Sequential Fault Detection and Isolation for Redundant Inertial Sensor Systems with Uncertain Factors†

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Abstract: We consider some problems of the Modified SPRT(Sequential Probability Ratio Test) method for fault detection and isolation of inertial redundant sensor systems and propose an Advanced SPRT method to solve the problems of the Modified SPRT method. One problem of the Modified SPRT method to apply to inertial sensor system comes from the effect of inertial sensor errors such as misalignment, scale factor error and sensor bias in the parity vector, which make the Modified SPRT method hard to be applicable. The other problem is due to the correlation of parity vector components which may induce false alarm. We use a two-stage Kalman filter to remove effects of the inertial sensor errors and propose the modified parity vector and the controlled parity vector which removes the effect of correlation of parity vector components. The Advanced SPRT method is derived form the modified parity vector and the controlled parity vector. Some simulation results are presented to show the usefulness of the Advanced SPRT method to redundant inertial sensor systems.

Keywords: fault detection and isolation, Modified SPRT, Advanced SPRT, two-stage Kalman filter, modified parity vector, controlled parity vector

1. INTRODUCTION

Redundancy has been used to improve reliability of systems, whether hardware redundancy or analytic redundancy depending on the characteristics of the systems. FDI(Fault Detection and Isolation) methods are essential to the systems which have redundancy to detect and isolate faults. We consider hardware redundancy for inertial sensor systems such as gyroscopes or accelerometers to provide improved performance and it is necessary to choose an appropriate FDI method for the inertial sensors.

Broadly, FDI method for hardware redundancy can be classified into two depending on whether we use a single measurement or a set of measurements. If we use a set of measurements, it is called a sequential FDI method, and if otherwise, a non-sequential FDI method. Non-sequential FDI method needs short time to detect faults but it cannot detect soft faults and may have much false alarms. On the other hand, sequential FDI method can reduce false alarms and detect soft faults but it takes a long time to detect faults.

The GLT(Generalized Likelihood Ratio Test)[3] and OPT(Optimal Parity Test)[4] are non-sequential FDI methods. The SPRT(Sequential Probability Ratio Test) method is a typical sequential FDI method which use sequentially independent parity components as observations of LLR(Log Likelihood Ratio) function. The Modified SPRT method[6] is proposed to solve the time delay problem.

Two problems must be solved to apply the Modified SPRT method to inertial sensor systems. First, the parity vectors of LLR function, which is a decision rule for the Modified SPRT method, must be sequentially independent to make the Modified SPRT method effective. But this condition cannot be satisfied because inertial sensors always have uncertain factors such as misalignment, scale factor error, and sensor bias, which make the parity vectors sequentially dependent. Therefore, the uncertain factors must be removed from parity vector. Second, a parity component of a faulty sensor must not affect other parity components. However in inertial sensor systems, a parity component of a faulty sensor affects other parity components of normal sensors because components of a parity vector are correlated each other and this correlation is seldom eliminated. Due to this problem, false alarm may occur for a normal sensor. To avoid the false alarm, parity vector must be adjusted as the correlation effect to be small.

In this paper, we propose an Advanced SPRT method for inertial sensor systems which contain uncertainty and solve the two problems above. To remove the uncertainty factors from parity vector, we use two-stage Kalman Filter(KF) and propose the modified parity vector. To reduce the effect of correlation among parity components, we propose the controlled parity vector.

In Section 2 and 3, the Modified SPRT method is introduced and the problems of this paper are formulated. In Section 4, the modified parity vector for uncertainty factors and the controlled parity vector for correlation of parity components are given and the Advanced SPRT method is proposed. Simulation results and conclusions are given in Section 5 and 6 respectively.

