A gain scheduling method for the vibration suppression servo controller of articulated robots

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Abstract: In this study we present a vibration controller for articulated robots that has flexible joints modeled as a 2-mass system. Most of articulated robots have time varying load inertias for each axis according to its motion. Moreover, the inertias vary drastically; for the base axis of articulated robots it may vary about 10 times of its minimum value. But, for industrial robots and many mechatronic devices, it is desirable to maintain control performance in spite of load inertia variation. So we propose a control gain adjustment rule considering the time-varying nature of load inertia. In this gain-adjusting algorithm, the pole locations are in proportion to the anti-resonance frequency of the 2-mass system. The simulation and experimental results show uniform properties in overshoot in spite of the variation of load.

Keywords: robot, 2-mass system, gain scheduling, uniform performance

1. INTRODUCTION

Vibration control is a very important issue for industrial robots because the vibration makes it difficult to achieve quick motion and might even damage the robot. In this paper we introduce a 2-mass system composed of two masses and an interconnecting spring element. This model represents the joint flexibility as a cause of the vibration of industrial robots. And we also introduce a vibration control algorithm using state feedback based on 2-mass system [1,2,3].

Most of articulated robots like fig. 1 have time varying load inertias for each axis according to its motion. Moreover, the inertias vary drastically; for the base axis of articulated robots it may vary about 10 times of its minimum value. And the dynamic characteristics of the 2-mass system are also considerably changed by its condition of load side inertia. We show the variations of the properties of the 2-mass system by the load inertia variation.

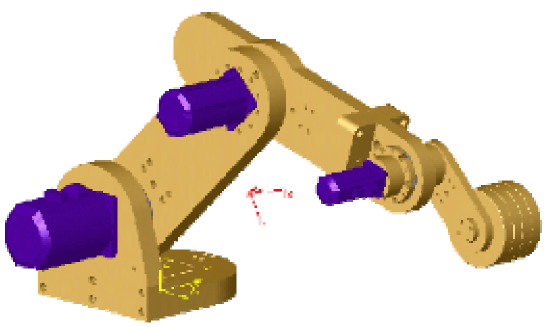


Fig. 1 Small size articulated robot

For industrial robots and many mechatronic devices, it is desirable to maintain control performance in spite of load inertia variation. Especially the maximum overshoot is the most important performance index of the motion control, because if it is changed to large or small value by varying load inertia, the user could not manipulate it with consistency. In our previous study [1,2], we used some rough gain-scheduling rules concerning the total inertia, sum of motor and load inertias. But the performance of the motion varied with change of load inertia.

To maintain the performance, the gain of the controller should be adequately adjusted by its time-varying nature of load inertia. In this study, we propose a control gain adjustment rule from a qualitative response analysis on state feedback control algorithm of the 2-mass system. The main idea of proposed gain adjusting strategy is “move fast for light load and move slowly for heavy load but it should maintain the prescribed maximum overshoot.” In this gain-adjusting algorithm, the pole locations are in proportion to the anti-resonance frequency of the 2-mass system. It is verified by mathematical analysis, simulation, and experiment.

For analytic verification, we derive the transfer function of the state feedback control system, and then derive the step response in time domain. And we show that the coefficients of the step response are maintained uniformly. We also verify this scheduling algorithm via simulation. We simulate some candidates of scheduling and show the proposed idea is the best one. If we fix the system poles to preserve the time constants, the system becomes vibratory in some cases; this comes from the typical existence of the zeros of the 2-mass system. And we also simulate some other candidates such as a method concerning the total inertia and a method proportional to the anti-resonance frequency of the system.

Finally, we show the experimental results on a real robot. We test this gain-adjusting algorithm using the HILS(Hardware In the Loop Simulation) system and an articulated robot shown in fig. 1 [4]. The experimental results show that the maximum overshoot maintained uniformly in spite of pose of the robot. But if we use another algorithm using the total inertia, the robot moves fast for large inertia pose, and move slowly for slightly small inertia and even become unstable for small inertia pose.

