hysterics material property, and back electro motive force in electrical actuators, precise identification of physical damping parameters through a very complicated dynamic characterization procedure [9] of haptic interface is required to achieve the optimized virtual coupling parameters for better performance. Moreover, in the case of the haptic interface with multiful degrees-of-freedom, the effect of physical parameter depends on configuration. In order to guarantee the stability in the whole workspace of the haptic interface, virtual coupling parameters should be chosen against the worst configuration and therefore excessively conservative design is inevitable.

Recently, Hannaford and Ryu [12] proposed a time-domain passivity control that is energy consumption algorithm based on the passivity theorem. The proposed algorithm does not require any knowledge about dynamic model of haptic interface. They proposed a passivity observer and a passivity controller to insure stability under a wide variety of operating conditions. The passivity observer can measure energy flow in and out of one or more subsystems in real-time, and the passivity controller, which is an adaptive dissipative element, absorbs exactly the net energy output measured by the passivity observer at each time sample. However, their time-domain passivity theorem is derived by considering whole haptic system as a discrete system and discards effects of the sample and hold operation. Therefore, it cannot manipulate the energy generated by sample and hold operation.

Besides, Gillespie and Cutkosky [16] explained several factors destabilizing haptic rendering of the virtual wall and used half sample prediction controller for compensating the effect of zero order hold. Stramigioli and Schaft [17] obtained an equivalence between the continuous time and discrete time energy flow and suggested the use of a clever book-keeping of the energy in excess supplied to the continuous time system or a continuous time damping circuit.

In this paper, we consider the haptic system as a sampled-data system and derive passivity conditions for stable haptic interaction. Besides, we focus on the energy generations of zero order hold and develop a novel stability algorithm, energy bounding algorithm, for stable haptic interaction. The energy bounding algorithm restricts energy generated by zero-order hold within consumable energy by physical damping which is energy consumption element in the haptic interface. The passivity condition can always be guaranteed by the energy bounding algorithm. The virtual coupling algorithm restricts the actuator force with respect to the penetration depth and restricts generated energy. In contrast, energy bounding algorithm restricts the change of actuator force with respect to time and restricts generated energy by zero-order hold. Therefore, much stiffer contact simulation can be implemented by the energy bounding algorithm. Moreover, the energy bounding algorithm is not computationally intensive and its implementation is very simple.

In chapter 2, passivity conditions of continuous, discrete, and sampled-data systems are summarized and the passivity condition of haptic simulation is described. In chapter 3, a novel stability algorithm, energy bounding algorithm, for stable haptic interaction is derived. Chapter 4 shows experimental results and conclusion and further study is presented in chapter 5.

## 2. BASIC THEORY

### 2.1 Passivity Condition of Continuous, Discrete, and Sampled-data systems

Passivity is an abstract formulation of the idea of energy dissipation and passivity theorem, which is based on the input-output point of view, deals with stability problem defined for linear as well as nonlinear systems. Passivity is closely associated with power dissipation. The passive systems, therefore, cannot generate energy. This guarantees stable behavior of those systems. In fact, passive systems are common in engineering. A system that is composed of resistors, capacitors, and inductors is one example in electrical engineering and a system with masses, springs, and dashpots is another example in mechanical engineering. Thus we treat those elements as passive elements. Passivity theorem says that a feedback connection of one passive system and one strictly passive system is stable, thus, it has many advantages for the analysis of coupled stability problems in robotics, teleoperation, and related research area. More recently, passivity is widely used for stability analysis of haptic system, which is coupled with human operator, haptic interface, and virtual environment.

Passivity condition is defined as follows;

*(i) for a continuous system*

$$\int_0^t \mathbf{F}_C(\tau)\cdot\mathbf{v}_C(\tau)\,d\tau \ge -\varepsilon_0\text{, for } t>0 \text{ and admissible } \mathbf{F}_C(t) \quad (1)$$

where $\mathbf{F}_C(\tau)$ and $\mathbf{v}_C(\tau)$ are continuous effort and flow variables, respectively and $\varepsilon_0$ is the initial energy of the system

*(ii) for a discrete system* during $0 \le t < nT$ (for an arbitrary constant time interval $T$ )

$$\sum_{k=0}^{n-1}\mathbf{F}_D(k)\cdot\mathbf{v}_D(k)\Delta t_k = T\sum_{k=0}^{n-1}\mathbf{F}_D(k)\cdot\mathbf{v}_D(k) \ge -\varepsilon_0 \quad (2)$$

where $\mathbf{F}_D(k)$ and $\mathbf{v}_D(k)$ are discrete effort and flow variables, respectively.