2. PRELIMINARIES

A typical measurement equation for redundant inertial sensors is represented as follows.

$$m(t) = H_n \omega(t) + f(t) + v(t) \tag{1}$$

where $m(t) = [m_1(t), \cdots, m_m(t)]^T \in R^m$ is the inertial sensor measurement, H_n is the nominal geometry matrix which depends on sensor configuration, $\omega(t) \in R^3$ is an input vector of gyroscopes or accelerometers, $f(t) \in R^m$ is the fault vector, and $v(t)$ is a measurement noise vector which is assumed to

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be Gaussian with zero mean and covariance matrix $\sigma^2 I_{m\times m}$.
The parity vector is obtained using a matrix V which satisfies $VH_n = 0$ as follows :

$$p(t) = Vm(t) = Vf(t) + Vv(t) \tag{2}$$

where $p(t) = [p_1(t), \cdots, p_m(t)]^T \in R^m$ is the parity vector. The parity vector is independent on the input vector, but sensitive to faults. Each component of the parity vector takes charge of the corresponding sensor fault.
Decision rule of the SPRT method for FDI is a LLR (log likelihood ratio) function :

$$\lambda_i(t_k) = \ln \frac{p(p_i(t_1), \cdots, p_i(t_k) | H_1)}{p(p_i(t_1), \cdots, p_i(t_k) | H_0)} \quad (i = 1, \cdots, m) \tag{3}$$

where $\lambda(t) = [\lambda_1(t), \cdots, \lambda_m(t)]^T \in R^m$, H_0 is a normal mode, H_1 a degradation mode, and $p_i(t)$'s are i'th component of the parity vector. If the parity vectors $\{p(t)\}$ are sequence of independent Gaussian random variable, LLR functions $\lambda(t)$ can be generated recursively as (4) [6] :

$$\lambda_i(t_k) = \lambda_i(t_{k-1}) + \frac{a_1}{\sigma^2}\left(|p_i(t_k)| - \frac{1}{2}a_1 \right) \quad (i = 1, \cdots, m) \tag{4}$$

where a_1 is the mean value of $p_i(t)$ in H_1 mode. If the parity component $p_i(t)$ is in H_0 mode, its normal distribution is $N(0, \sigma)$ and in H_1 mode, its normal distribution is $N(a_1, \sigma)$.

It has some problems to detect a fault using $\lambda_i(t)$ of the SPRT method. In case that a fault occurred after long normal mode, the SPRT method will undergo time delay problem to detect a faulty sensor using $\lambda_i(t)$ defined by (4). To overcome this time delay problem, the Modified SPRT method uses a control signal $\xi(\lambda_i(t))$ to reduce the time delay. Define new variable $\lambda^*(t)$ where $\lambda^*(t) = [\lambda_1^*(t), \cdots, \lambda_m^*(t)]^T \in R^m$:

$$\lambda_i^*(t_k) = \lambda_i(t_k) + \xi(\lambda_i(t_k)) \quad (i = 1, \cdots, m) \tag{5}$$

where

$$\xi(\lambda_i(t_k)) = \begin{cases} -\lambda_i(t_k), & \lambda_i(t_k) < 0 \\ 0, & \lambda_i(t_k) \geq 0 \end{cases} \tag{6}$$

The control signal $\xi(\lambda_i(t))$ makes LLR function $(\lambda_i(t))$ reset to initial condition whenever $\lambda_i(t)$ is negative. This means that detection of H_1 mode is more important than detection of H_0 mode.

The decision boundary of H_1 mode must be adjusted because $\lambda_i(t)$ is transformed into $\lambda_i^*(t)$ by control signal $\xi(\lambda_i(t))$. The new decision boundary is represented as (7).

$$B = \ln\left(\frac{a_1^2}{2\sigma^2} \cdot T \right) \tag{7}$$

where T is a guaranteed time that there is no false alarm occurred.

3. PROBLEM DEFINITIONS

A. The Effect of Inertial Sensor System Uncertainty

The Modified SPRT method uses sequential parity vector. This makes it possible to detect soft faults and to reduce false alarms. To use this method, the condition has to be satisfied that the parity vectors $\{p(t)\}$ are sequentially independent. But the condition is not satisfied in inertial sensor systems because there are always inertial sensor errors. The effect of inertial sensor errors such as misalignment, scale factor, and bias keeps parity vectors from being sequentially independent and (3) from being transferred to (4). These make the Modified SPRT method to be impossible. Therefore, we must eliminate the effect of inertial sensor errors from parity vector. Consider the measurement equation as follows.