2. TWO-MASS SYSTEM

The joint flexibility between driving motor and link can be modeled as a torsional spring as can be seen in Fig. 2 [5,6]. Conventional industrial robots adopt PID type semi-closed servo control loop with measurement of motor angle. Accordingly, it is the motor angle, instead of link angle, that the controller makes it track the desired position command.

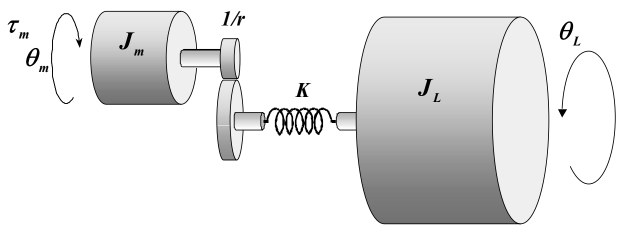


Fig. 2 Two-mass system

If the motor angle tracks the command faithfully, the link angle tracks the motor angle with vibratory behavior because of the role of torsional spring. Its block diagram is shown in Fig. 3.

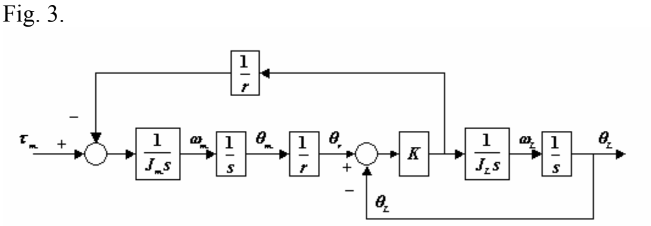


Fig. 3 Block diagram of two-mass system

The motion of two-mass system can be expressed by the following equations.

$$\tau_m = J_m \frac{d^2\theta_m}{dt^2} + \frac{1}{r} K(\frac{\theta_m}{r} - \theta_L) \tag{1}$$

$$K(\frac{\theta_m}{r} - \theta_L) = J_L \frac{d^2\theta_L}{dt^2}, \tag{2}$$

where, J_m and J_L are the moments of inertia, θ_m and θ_L are the angles, of the motor and link, respectively, K is the stiffness of the joint, r is the reduction ratio, τ_m is the motor torque, and frictions are neglected for simplicity. The state space equation of two-mass system can be represented as follows.

$$\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} + \mathbf{B}u$$
$$y = \mathbf{C}\boldsymbol{x} \tag{3}$$

where,

$$\boldsymbol{x} = \begin{bmatrix} \omega_m \\ \theta_s \\ \omega_L \end{bmatrix}, \quad y = \omega_m, \quad u = \tau_m, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} 0 & -\frac{1}{r}\frac{K}{J_m} & 0 \\ \frac{1}{r} & 0 & -1 \\ 0 & \frac{K}{J_L} & 0 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} \frac{1}{J_m} \\ 0 \\ 0 \end{bmatrix}.$$

The transfer functions of the two-mass system can be derived from (1) and (2).

$$\frac{\omega_m}{\tau_m} = \frac{J_L s^2 + K}{J_m J_L s^3 + K(J_m + \frac{1}{r^2} J_L)s} \tag{4}$$

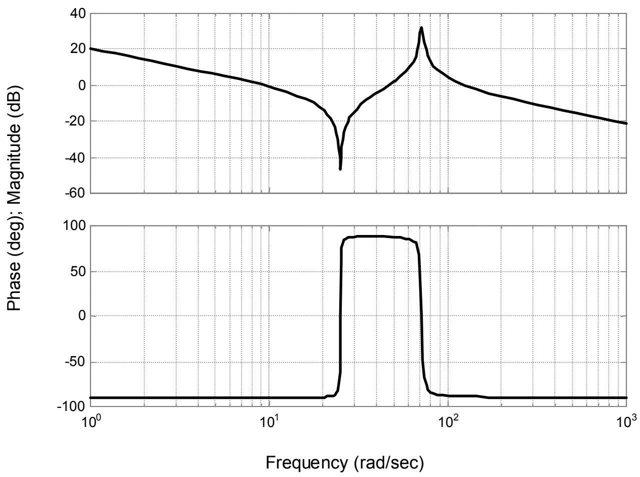
$$\frac{\omega_L}{\tau_m} = \frac{\frac{1}{r} K}{J_m J_L s^3 + K(J_m + \frac{1}{r^2} J_L)s} \tag{5}$$

Fig. 4 depicts their Bode plots. It can be seen that there is a resonance phenomenon at a certain frequency. The resonance frequency and the anti-resonance frequency are expressed as follows.