*(iii) for a sample-data system* during $0 \le t < nT$

$$\int_0^{nT}\mathbf{F}_C(t)\cdot\mathbf{v}_C(t)\,dt + T\sum_{k=0}^{n-1}\mathbf{F}_D(k)\cdot\mathbf{v}_D(k) \ge -\varepsilon_0 \quad (3)$$

### 2.2 Passivity Conditions of Haptic Simulation

In general, haptic simulation is composed of human operator, haptic interface, controller, and virtual environment as indicated in Figure 1, which creates kinesthetically immersive feelings. The haptic interface contains a haptic mechanism, actuators, and sensors. The controller part is involved with calculations for operating haptic interface such as forward kinematics, inverse kinematics and Jacobian calculations. It may contain inverse dynamics, gravity compensation, and friction compensation algorithms. It may also contain stability algorithms such as virtual coupling or time-domain passivity algorithm.

If the human operator is passive and remaining components (*H*: haptic interface, *S*: sample & hold operation, *C*: controller, and *V*: virtual environment) are also passive, then total haptic simulation becomes stable. We assume that human operator is passive at frequencies of interest [15]. Stability problem, therefore, falls into passivity of the combined HSCV system as follows:

$$\int_0^t F_h(\tau)v_h(\tau)\,d\tau + \varepsilon_0 \ge 0\text{, for } t>0 \text{ and admissible } F_h(t) \quad (4)$$

where $\varepsilon_0$ is the initial energy. For the haptic simulation which satisfies the above passivity condition, the combined HSCV system consumes energy and human operator can never extract energy from it. Therefore, the sum of energy generation of haptic interface, sample & hold, controller, and virtual environment should be negative for stable haptic simulation. Let's consider the passivity relations for them respectively.

The haptic interface (Figure 2. (a)) is a continuous subsystem and thus its passivity during $0 \le t < nT$ is derived by Eq.(1).

$$P_H(n) = \int_0^{nT} F_h(t)v_h(t) - F_d^h(t)v_d(t)\,dt \quad (5)$$

Since haptic interface is composed of passive elements, it cannot generate energy by itself. Practically, every mechanical haptic interface satisfies passivity condition since it has energy consumption elements such as friction, hysterics material property, and back electro motive force in electrical actuators. The energy consumption element in haptic interface gives some allowance of energy generation in other components. Especially, the energy consumption element in haptic interface is very important to guarantee stable haptic interaction since there is inevitable energy generation in hold operation which has inherent phase lag.

The combined CV system (Figure 2. (c)), composed of the controller and the virtual environment, is discrete subsystem and thus their passivity, $P_{CV}$, during $0 \le t < nT$ is derived by Eq.(2).

$$P_{CV}(n) = T\sum_{k=0}^{n-1}F_d(k)v_{d,k} \quad (6)$$

There are several energy generation factors in the CV subsystem. Explicit numerical integrations in virtual environment, phase lags in the position or velocity filter, communication delays when haptic interface is connected to virtual environ-