$$m(t) = H\omega(t) + b_s + f(t) + v(t) \tag{8}$$

In this paper, we consider the error factors such as misalignment, scale factor error and sensor bias.
The measurement equation with error factors can be rewritten as (9)

$$\begin{aligned} m(t) &= (1+\varepsilon_{SF})((H_n + H_m)\omega(t) + b_s) + f(t) + v(t) \\ &= (1+\varepsilon_{SF})(H_n + H_m)\omega(t) + (1+\varepsilon_{SF})b_s + f(t) + v(t) \end{aligned} \tag{9}$$

where H_m is the misalignment matrix. b_s is the sensor bias vector, and ε_{SF} is the scale factor error.
The parity equation for FDI is represented as follows.

$$\begin{aligned} p(t) &= Vm(t) \\ &= (1+\varepsilon_{SF})VH_m\omega(t) + (1+\varepsilon_{SF})Vb_s + Vf(t) + Vv(t) \end{aligned} \tag{10}$$

In this paper, $(1+\varepsilon_{SF})H_m$ is taken as a misalignment term and $(1+\varepsilon_{SF})b_s$ is taken as a sensor bias term for convenience.
From (10), we can know that if inertial sensor errors are presented, parity vector is sequentially dependent. As mentioned above, the parity vector can not be applied to (4) because the condition of the SPRT method is not satisfied. If parity vector (10) is used to LLR function (4) without correction, the effect of inertial sensor errors keeps on being accumulated and $\lambda^*(t)$ in (5) will be increased. This may cause a false alarm even though there is no fault actually.

In the Modified SPRT method, these inertial sensor error factors may cause to determine normal sensors as fault sensors, and thus normal sensors may be disconnected, which makes GN&C(Guidance, Navigation and Control) performance worse. Therefore error factors must be filtered out to improve reliability of the inertial navigation system.

B. The Effect of Correlation among Parity Components

In the Modified SPRT method, recursive LLR function $(\lambda_i(t))$ will be increased if $|p_i(t)|$ is greater than $0.5a_1$. In case that the mean value of $|p_i(t)|$ is greater than $0.5a_1$, the LLR value $(\lambda_i(t))$ will be increased continuously because LLR function is a recursive form and the LLR value will cross the decision boundary. Therefore the sensor whose LLR value is over decision boundary will be regarded as a fault sensor and will be isolated. This means that we can detect a fault that makes a parity component to be greater than $0.5a_1$ and we cannot detect a fault that makes a parity component to be smaller than $0.5a_1$ in the Modified SPRT method. There is a characteristic of recursive LLR function that judge whether a sensor is faulty or not.

$$\left[\begin{array}{lll} E\{|p_i(t)|\} > \frac{1}{2}a_1 & \Rightarrow & E\{\Delta\lambda_i(t)\} > 0 \\ E\{|p_i(t)|\} = \frac{1}{2}a_1 & \Rightarrow & E\{\Delta\lambda_i(t)\} = 0 \\ E\{|p_i(t)|\} < \frac{1}{2}a_1 & \Rightarrow & E\{\Delta\lambda_i(t)\} < 0 \end{array} \right. \tag{11}$$

The sensitivity of fault detection is dependent upon a_1 which is decided by false alarm probability.

One problem of using the Modified SPRT method for redundant inertial sensor systems which have uncertainty is that a large fault can make a parity component of normal sensor to be greater than $0.5a_1$ and thus the normal sensor may be isolated.

This problem arises due to matrix V which is used to generate parity vector from measurement. Though matrix V makes parity vector to be independent from the input vector, it makes components of the parity vector correlated from each other. Therefore, a fault affects all of the parity components. In the presence of fault, not only the parity component of fault sensor but also the parity components of normal sensors have values which are proportional to fault magnitude due to effect of correlation among parity components. So, if the absolute value of normal sensor's parity component is greater than $0.5a_1$, the value of $\lambda_i^*(t)$ is increased and false isolation can be occurred.

In the Modified SPRT method, the effect of correlation among parity components causes false isolation. To avoid false isolation, the correlation among parity components must be eliminated for which we propose a method. But in INS, the correlation among parity components cannot be isolated. Therefore, to avoid false isolation in the Modified SPRT FDI method, the effect of correlation among parity components must be reduced.

4. ADVANCED SPRT METHOD

A. Two-stage Kalman Filter for Error Compensation

We use a two-stage Kalman Filter(KF) to compensate inertial sensor errors. The two-stage KF has some advantages in computation burden over the augmented state KF, and keeps the matrix condition number from being increased. The two-stage KF consists of two filters, one being a modified bias-free filter which estimates misalignment term taking into account bias term, and the other being a bias filter which estimates sensor bias term.