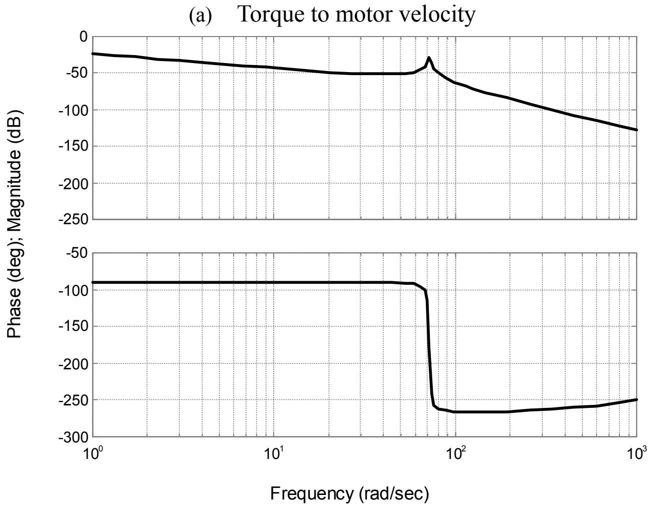
$$\omega_n = \sqrt{K\left(\frac{J_m + \frac{1}{r^2} J_L}{J_m J_L}\right)} \tag{6}$$

$$\omega_a = \sqrt{\frac{K}{J_L}}\ . \tag{7}$$

The frequencies determined by the torsional spring constant K and the inertias J_m and J_L.



(a) Torque to motor velocity



(b) Torque to link velocity

Fig. 4 Bode plots of two-mass system.

3. GAIN SCHEDULING METHOD

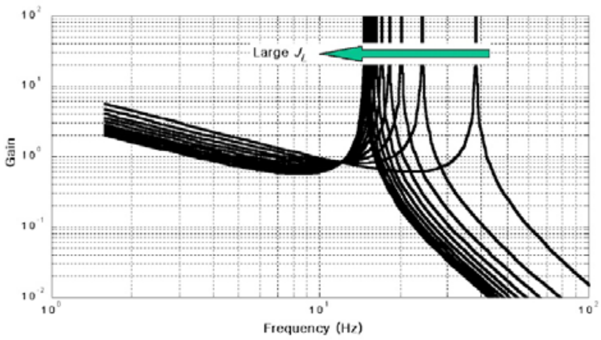
3.1 Variation of load side inertia

Most of industrial robots have time varying load inertias according to its motion. The amount of variation can be hundreds of percents between minimum and maximum inertia. The load inertia of a joint is the function of the joint angles. As a robot manipulator moves, dynamics governs its movement. The dynamic equation of the robot manipulator is as follows [7].

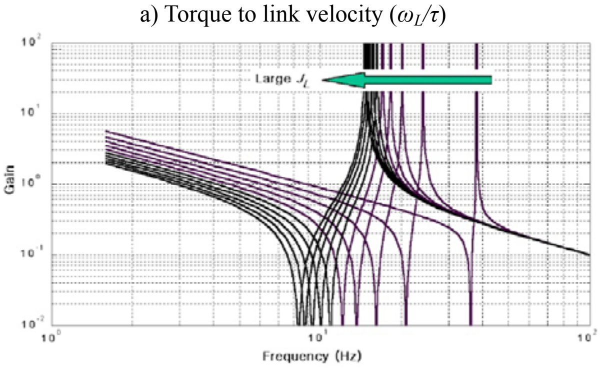
$$\boldsymbol{D}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{H}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{G}(\boldsymbol{q}) = \tau \tag{8}$$

where q is the generalized coordinate vector, $D(q)$ is the generalized inertia matrix, $H(q,\dot{q})$ is the Coriolis and centrifugal torque vector, $G(q)$ is the gravity torque vector, and τ is the generalized torque vector. The load inertias to be calculated are the diagonal terms of $D(q)$.