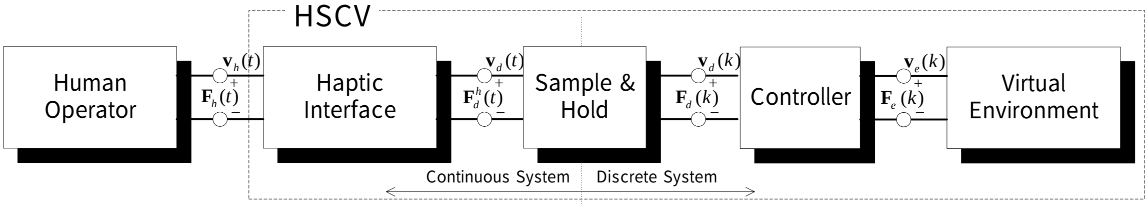
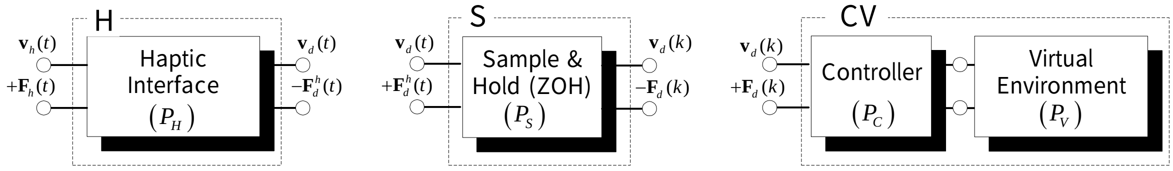


Figure 1. Overall configuration of haptic simulation of sampled-data system.



(a) (b) (c)

Figure 2. Each component of haptic system: (a) haptic interface, (b) sample & hold operation, (c) controller and virtual environment

ment by network, and gravity or friction compensation algorithm can generate energy and cause unstable behavior of the haptic simulation. Therefore, the generated energy in CV subsystem should be eliminated properly in order to guarantee stable haptic interaction. Energy consumption algorithm known as time-domain passivity algorithm [12] can effectively dissipate the generated energy in discrete system.

The sample and hold operation (Figure 2. (b)) is the sampled data system and thus its passivity during $0 \le t < nT$ is derived by Eq.(3).

$$P_S(n) = \int_0^{nT} F_d^h(t) v_d(t) dt - T \sum_{k=0}^{n-1} F_d(k) v_{d,k} \tag{7}$$

Usually zero order hold is used. In that case, passivity of the sample and hold operation is expressed as

$$\begin{aligned} P_S(n) &= \sum_{k=0}^{n-1}\left(\int_{kT}^{(k+1)T} F_d^h(t) v_d(t) dt\right) - T\sum_{k=0}^{n-1} F_d(k) v_{d,k} \\ &= \sum_{k=0}^{n-1}\left(F_d(k)\int_{kT}^{(k+1)T} v_d(t) dt\right) - T\sum_{k=0}^{n-1} F_d(k) v_{d,k} \\ &= \sum_{k=0}^{n-1} F_d(k)\left(x_{d,k+1} - x_{d,k}\right) - T\sum_{k=0}^{n-1} F_d(k) v_{d,k} \end{aligned} \tag{8}$$

where, using the unit step function $1(t)$,

$$F_d^h(t) = \sum_{k=0}^{\infty} F_d(k)\left[1(t-kT) - 1(t-(k+1)T)\right]$$

In many applications, the position is sensed by the encoder and the velocity is calculated by the backward rectangular rule. In that case, passivity of the sample and hold operation is expressed as

$$P_S(n) = T\sum_{k=0}^{n-1} F_d(k) v_{d,k+1} - T\sum_{k=0}^{n-1} F_d(k) v_{d,k} \tag{9}$$

The Eq.(9) indicates that force or velocity change may cause energy generation even though we make the controller and the virtual environment, the CV subsystem, passive. Since every haptic system has sample and hold operation, it becomes fundamental factor for the stable haptic interaction.

Finally, the passivity of the combined SCV system is simply derived as follows.

$$P_{SCV}(n) = P_S(n) + P_{CV}(n) = T\sum_{k=0}^{n-1} F_d(k) v_{d,k+1} \tag{10}$$

Note that passivity observed by Eq.(10) contains energy generations in controller, virtual environment as well as zero order hold. Therefore, whole haptic system becomes stable when the $P_{SCV}$ has positive value or greater value than $-P_H$.