1. Measurement Equation

Measurement equation of two-stage KF, which is the parity equation (10), takes the form as follows :

$$p(t_k) = C(t_k)x(t_k) + Vb_s^* + Vf(t_k) + Vv(t_k) \tag{12}$$

where $C(t_k)x(t_k) = VH_m^*\omega(t_k)$, $b_s^* = (1+\varepsilon_{SF})b_s$

The state $x(t_k)$ is a vector composed of elements of VH_m^* such as

$$x(t) = [VH_{m(11)}^*, VH_{m(12)}^*, VH_{m(13)}^*, VH_{m(21)}^*, \cdots, VH_{m(n3)}^*]^T \tag{13}$$

Measurement matrix $C(t_k)$ can be expressed as (14) :

$$C(t_k) = \begin{bmatrix} \omega^T(t_k) & 0_{1\times3} & \cdots & 0_{1\times3} \\ 0_{1\times3} & \omega^T(t_k) & & 0_{1\times3} \\ \vdots & & \ddots & \vdots \\ 0_{1\times3} & 0_{1\times3} & \cdots & \omega^T(t_k) \end{bmatrix} \tag{14}$$

The element $\omega^T(t_k)$ in matrix $C(t_k)$ is an unknown value, and thus the least square estimate value $\hat{\omega}(t_k)$ is used instead of $\omega(t_k)$.

$$\hat{\omega}(t_k) = (H_n^T H_n)^{-1} H_n^T m(t_k) \tag{15}$$

Therefore $C(t_k)$ in (12) is replaced with $\hat{C}(t_k)$.
Suppose a fault exist as a bias form, then the measurement equation (12) can be simplified as (16)

$$p(t_k) = \hat{C}(t_k)x(t_k) + b(t_k) + Vv(t_k) \tag{16}$$

where $b(t_k) = Vb_{sf}(t_k) = Vb_s^* + Vf(t_k)$, $x(t_k) \in R^n$ is the state containing misalignment term and $b(t_k) \in R^m$ is the state containing bias term.

2. The Error Dynamics Model

The two-stage KF needs two error state models. We assume that the two errors can be modeled as a discrete-time Markov process. The error dynamic model is given as

$$x(t_{k+1}) = Ax(t_k) + w_x(t_k) \tag{17}$$

$$b(t_{k+1}) = b(t_k) + w_b(t_k) \tag{18}$$

where process noise $w_x(t_k)$ and $w_b(t_k)$ are zero-mean white Gaussian sequences and their covariances are $cov(w_x(t_k)) = Q_x(t_k)$ and $cov(w_r(t_k)) = Q_r(t_k)$. Matrix A is the state transition matrix for the misalignment term $(x(t_k))$ as follows

$$A = exp(-\Delta t/\tau)I_{n\times n} \tag{19}$$

where Δt is a sample time and τ is a time constant of the Markov process. The state $b(t_k)$ in (18) denotes a bias term of measurement and is assumed as a random walk.

3. Two-stage Kalman Filter

The structure of two-stage KF[7] is shown in Fig. 1.

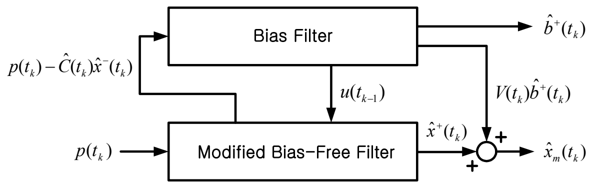

Fig. 1 Block diagram of the two-stage KF.

where $\hat{b}^+(t_k)$ is the bias filter output and the estimated value of bias term, $\hat{x}^+(t_k)$ is the output of a modified bias-free filter, and $\hat{x}_m(t_k)$ is the estimated value of misalignment term.

The two-stage KF consists of the bias filter and the modified bias-free filter. The estimated value of bias term can be directly derived from the bias filter and the estimated value of misalignment term can be derived from the bias filter and the modified bias-free filter.

To correctly estimate a misalignment term, the modified bias-free filter must compensate bias effect. $u(t_{k-1})$ and $V(t_k)$, which are the external information, are used to compensate bias effect. From those, this filter can estimate misalignment term which does not include bias term even in the case of using the parity vector with bias term.