The variation of dynamic properties of the 2-mass system can be analyzed by its frequency response. Fig.5 shows the gain plot of the 2-mass system along the change of the load side inertia, J_L. This plot represents the variation of J_L for the 1st axis of an articulated robot with the variation of its pose.



a) Torque to link velocity (ω_L/τ)



b) Torque to motor velocity (ω_m/τ)

Fig. 5 Variation of frequency response

The change of J_L brings dramatic variations on the resonance and anti-resonance frequency of the 2-mass system. Especially the anti-resonance frequency, Eq. (7), is more widely changed. The variation on load inertia affects many dynamic characteristics such as time response, driving torque, and so on.

But, for industrial robots and many mechatronic devices, it is desirable to maintain control performance in spite of load inertia variation. To maintain control performance, the gain of the controller should be adequately adjusted by its time-varying nature of load inertia. Now, we can consider some candidates of gain adjusting method for the control system like Fig. 6. Firstly, we may fix the location of the system poles in complex plane to preserve the time response.

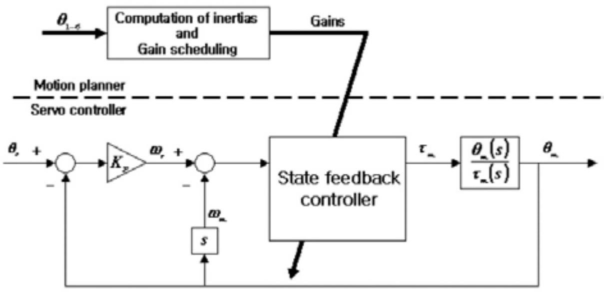


Fig. 6 Gain-adjusting scheme

Fig. 7 shows the simulation result of a state feedback system for the velocity control of a 2-mass system when we fix the desired poles. The response for a trapezoidal velocity command was simulated for some different values of load inertia. The responses of the system have undesirable overshoot and oscillation for many load conditions. This comes from that the changed load side inertia moves the system zeros and this changes the system overshoot characteristics and then makes system response undesirably.

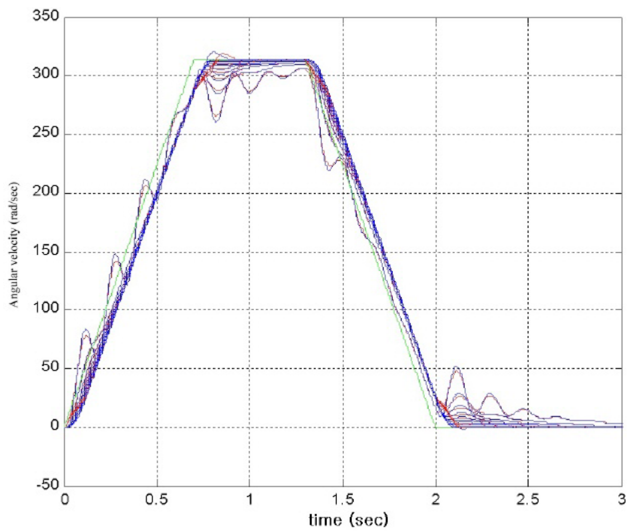


Fig. 7 Responses of fixed-pole case

So, a new gain adjusting strategy is needed to ***move fast for light load and move slowly for heavy load but it should maintain some performance indices*** instead of keeping the hole dynamic properties. Especially the maximum overshoot is the most important performance index of the motion control, because if it is changed to large or small value by varying load inertia, the user could not manipulate it with consistency. In our previous study, we used a rough gain-scheduling rule concerning the total inertia, sum of motor and load inertias. But the performance of the motion varied with change of load inertia [1].

In this study, we propose a control gain adjustment rule from a qualitative response analysis on state feedback control algorithm of the 2-mass system. This gain adjusting algorithm is that the pole locations are in inverse proportion to the square root of load inertia, in other words in proportion to the variation of the anti-resonance frequency ω_a. It is verified by mathematical analysis, simulation, and experiment.