### 2.3 Example: Virtual Wall Simulation

We will consider the virtual wall simulation for 1-dof haptic interface as an example for better understanding of energy relation of each subsystem for basic virtual element, damping and stiffness. The virtual wall simulation has been widely used as a benchmark example revealing many fundamental issues for stable haptic interaction. For simplicity, we will not consider any control algorithm ($F_d(k) = F_e(k) \equiv F(k)$, $x_{d,k} = x_{e,k} \equiv x_k$) and we use ZOH for hold operation and backward rectangular rule for velocity estimation. Consider the following virtual wall.

$$F(k) = Kx_k + Bv_k \tag{11}$$

Through Eq.(6), the passivity of CV subsystem for the virtual wall becomes

$$\begin{aligned} P_{CV}(n) &= K\sum_{k=0}^{n-1} x_k(x_k - x_{k-1}) + TB\sum_{k=0}^{n-1} v_k^2 \\ &= \frac{1}{2}K\sum_{k=0}^{n-1}(x_k^2 - x_{k-1}^2) + \frac{1}{2}K\sum_{k=0}^{n-1}(x_k - x_{k-1})^2 + TB\sum_{k=0}^{n-1} v_k^2 \\ &\ge \frac{1}{2}T^2K\sum_{k=0}^{n-1} v_k^2 + TB\sum_{k=0}^{n-1} v_k^2 \end{aligned} \tag{12}$$

and it satisfies passivity condition. This means virtual spring and damper elements does not generate energy in CV subsystem and passivity observed by Eq.(6) always gives positive value. Through Eq.(10) and 1 and 2 of appendix, the passivity of SCV subsystem becomes

$$\begin{aligned} P_{SCV}(n) &= P_S(n) + P_{CV}(n) = K\sum_{k=0}^{n-1} x_k(x_{k+1} - x_k) + TB\sum_{k=0}^{n-1} v_k v_{k+1} \\ &= \frac{1}{2}K\sum_{k=0}^{n-1}(x_{k+1}^2 - x_k^2) - \frac{1}{2}K\sum_{k=0}^{n-1}(x_{k+1} - x_k)^2 + TB\sum_{k=0}^{n-1} v_k v_{k+1} \\ &\ge -\frac{1}{2}T^2K\sum_{k=0}^{n-1} v_{k+1}^2 - TB\sum_{k=0}^{n-1} v_{k+1}^2 \end{aligned} \tag{13}$$

and it does not satisfy passivity condition any more because of the sample and hold operation. The spring element always generates energy when any movement occurs, and damping element generates energy when the velocity change occurs. Note that in order to observe the generated energy due to the sample and hold operation, we have to use Eq.(10) instead of Eq.(6). The lower bound of $P_{SCV}$, indicated in Eq.(13), means the maximum value of energy which virtual spring and damper can generate. In order to guarantee stable haptic interaction, the generated energy should be consumed by the remaining subsystem, the haptic interface. Let's consider the following 1 DOF haptic interface in Figure 3.

$$F_h(t) = m\dot{v}_h(t) + bv_h(t) + F_d^h(t) \tag{14}$$

$$v_h(t) = v_d(t) \tag{15}$$

Through Eq.(5) and appendix 3, the passivity of the haptic interface becomes

$$P_H(n) = \int_0^{nT}\left(mv\dot{v} + bv^2\right)dt \;\ge\; \int_0^{nT} bv^2 dt \;\ge\; T\sum_{k=0}^{n-1} bv_{k+1}^2 \tag{16}$$

It has nonnegative value for nonnegative damping coefficient and its lower bound value is definitely related to energy consuming damping component in the haptic interface. Finally, the passivity of the combined HSCV system becomes

$$\begin{aligned} P_{HSCV}(n) &= P_H(n) + P_{SCV}(n) \\ &\ge T\sum_{k=0}^{n-1} bv_{k+1}^2 - \frac{1}{2}T^2K\sum_{k=0}^{n-1} v_{k+1}^2 - TB\sum_{k=0}^{n-1} v_{k+1}^2 \end{aligned} \tag{17}$$

In order to make stable haptic interaction, the passivity of the combined HSCV system ($P_{HSCV}$) has nonnegative value, and thus virtual damping and stiffness value should satisfy the

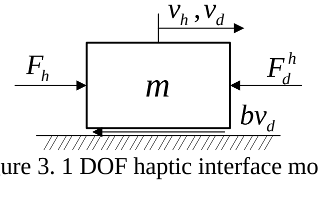


Figure 3. 1 DOF haptic interface model

following condition.

$$b \geq \frac{KT}{2} + B \tag{18}$$

This is a very well-known stability condition for virtual wall simulation studied by Colgate Schenkel [3]. In the energy point of view, above condition means that the generated energy by the virtual spring and damping parameter value should be constrained within the consumable energy by the haptic interface.