The estimated values of the two-stage KF are completely same as those of the augmented state KF.

B. Modified Parity Vector

In the absence of inertial sensor fault, we can estimate the states of sensor bias term and misalignment term using the two-stage KF. Therefore the uncertainty factors can be removed from the original parity vector (10) by the estimated states. The compensated parity vector[8] is defined by

$$\begin{aligned} p^c(t_k) &= p(t_k) - \left(\hat{C}(t_k)\hat{x}_m(t_k) + \hat{b}^+(t_k)\right) \\ &= VH_m^*\omega(t_k) + Vb_s^*(t_k) + Vv(t_k) - \left(\hat{C}(t_k)\hat{x}_m(t_k) + \hat{b}^+(t_k)\right) \\ &\approx Vv(t_k) \end{aligned} \tag{20}$$

In case that there are no faults in inertial sensors, the compensated parity vector is a function of measurement noise term. At this time, the estimated value $\hat{b}^+(t_k)$ from bias filer becomes the sensor bias term. However, (20) has a serious problem to be used for FDI. In case that a fault occurs as a form of bias, the estimated value $\hat{b}^{+}(t_k)$ is the sum of the sensor bias term and the fault bias term. Therefore the fault bias term as well as the sensor bias term is removed from compensated parity vector $p^{c}(t_k)$. To use $p^{c}(t_k)$ for FDI, only sensor bias term should be removed and fault bias term should remain in $p^{c}(t_k)$ in the presence of fault. The compensated parity vector which is proposed by M.I.T. paper [8] is not suitable for FDI. So, the compensated parity vector must be modified as another form.

In this paper, we define the modified parity vector which removes the sensor bias only from compensated parity vector $p^{c}(t_k)$ in the presence of a fault bias of inertial sensor. Two conditions are assumed as follow.

ⓐ There is no fault at initial time.
ⓑ The sensor bias term is a constant.

Under the condition ⓐ, $\hat{b}^{+}(t_k)$ is the estimated sensor bias term and reaches a steady state for the initial time. When $\hat{b}^{+}(t_k)$ reaches a steady state, $\hat{b}^{+}(t_k)$ is defined as $\hat{b}_{fixed}$. Under the condition ⓑ, $\hat{b}_{fixed}$ is a vector whose element is a constant. Therefore, $\hat{b}_{fixed}$ will be sensor bias term.
We can define the modified parity vector as (21).

$$\begin{aligned} p^{c}(t_k) &= p(t_k) - (\hat{C}(t_k)\hat{x}_m(t_k) + \hat{b}_{fixed}) \\ &\approx Vf(t_k) + Vv(t_k) \end{aligned} \tag{21}$$

In the modified parity vector (21), the sensor bias term and the misalignment term are removed and the fault bias term remains.

C. Controlled Parity Vector

The modified parity vector (21) satisfies the condition that parity vector must be sequentially independent. Therefore we can use recursive LLR function (4) of the Modified SPRT method. Although the modified parity vector is almost same as the ideal parity vector (2), the modified parity vector can cause a false isolation in the Modified SPRT method because components of the modified parity vector are correlated. We must reduce the effect of correlation among components of the modified parity vector to avoid false isolation.

In this section, we propose the controlled parity vector to reduce the effect of correlation among parity components. The controlled parity vector is defined under the following assumption.

ⓐ Only single sensor fault can take place at a time.

Under condition ⓐ, controlled parity vector can be generated by adjusting modified parity vector. We can reduce the effect of correlation among parity components by using controlled parity vector as observation of LLR function.
We can represent controlled parity vector as follows.

$$p_i^{*}(t_k) = p_i^{c}(t_k) + \varphi(p_i^{c}(t_k)) \quad (i = 1, \cdots, \mathrm{m}) \tag{22}$$

where

$$j = \arg\max_i |\, p_i^{c}(t_k) \,| \tag{23}$$

$$\varphi(p_i^{c}(t_k)) = \begin{cases} 0 & , \quad i = j \\ 0.5a_1 - p_i^{c}(t_k) & , \quad i \neq j,\ p_i^{c}(t_k) > 0.5a_1 \\ 0 & , \quad i \neq j,\ p_i^{c}(t_k) \leq 0.5a_1 \end{cases} \tag{24}$$