3.2 State feedback controller and gain scheduling method

When one tries to control the two-mass system without vibration using PID type controller, there might be some drawbacks in the performance of the robot [1,8]. It is because the controller cannot feedback the state of the link and consequently the system poles cannot be located freely. In this study, we consider a vibration suppression velocity controller using state feedback.

The dynamics of the augmented state feedback controller for the 2-mass system is as follows.

$$\begin{aligned} \dot{x} &= Ax + B\tau \\ \dot{\xi} &= r - y, \\ \tau &= -Kx + k_I \xi \end{aligned} \tag{9}$$

$$\begin{bmatrix} \dot{\omega}_m \\ \dot{\theta}_s \\ \dot{\omega}_L \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} -\frac{k_1}{J_m} & -\frac{K_n}{J_m} - \frac{k_2}{J_m} & -\frac{k_3}{J_m} & \frac{k_I}{J_m} \\ 1 & 0 & -1 & 0 \\ 0 & \frac{K_n}{J_{Ln}} & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_m \\ \theta_s \\ \omega_L \\ \xi \end{bmatrix} + \begin{bmatrix} \frac{k_1}{J_m} \\ 0 \\ 0 \\ 1 \end{bmatrix} r \tag{10}$$

where,

$K_n = \dfrac{K}{r^2}$, $J_{Ln} = \dfrac{J_L}{r^2}$.

The transfer functions of ω_m and ω_L for the angular velocity command ω_r can be get as follows.

$$\frac{\omega_m}{\omega_r} = \frac{(s^2 + K_n/J_{Ln})(k_1/J_m s + k_I/J_m)}{s^4 + (k_1/J_m)s^3 + (k_I/J_m + k_2/J_m + K_n/J_m + K_n/J_{Ln})s^2 + (k_1/J_m + k_3/J_m)(K_n/J_{Ln})s + (k_I K_n/J_m J_{Ln})} \tag{11}$$

$$\frac{\omega_L}{\omega_r} = \frac{(K_n/J_{Ln})(k_1/J_m s + k_I/J_m)}{s^4 + (k_1/J_m)s^3 + (k_I/J_m + k_2/J_m + K_n/J_m + K_n/J_{Ln})s^2 + (k_1/J_m + k_3/J_m)(K_n/J_{Ln})s + (k_I K_n/J_m J_{Ln})} \tag{12}$$

This is a 4th order system, so we let the system characteristic equation have two real roots and two complex poles.

$$\begin{aligned} &(s^2 + 2\varsigma\omega_n s + \omega_n^2)(s+\tau_1)(s+\tau_2) \\ &= s^4 + (\tau_1 + \tau_2 + 2\varsigma\omega_n)s^3 + (\tau_1\tau_2 + \omega_n^2 + 2\varsigma\omega_n(\tau_1+\tau_2))s^2 \\ &\quad + (2\varsigma\omega_n\tau_1\tau_2 + (\tau_1+\tau_2)\omega_n^2)s + \omega_n^2\tau_1\tau_2 \end{aligned} \tag{13}$$

In this paper, for simplicity, we only consider the load side angular velocity ω_L. As you can see in the following equation, although you may allocate the desired system poles as you wish, but there are system zeros affected by the anti-resonance frequency. If the system stiffness is unchanged, the zeros are affected by the load inertia J_{Ln}.

$$\frac{\omega_L}{r} = \frac{(K_n/J_{Ln})(\tau_1 + \tau_2 + 2\varsigma\omega_n)s + \omega_n^2\tau_1\tau_2}{(s^2 + 2\varsigma\omega_n s + \omega_n^2)(s+\tau_1)(s+\tau_2)} \tag{14}$$

Now, we analysis this control system in time domain for the step input. The load side angular velocity becomes as follows.