## 3. ENERGY BOUNDING ALGORTHM

### 3.1 Energy bounding algorithm

As shown in the chapter 2, the combined HSCV subsystem should satisfy passivity condition for stable haptic interaction. The haptic interface does not generate energy but the sample and hold operation and the CV subsystem can generate energy. The energy generated by CV subsystem, which is discrete system, can be effectively dissipated by energy consumption algorithm known as time-domain passivity algorithm. In order to dissipate the energy generated by SCV subsystem, the energy consumption algorithm requires prediction about the position at the next sampling.

For example, the passivity observer detects 10Nm energy generation at *t=kT*. Then, passivity controller should determine the compensation force which will dissipate 10Nm energy. The compensation force will be applied during $kT \leq t < (k+1)T$ , and thus the real work by the compensation force depends on $x_{k+1}$. Therefore, we should know $x_{k+1}$ in order to determine the compensation force at *t=kT*. However, it is not easy to predict the position since haptic interaction includes human operator that have many uncertainties. Therefore, we can make CV subsystem passive but cannot make sample and hold operation passive by the energy consumption algorithm.

In order to overcome this problem, this paper proposes a novel control algorithm, energy bounding algorithm, for stable haptic interaction. The energy bounding algorithm restricts energy generated by the zero-order hold within consumable energy by the physical energy consumption elements in the haptic interface. To do so, we focus on the passivity of sample and hold subsystem, Eq.(9), especially when zero order hold is used. It can be rewritten as

$$\begin{aligned} P_S &= T\sum F_d(k)v_{d,k+1} - T\sum F_d(k)v_{d,k} \\ &= T\sum\left(F_d(k-1) - F_d(k)\right)v_{d,k} \end{aligned} \tag{19}$$

It means that the energy is determined by velocity and change of actuating force. For impedance display, we can adjust the actuating force properly. Therefore, the energy bounding algorithm, which is composed of control law and bounding law, restricts change of the actuating force and bounds energy generation within consumable energy by physical energy consumption element in the haptic interface. The control law determined actuating force, $F_d(k)$, with respect to the virtual environment force, $F_e(k)$, and is represented as

$$\begin{aligned} &F_d(k) = F_d(k-1) + \beta(k)v_{d,k} + \alpha(k) \\ &\beta(k) = \begin{cases} \dfrac{F_e(k) - F_d(k-1)}{v_{d,k}} & \text{if } v_{d,k} \neq 0 \\ 0 & \text{otherwise} \end{cases} \\ &\alpha(k) = \begin{cases} 0 & \text{if } v_{d,k} \neq 0 \\ F_e(k) - F_d(k-1) & \text{otherwise} \end{cases} \end{aligned} \tag{20}$$

If there is no bounding law on $\beta(k)$ and $\alpha(k)$, then the actuating force becomes the virtual environment force directly. The bounding law modulates $\beta(k)$ and $\alpha(k)$. As a result it restricts energy generation within consumable energy by physical energy consumption element in the haptic interface. For the 1 degree of freedom haptic interface, Eq.(16), we use following bounding law.

$$\begin{aligned} &\text{if } \beta(k) > b \text{ then } \beta(k) = b \\ &\text{else if } \beta(k) < -b \text{ then } \beta(k) = -b \end{aligned} \tag{21}$$

Then, the passivity of zero order hold becomes

$$P_S = -T\sum \beta(k)v_{d,k}^2 \geq -T\sum b v_{d,k}^2 \tag{22}$$

Accordingly, the generated energy by ZOH can be entirely consumed by the haptic interface. Thus, it enables stable haptic interaction. Theoretically, we don't need any bounding law of $\alpha(k)$ because it is applied when velocity is zero. Practically, however, it is better to use bounding law of $\alpha(k)$ since the velocity is obtained by the encoder with some level of resolution. The velocity obtained by the encoder may indicate zero value for the movement within encoder resolution. In our case, the bounding value of $\alpha(k)$ is chosen by experiment.