If we use this controlled parity vector in the Modified SPRT method, the LLR value $\lambda_j^{*}(t)$ of a sensor which has maximum possibility for fault will be increased and the LLR value $\lambda_i^{*}(t)$ of other sensors which have little possibility will not be increased. In case that the value of faulty sensor's parity component is not the greatest value due to the effect of measurement noise at this sampling point, time delay problem for detection of fault sensor will not be occurred because LLR value $\lambda_i^{*}(t)$ of fault sensor is not decreased. By using controlled parity vector in the Modified SPRT method, the LLR value $\lambda_j^{*}(t)$ of faulty sensor will firstly reach a decision boundary and the fault sensor will be isolated. So, the possibility of false isolation will be decreased.

We call the Modified SPRT method using the controlled parity vector as the Advanced SPRT method. The block diagram of the Advanced SPRT method is represented in Fig. 2.

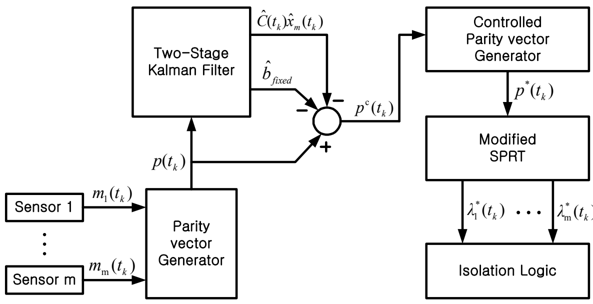

Fig. 2 Block diagram of Advanced SPRT method.

5. SIMULATIONS

We analyze the effect of main inertial sensor errors and the correlation effect on the Modified SPRT method, and then verify the performance of the Advanced SPRT method in these simulations.

We consider the six single degree of freedom(SDOF) gyroscopes as inertial sensors and the symmetric configuration of six gyroscopes mounted on the surface of a dodecahedron. This configuration has an excellent performance on GN&C and FDI compared with other configurations of the six SDOF gyroscopes.

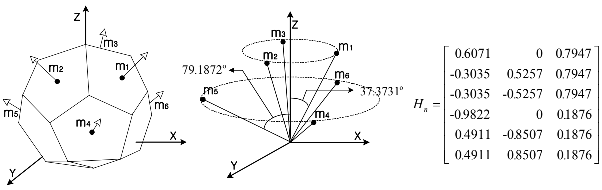

Fig. 3 Dodecahedron Configuration of 6 SDOF sensors.

Table 1 Gyroscope parameters.

Parameter	Value
Misalignment	5 x 10^{-4} rad
Sensor Bias	1 deg/h
Scale-Factor Error	5 ppm

Table 2 Dynamic Trajectory.

Time (sec)	Angular velocity
0 – 80	10sin(2 π t/20) (deg/sec) (단, t (sec))
80 – 130	0 (deg/sec)
130 – 180	30 (deg/sec)

Gyroscope parameter values are given Table 1 and the dynamic trajectory is defined in Table 2 and we set the sampling frequency 50Hz. In these simulations, we assume that false alarm probability is 1% and guaranteed time (T) is 3 years. From these parameters, we can set the threshold as 25.0813.

A. The effect of inertial sensor errors

To analysis the effect of inertial sensor errors, we perform the simulations in the absence of fault. In the absence of fault, the mean value of ideal parity vector has to be zero because ideal parity vector is a function of measurement noise.

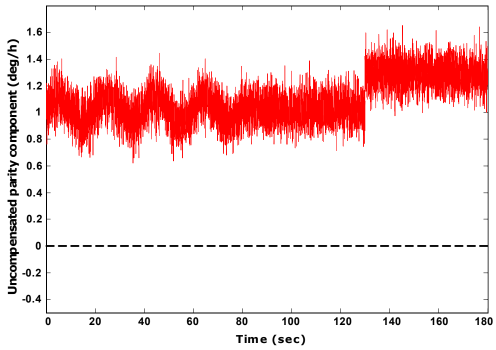

Fig. 4 Uncompensated parity component.