$$\begin{aligned} \omega_L(t) = 1 &+ \left(\left(\frac{K_n}{J_{Ln}}\right)G_{11} + K_{11}\right)\cos\omega_d t + \left(\left(\frac{K_n}{J_{Ln}}\right)G_{12} + K_{12}\right)\sin\omega_d t \\ &+ \left(\left(\frac{K_n}{J_{Ln}}\right)G_2 + K_2\right)e^{-\tau_1 t} + \left(\left(\frac{K_n}{J_{Ln}}\right)G_3 + K_3\right)e^{-\tau_2 t} \end{aligned} \tag{15}$$

where,

$$\begin{aligned} \left(\frac{K_n}{J_{Ln}}\right)G_{11} + K_{11} &= \left(\frac{K_n}{J_{Ln}}\right)\frac{(-(\tau_1+\tau_2-2\varsigma\omega_n)(\tau_1+\tau_2+2\varsigma\omega_n))}{(\omega_n^2+\tau_1^2-2\varsigma\omega_n\tau_1)(\omega_n^2+\tau_2^2-2\varsigma\omega_n\tau_2)} \\ &+ \frac{(\tau_1\tau_2(-\tau_1\tau_2+\omega_n(\omega_n+2(\tau_1+\tau_2)\varsigma-4\omega_n\varsigma^2)))}{(\omega_n^2+\tau_1^2-2\omega_n\tau_1\varsigma)(\omega_n^2+\tau_2^2-2\omega_n\tau_2\varsigma)} \\ \left(\frac{K_n}{J_{Ln}}\right)G_{12} + K_{12} &= \left(\frac{K_n}{J_{Ln}}\right)\frac{(\tau_1+\tau_2+2\varsigma\omega_n)(\tau_1\tau_2-\omega_n(\tau_1+\tau_2)\varsigma+\omega_n^2(2\varsigma^2-1))}{(\omega_n^2+\tau_1^2-2\varsigma\omega_n\tau_1)(\omega_n^2+\tau_2^2-2\varsigma\omega_n\tau_2)\omega_d} \\ &+ \frac{-\tau_1\tau_2(\tau_1\tau_2\varsigma-\omega_n(\tau_1+\tau_2)(-1+2\varsigma^2)+\omega_n^2\varsigma(-3+4\varsigma^2))}{(\omega_n^2+\tau_1^2-2\omega_n\tau_1\varsigma)(\omega_n^2+\tau_2^2-2\omega_n\tau_2\varsigma)\sqrt{1-\varsigma^2}} \end{aligned}$$

$$\begin{aligned} \left(\frac{K_n}{J_{Ln}}\right)G_2 + K_2 &= \left(\frac{K_n}{J_{Ln}}\right)\frac{(-(\tau_1+\tau_2+2\varsigma\omega_n))}{(\tau_1-\tau_2)(\omega_n^2+\tau_1^2-2\tau_1\varsigma\omega_n)} \\ &+ \frac{\omega_n^2\tau_2}{(\tau_1-\tau_2)(\omega_n^2+\tau_1^2-2\omega_n\tau_1\varsigma)} \\ \left(\frac{K_n}{J_{Ln}}\right)G_3 + K_3 &= \left(\frac{K_n}{J_{Ln}}\right)\frac{\tau_1+\tau_2+2\varsigma\omega_n}{(\tau_1-\tau_2)(\omega_n^2+\tau_2^2-2\tau_2\varsigma\omega_n)}. \\ &+ \frac{-\omega_n^2\tau_1}{(\tau_1-\tau_2)(\omega_n^2+\tau_2^2-2\omega_n\tau_2\varsigma)} \end{aligned}$$

To maintain the performance, we propose a control gain adjustment rule that the pole locations are in proportion to the anti-resonance frequency, for fixed stiffness case, in inverse proportion to the square root of load inertia.

We let the load side inertia for a reference pose of a robot as J_{Lnref}, the variation of the anti-resonance frequency for changed inertia as $\Delta\omega_a = \sqrt{\dfrac{K_n/J_{Ln}}{K_n/J_{Lnref}}} = \sqrt{\dfrac{J_{Lnref}}{J_{Ln}}}$, and the parameters of the pole location at the reference pose as τ_{1ref}, τ_{2ref}, and ω_{nref} for two real poles and two complex poles respectively.

When we let the location of the poles τ_1, τ_2, and ω_n for the changed load inertia as follows.

$$\tau_1 = \tau_{1ref}\Delta\omega_a,$$
$$\tau_2 = \tau_{2ref}\Delta\omega_a, \quad (16)$$
$$\omega_n = \omega_{nref}\Delta\omega_a$$

The time responses of the reference pose and the changed pose are as follows.