### 3.2 Comparison of Energy bounding algorithm with virtual coupling algorithm

The virtual coupling algorithm and the energy bounding algorithm have some similarity. Both algorithms deal with stability problem caused by sample and hold operation which is structural and fundamental problem of the haptic simulation (the sampled-data system), and they use physical damping in the haptic interface for consuming energy generated by the sample and hold operation. However, the virtual coupling algorithm restricts the actuator force with respect to the penetration depth and restricts generated energy in the end. On the contrary, energy bounding algorithm restricts the change of actuator force with respect to time and restricts generated energy eventually. This difference between them makes discrepancy of ability for available impedance of contact simulation. The energy bounding algorithm enables to apply bigger force at the same penetration depth. Instead, it requires a little time to reach the force.

For example, let's assume that we have 1 degree of freedom haptic interface with physical damping *b*=1Nsec/m and 0.1mm encoder resolution. If the haptic rate is 1KHz (*T* = 0.001sec), the maximum equivalent stiffness of the virtual wall which can be achieved by the virtual coupling algorithm is 2kN/m. However, energy bounding algorithm enables much stiffer virtual wall simulation. The virtual wall with 10kN/m stiffness requires 1N virtual environment force for the 0.1mm penetration depth (encoder resolution). The energy bounding algorithm can generate 1N actuating force for that penetration depth without loss of stability. Instead, it requires maximum 10 sampling time (0.01 sec) to reach 1N actuating force. Therefore, much stiffer contact simulation can be implemented by the energy bounding algorithm.

In addition, the energy bounding algorithm is very simple and doesn't require lots of computations. Moreover, it is very easy to be implemented. Once virtual environment force is given, the actuating force is automatically determined by the bounding law. For contact simulation, virtual coupling algorithm requires penetration depth calculation. When multiple contacts occur, moreover, virtual coupling parameter should be divided by the number of contact point. However, the energy bounding algorithm does not require those additional calculations.

# 4. EXPERIMENTS

In our experiment Impulse engine 2000 which is well-developed commercial haptic interface is used with expectation that it may contain less instability factors comparing to developing prototype haptic interface. Impulse Engine 2000 is high-quality, tactile force-feedback device and was developed by Immersion Corporation for military, robotic, medical, and other various research applications. It uses gimbal structure which enables two degrees of freedom angular motion and capstan mechanism, that is cable transmission instead of gear, which enhances back-drivability. Workspace is 6"x6" (about ±45°) along x and y axes. Maximum force output at handle is 2 lbs. (8.9 N) and encoder resolution is 0.0008" that is 0.254mm through translational transformation. Therefore, maximum stiffness which the haptic interface can produce is about 35KN/m.

Figure 4 shows the results of the virtual wall simulation without any stabilizing control algorithm. The human operator feels unwanted permanent oscillations (Figure 4. (a)) because of energy generated by ZOH. Since virtual spring and damper does not generate energy within the controller and virtual environment subsystem, as shown in Eq. (12), the passivity observer implemented by Eq. (6) gives positive value (Figure 4. (c)). That means the passivity observer does not detect the energy generation even though there exists unwanted permanent oscillations and the time domain passivity algorithm cannot manipulate the instability caused by the ZOH.

Figure 5 shows the results of the virtual wall simulation with virtual coupling algorithm. By virtue of the virtual coupling, unwanted permanent oscillations are almost eliminated (Figure 5. (a)). Since we achieve stable behavior at the cost of stiffness (Figure 5. (c)), the level of oscillation is increased when more stiff wall is used. When we use the virtual wall with about 35KN/m stiffness, the level of oscillation is same as virtual wall simulation without any stabilizing algorithm, as shown in Figure 4 (a).

Figure 6 shows the results of the virtual wall simulation with energy bounding algorithm. By virtue of the energy bounding algorithm, the unwanted permanent oscillations are almost eliminated (Figure 6. (a)). Moreover, it also gives very stiff haptic sense (maximum stiffness which our haptic interface can produce) as shown in Figure 6 (c).