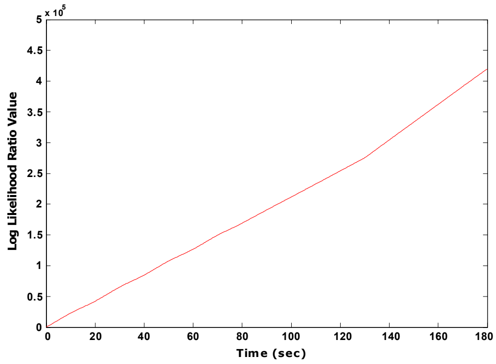

Fig. 5 LLR value using the uncompensated parity vector.

Fig. 4 shows that the mean value of parity vector is not zero and the uncertain factors influence on the parity components.

Fig. 5 shows that LLR value for the Modified SPRT method is increased due to the effect of uncompensated parity vector and this leads to a false isolation.

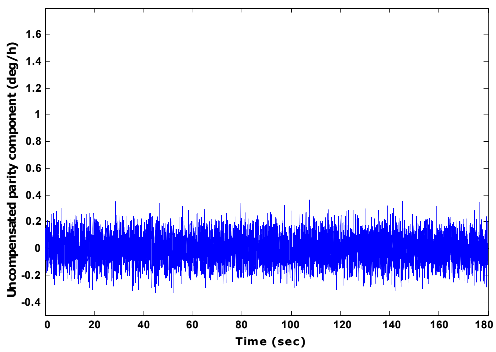

Fig. 6 Modified parity component.

Fig. 6 shows the value of the modified parity vector as in (21) for a sensor in the absence of fault. In the absence of fault, the parity vector is compensated by the two-stage KF because the mean value of the parity vector is zero regardless of the angular velocity. Therefore, the LLR function $\lambda_i^*(t)$ for the Modified SPRT method using the modified parity vector is not increased, which is shown in Fig. 7. Here, the dotted line presents threshold value.

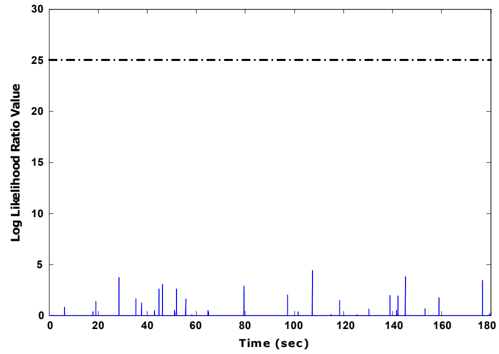

Fig. 7 LLR value using the modified parity vector.

Figs. 4~7 show that the modified parity vector has no uncertainty factors. Therefore, a false isolation due to the uncertainty factors can be avoided by using the modified parity vector.

B. The effect of correlation among parity components

To analysis the correlation effects of two FDI methods, we perform some simulations under degradation mode. The simulation condition is that sensors are operating in normal mode during 0~100 sec and one sensor has a fault bias whose magnitude is 10σ after 100 sec.

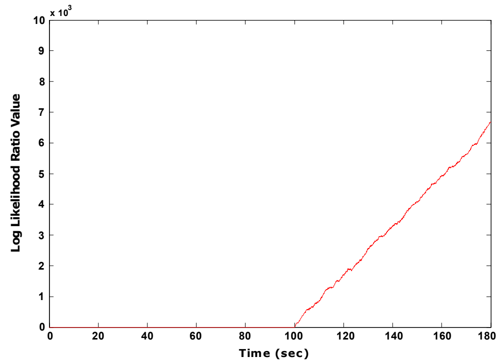

Fig. 8 LLR value using the uncontrolled parity vector.

Fig. 8 shows the LLR value $\lambda_j^*(t)$ of the Modified SPRT method for a normal sensor. In ideal case, the LLR value for normal sensor must not be increased but it is increased after 100 sec. This is due to the correlation effect among parity components and may induce a false isolation. According to this simulation result, the LLR value exceeds threshold within 0.4 sec after a fault occurrence.

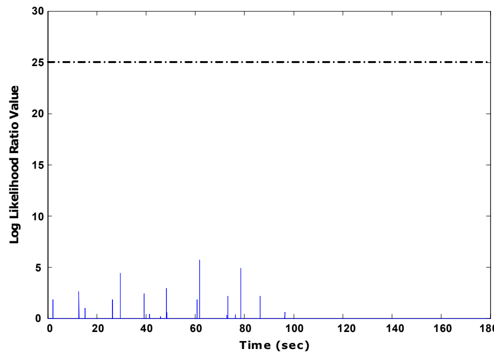

Fig. 9 LLR value using the controlled parity.