Time response for reference inertia:

$$\begin{aligned}\omega_L(t) = 1 &+ \left(\left(\frac{K_n}{J_{Lnref}}\right)G_{11} + K_{11}\right)\cos(\omega_{nref}\sqrt{1-\varsigma^2}t) \\ &+ \left(\left(\frac{K_n}{J_{Lnref}}\right)G_{12} + K_{12}\right)\sin(\omega_{nref}\sqrt{1-\varsigma^2}t) \\ &+ \left(\left(\frac{K_n}{J_{Lnref}}\right)G_2 + K_2\right)e^{-\tau_{1ref}t} + \left(\left(\frac{K_n}{J_{Lnref}}\right)G_3 + K_3\right)e^{-\tau_{2ref}t}\end{aligned} \quad (17)$$

Time response for changed inertia:

$$\begin{aligned}\omega_L(t) = 1 &+ \left(\left(\frac{K_n}{J_{Lnref}}\right)G_{11} + K_{11}\right)\cos(\omega_{nref}\Delta\omega_a\sqrt{1-\varsigma^2}t) \\ &+ \left(\left(\frac{K_n}{J_{Lnref}}\right)G_{12} + K_{12}\right)\sin(\omega_{nref}\Delta\omega_a\sqrt{1-\varsigma^2}t) \\ &+ \left(\left(\frac{K_n}{J_{Lnref}}\right)G_2 + K_2\right)e^{-\tau_{1ref}\Delta\omega_a t} + \left(\left(\frac{K_n}{J_{Lnref}}\right)G_3 + K_3\right)e^{-\tau_{2ref}\Delta\omega_a t}\end{aligned} \quad (18)$$

Using proposed method, the coefficients of the time response are maintained equally to the reference pose's one. The time constants in the sinusoidal and exponential function are changed, this means to move fast or slow by the size of the load.

In this part, we analyzed the step response of the velocity control system of 2-mass system, and showed the fact that using proposed pole allocation method, the coefficients of the time response maintained uniformly in spite of the load inertia variation. Maintaining the coefficients means that the proposed method guarantees the uniform maximum overshoot in step response.

4. SIMULATIONS AND EXPERIMENTS

4.1 Simulation results

Now, we make some verifications of the proposed method by simulations and experiments. In the verification, we compare 2 methods: **method 1** is that the pole locations are in inverse proportion to the variation of the square root of the total inertia and **method 2** is the proposed one that the pole locations are in proportion to the variation of the anti-resonance frequency. The pole location adjustment rules to compare are as follows.

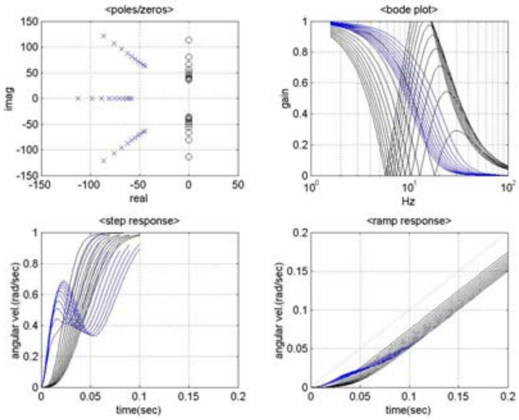
Method 1:

$$G_r = \sqrt{\frac{J_{total_{ref}}}{J_{total}}} = \sqrt{\frac{J_m + J_{Lnref}}{J_m + J_{Ln}}} \quad (19)$$