# 5. CONCLUSION AND FUTHER STUDY

In this paper, we consider a haptic system as a sampled-data system and derived passivity conditions for stable haptic interaction. The combined HSCV subsystem should satisfy passivity condition for stable haptic interaction. The haptic interface does not generate energy but the sample and hold operation and the CV subsystem can generate energy. The energy generated by CV subsystem, that is discrete system, can be effectively dissipated by energy consumption algorithm known as time-domain passivity algorithm. However, the energy consumption algorithm cannot exactly compensate the energy generated by sample and hold operation. Therefore, we focus on the energy generations by the sample and hold operation and develop a novel stability algorithm, energy bounding algorithm, for stable haptic interaction. The energy bounding algorithm restricts energy generated by zero-order hold within consumable energy by physical damping which is energy consumption element in the haptic interface. The passivity condition can always be guaranteed by the energy bounding algorithm. The virtual coupling algorithm restricts the actuator force with respect to the penetration depth and restricts generated energy. On the contrary, energy bounding algorithm restricts the change of actuator force with respect to time and restricts generated energy by zero-order hold. Therefore, much stiffer contact simulation can be implemented by the energy bounding algorithm. Moreover, the energy bounding algorithm doesn't require high computation load and its implementation is very simple. In future, we will develop the energy bounding algorithm for admittance display and find duality. In addition, we will apply the algorithm to the volumetric haptic rendering algorithm.

## ACKNOWLEDGMENTS

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# APPENDIX

1. $\sum_{k=0}^{n-1} v_k v_{k+1} \ge -\sum_{k=0}^{n-1} v_{k+1}^2$ for zero initial condition.

Proof:

Since $\sum_{k=0}^{n-1} \left(v_k + v_{k+1}\right)^2 \ge 0$, $2\sum_{k=0}^{n-1} v_k v_{k+1} \ge -\sum_{k=0}^{n-1} v_k^2 - \sum_{k=0}^{n-1} v_{k+1}^2$ is satisfied.

Since $\sum_{k=0}^{n-1} v_k^2 = \left(v_0^2 - v_n^2\right) + \sum_{k=0}^{n-1} v_{k+1}^2$ , $\sum_{k=0}^{n-1} v_k v_{k+1} \ge -\sum_{k=0}^{n-1} v_{k+1}^2 + \frac{\left(v_n^2 - v_0^2\right)}{2} \ge -\sum_{k=0}^{n-1} v_{k+1}^2$ is satisfied.

2. $\sum_{k=0}^{n-1} x_k (x_{k+1} - x_k) \ge -\frac{1}{2} T^2 \sum_{k=0}^{n-1} v_{k+1}^2$ for zero initial condition.

Proof:

$$\begin{aligned}\sum_{k=0}^{n-1} x_k (x_{k+1} - x_k) &= \frac{1}{2}\sum_{k=0}^{n-1}(x_{k+1}^2 - x_k^2) - \frac{1}{2}\sum_{k=0}^{n-1}(x_{k+1} - x_k)^2 \\ &= \frac{1}{2}(x_n^2 - x_0^2) - \frac{1}{2}T^2\sum_{k=0}^{n-1} v_{k+1}^2 \ge -\frac{1}{2}T^2\sum_{k=0}^{n-1} v_{k+1}^2\end{aligned}$$

3. $\int_0^{nT} v^2(t)dt \ge T\sum_{k=0}^{n-1} v_{k+1}^2$

Proof:

Since

$$\begin{aligned}&\int_{kT}^{(k+1)T} v^2(t)dt - Tv_{k+1}^2 = \int_{kT}^{(k+1)T}\left[v^2(t) - v_{k+1}^2\right]dt = \int_{kT}^{(k+1)T}\left[v^2(t) - v_{k+1}v(t)\right]dt \\ &= \int_{kT}^{(k+1)T}\left[2\left\{v^2(t) - v_{k+1}v(t)\right\} - \left\{v^2(t) - v_{k+1}^2\right\}\right]dt = \int_{kT}^{(k+1)T}\left[\left\{v(t) - v_{k+1}\right\}^2\right]dt \ge 0,\end{aligned}$$

$\int_{kT}^{(k+1)T} v^2(t)dt \ge Tv_{k+1}^2$ is satisfied. Therefore, $\sum_{k=0}^{n-1}\int_{kT}^{(k+1)T} v^2(t)dt \ge T\sum_{k=0}^{n-1} v_{k+1}^2$ is derived.