Fig. 9 shows the LLR value $\lambda_j^*(t)$ of the Advanced SPRT method for a normal sensor. As shown Fig. 9, the LLR value does not cross the decision boundary and it does not have the rising tendency. This means that the effect of correlation among parity components is reduced by controlled parity vector and we can avoid the false isolation.

C. Performance Evaluation

The performance of the Advanced SPRT method is evaluated by the comparison with the performance of the Modified SPRT method. In this paper, the 10,000 Monte Carlo simulation results are presented for application of the Modified SPRT method and the Advanced SPRT method in the absence of uncertain factors. Fig. 10 and 11 represent the correct isolation percentage and the false isolation percentage of the Advanced SPRT method and the Modified SPRT method according to F/N(Fault/Noise) ratio respectively.

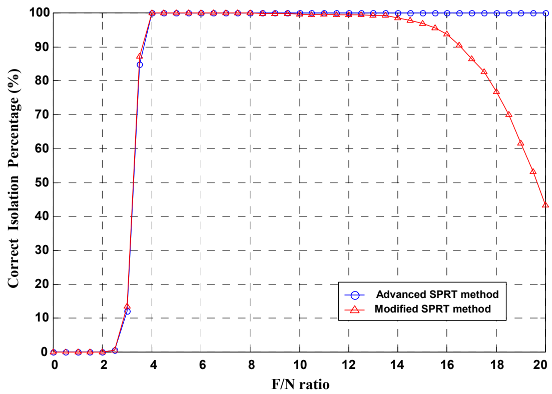

Fig. 10 Correct isolation percentages of the Advances SPRT method and the Modified SPRT method.

The correct isolation percentage of the Advanced SPRT method is similar to the correct isolation percentage of the Modified SPRT method at small F/N ratio. The correct isolation percentage f the Advanced SPRT method is nearly 100% after 4 F/N ratio but the correct isolation percentage of the Modified SPRT method is decreased according to an increase of F/N ratio.

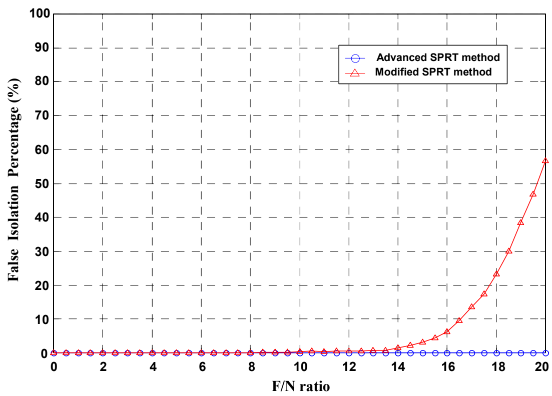

Fig. 11 False isolation percentages of the Advances SPRT method and the Modified SPRT method.

The false isolation does not happen at small F/N ratio. The false isolation percentage of the Modified SPRT method is increased according to an increase of F/N ratio. This leads the correct isolation percentage of the Modified SPRT method to be decreased. From Fig. 10 and 11, we can verify that the Advanced SPRT method has a good performance.

6. CONCLUSIONS

The Modified SPRT method can detect soft faults and reduce false alarms and detection time. However, we need to solve some problems to obtain performances in redundant inertial sensor systems. First, parity vectors of the Modified SPRT method must be independent each other. Unfortunately, this condition cannot be satisfied for inertial sensor systems due to uncertainty factors such as misalignment, scale factor error, and sensor bias. We use two-stage Kalman filter to eliminate the effect of uncertainty factors and thus obtain the modified parity vector without uncertainty factors. Second, components of the parity vector must be independent each other, which can not be satisfied for inertial sensor systems. So, we use the controlled parity vector which adds a control vector to modified parity vector to reduce the correlation effect of inertial sensors.

We compared the performance of the Advanced SPRT method using the controlled parity vector with that of the original Modified SPRT method by simulation and obtained good results to verify good performance of the Advanced SPRT method.

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