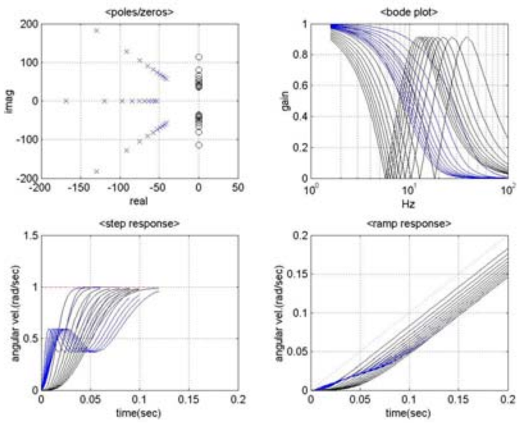
Method 2:

$$G_r = \Delta\omega_a = \frac{\omega_a}{\omega_{a_0}} = \sqrt{\frac{J_{Lnref}}{J_{Ln}}} \quad (20)$$

Fig. 8 depicts the simulation results of each method applied to the velocity control system for the 2-mass system. For each sub-figure, the first part <poles/zeros> shows the allocated poles and zeros in the complex plane, the second part <bode plot> shows the frequency response of the designed control system for the motor side and link side, the third one <step response> and finally the forth one <ramp response> show the step and ramp response of the system respectively.



a) Simulation results of method 1



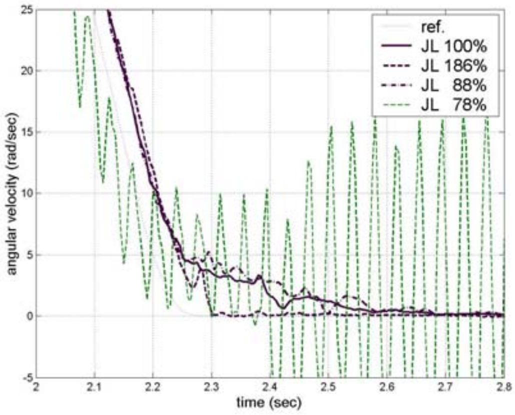
b) Simulation results of method 2(proposed)

Fig. 8 Simulation on 2-mass system with scheduling laws

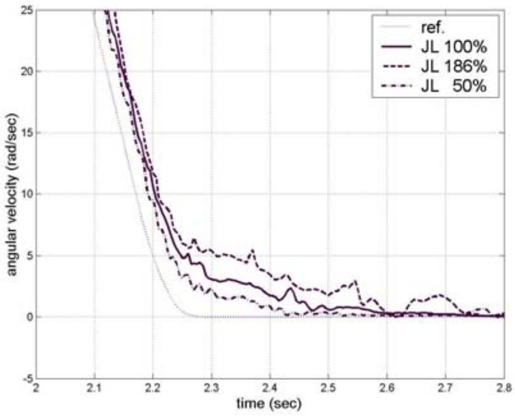
The pole-zero map shows that the proposed method more broadly varies the closed loop pole locations. Bode plot shows uniform gain at the peak frequency. And the step responses have uniform overshoot characteristics. From these results, we can say that the proposed method possesses more uniform performance. But the proposed method shows wider range of time constant along the load inertia.

4.2 Experimental results

Fig. 9 shows the experimental results of verification. The experiment was fulfilled using the HILS system constructed by Matlab and dSPACE equipments, and small size articulated robot handling 3kg with 4 axes as shown in Fig. 1 [4]. For some poses of the robot, the effects of the two gain scheduling methods are compared. For Method 1, the robot moves fast for heavy load, moves slow for light load and even becomes unstable for smaller than 80% of the reference load. On the other hand, the proposed method, method 2, works effectively. In spite of the varying load inertia, it shows uniform overshoot characteristics, and moves fast for light load and slow for heavy load. And it works well even for quite small value of 50% of reference load.



a) experimental results of method 1



b) experimental results of method 2(proposed)
Fig. 9 Experiments on real robot manipulator

5. CONCLUSION

In this study we presented a vibration controller for an industrial robot that has flexible joints. And we proposed a control gain adjustment rule considering time-varying nature of load inertia. The simulation result showed uniform properties in overshoot in spite of the variation of load. The experimental results also showed uniform properties in performance such as overshoot in spite of the variation of load. For the future work, it is needed to develop methods to maintain the time constant in addition to the maximum overshoot. And for the further future, it is needed to make the system keep as many as possible performance indexes uniform. The result of this study can be applied to the appropriate gain selection for industrial robots and many other mechatronic devices that have the 2-mass system with varying load side inertia.

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