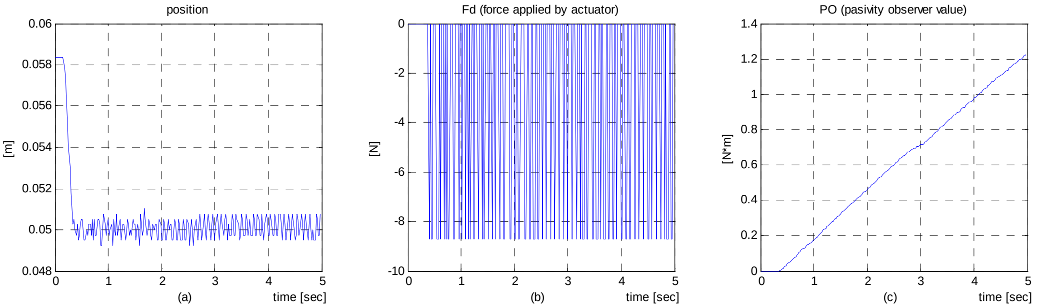

Figure 4. Virtual wall simulation when no stabilizing algorithm is used. (a) Position of haptic interface. The virtual wall is located at 0.05m. (b) Actuating force. (c) Passivity observer value defined by Hannaford [12].

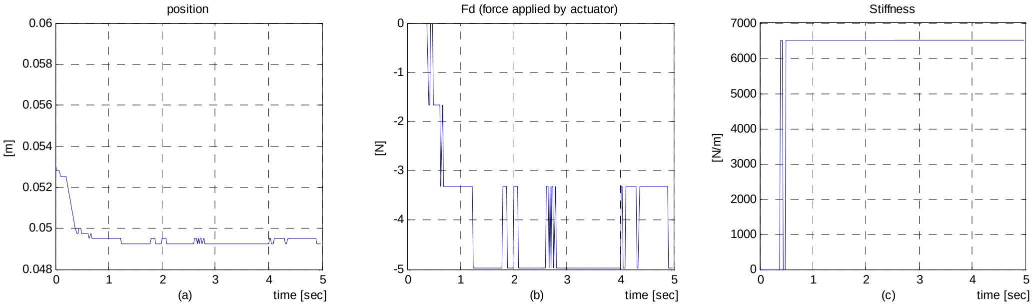

Figure 5. Virtual wall simulation when the virtual coupling is used. (a) Position of haptic interface. (b) Actuating force. (c) Stiffness value (actuating force / penetration depth).

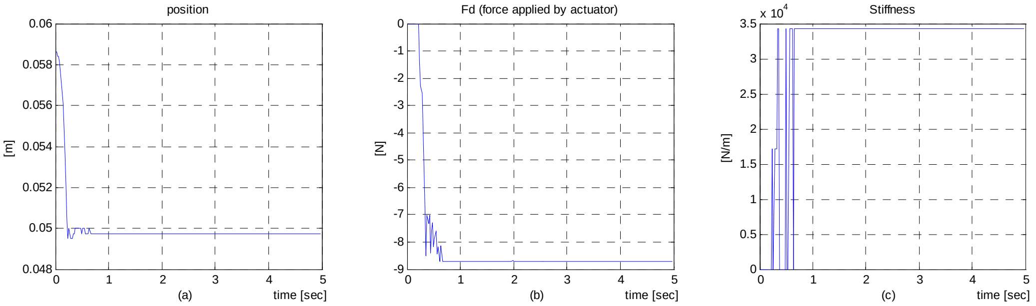

Figure 6. Virtual wall simulation when energy bounding algorithm is used. (a) Position of haptic interface. (b) Actuating force. (c) Stiffness value (actuating force / penetration depth).