Fault Detection and Isolation using navigation performance-based Threshold for Redundant Inertial Sensors †

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Abstract: We consider fault detection and isolation (FDI) problem for inertial navigation systems (INS) which use redundant inertial sensors and propose an FDI method using average of multiple parity vectors which reduce false alarm and wrong isolation, and improve correct isolation. We suggest optimal isolation threshold based on navigation performance, and suggest optimal sample number to obtain short detection time and to enhance detectability of faults little larger than threshold.

Keywords: fault detection and isolation, inertial sensors, parity equation, optimal threshold

1. INTRODUCTION

Real systems are often subjected to faults and thus many researchers have studied on fault detection problem from many viewpoints. Fault may be defined as unexpected system change which degrades the system performance. We can classify the fault into three parts such as main system, actuator and sensor. So studies on fault detection and isolation (FDI) for these parts are necessary to improve the reliability of the systems.

To detect faults, parity method is used usually for both analytic and hardware redundancy. Inertial navigation systems (INS) use three accelerometers and gyroscopes to calculate navigation information such as position, velocity and attitude. To obtain reliability and to enhance navigation accuracy, INS can use redundant sensors. So a lot of studies on FDI for the redundant sensors have been performed so far. There are many papers for FDI such as SE[2], GLT[4] and OPT[5] for hardware redundancy. These methods consist of three procedures such as parity equation generation, fault detection and isolation. The parity equation is obtained from residual or using vectors of null space of measurement matrix. And fault detection is performed by comparing the parity value with threshold. These methods are adequate for large fault detection but not for small fault. The reason is that small threshold should be used for small fault detection and thus false alarm and wrong isolation probability increases because of effect of measurement noise.

In this paper, we propose a new FDI method for redundant inertial sensors using average of multiple parity vectors. For the proposed FDI method, false alarm, miss isolation probability and wrong isolation probability are decreased.

Also in this paper, we determine optimal threshold based on the analysis of navigation performance, not on false alarm and propose an optimal sample number through the analysis of the FDI performance with respect to sample number.

This paper is consisted of as follows. In section 2, averaged parity vector(APV) method using multiple parity vectors is proposed. In section 3, we analyze characteristics of the APM method. In section 4, we analyze navigation performance with respect to fault size and determine optimal threshold. In section 5, we determine optimal sample number. Lastly, we analyze the performance of the APV method through Monte-Carlo simulation and give conclusion in section 6and 7.

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Nomenclature

$m(t)$: n×1 measurement vector

$x(t) \in R^3$: triad-solution(acceleration or angular rate)

Th : threshold for fault isolation

F : the event of a failure (subscript i indicates i th sensor)

$H = [h_1 \ \cdots \ h_n]^T$: n×3 measurement matrix with rank(H)=3

h_i : 3×1 vector

$f(t)$: fault signal(scalar)

I_n : n×n identity matrix

$N(x, y)$: normal probability density function with mean x and standard deviation y

$\varepsilon(t) \sim N(0_n, \sigma I_n)$: normal distribution of measurement noise(white noise)

$p(t)$: (n-3)×1 parity vector

$V_{Fi} = [0 \ \cdots \ 0 \ 1 \ 0 \ \cdots \ 0]^T$: n×1 vector, i th element is one only.

2. AVERAGED PARITY VECTOR (APV) METHOD

In this section we propose the averaged parity vector (APV) method to detect and isolate faults. We assume that one fault occurs at a time. Suppose that i-th sensor has fault with magnitude f(t) among n sensors, then we have the following sensor measurement equation.

$$m(t) = H\ x(t) + V_{Fi}\, f(t) + \varepsilon(t) \tag{1}$$

Multiplying V on the left-hand side satisfying $VH = 0$ and $V \in R^{(n-3)\times n}$, we obtain the parity vector

$$p(t) = Vm(t) = v_i\, f(t) + V\varepsilon(t) \tag{2}$$

where $V = [v_1 \ \cdots \ v_n]$ and $VV_{Fi} = v_i$.

Parity vector $p(t)$ has probabilistic characteristics such as

$$E[\dot{p}(t)] = v_i f(t) \quad ,$$

$$E[(p(t) - v_i f(t))(p(t) - v_i f(t))^T] = \sigma^2 VV^T \qquad (3)$$

Parity vector $p(t)$ depends on noise as well as fault. If we want to detect small fault and thus decrease threshold, then the fault detection probability gets lower and the correct isolation probability also decreases.

The FDI problem can be stated as follows:

Problem Definition

Consider a measurement equation for redundant sensors including fault as (1). Find a FDI method which has as much correct isolation probability as users want and has short detection time even though the fault is small.

To obtain the performance described above, we use multiple measurements.

Suppose that we use q samples of measurements at $t_{k+1}, \ldots, t_{k+q}$, then the j-th sensor component of averaged parity vector can be described as

$$\hat{f}_j(t_{k+q}) = \frac{v_j^T}{q\|v_j\|^2}(p(t_{k+1}) + p(t_{k+2}) + \cdots + p(t_{k+q})) \text{ ,(j=1,…,n)} \qquad (4)$$

which is the projection of averaged parity vector onto vector v_i, i-th column vector of matrix V.

Eq.(4) can be converted to the recursive form as follows.

$$\hat{f}_j(t_{k+q}) = \hat{f}_j(t_{k+q-1}) + \frac{v_j^T}{q\|v_j\|^2}(p(t_{k+q}) - p(t_k)) \text{ , (j=1,…,n)} \qquad (5)$$

The fault detection and isolation method we propose is described as follows.

APV Algorithm

(1) Compute parity vectors $p(t_k), p(t_{k+1}), \cdots, p(t_{k+q})$ at $t = t_k, t_{k+1}, \cdots, t_{k+q}$

where $p = Vm$, $VH = 0$, $VV^T = I$, $V \in R^{(n-3)\times n}$.

(2) Compute n test functions according to each sensor.

$$\hat{f}_j(t_{k+q}) = \hat{f}_j(t_{k+q-1}) + \frac{v_j^T}{q\|v_j\|^2}(p(t_{k+q}) - p(t_k)) \text{ , (j=1,…,n)}$$

(3) Find the sensor index for which the test function has maximum absolute value among n values.

$$r = \arg\max_j \left|\hat{f}_j(t_{k+q})\right| \text{ , (j=1,…,n)}$$

(4) Compare the maximum value of the test function found in step (3) with the threshold Th. If $|\hat{f}_r(t_{k+q})| > Th$, then fault occurs at r-th sensor. Otherwise, set $t = t_{k+q+1}$ and go to procedure 1.

3. CHARACTERISTICS OF THE AVERAGED PARITY VECTOR METHOD

In this section we consider some characteristics of APV method such as false alarm, correct isolation probability (or miss isolation probability), wrong isolation probability, and detection time.

Let $\hat{f}_j(t_k)$ be the projection of averaged parity vector onto vector v_j described as

$$\hat{f}_j(t_k) \equiv \frac{v_j^T}{k\|v_j\|^2}(p(t_1) + p(t_2) + \cdots + p(t_k)) \text{ , (j=1,…,n)} \qquad (6)$$

$$= \frac{v_j^T v_i}{k\|v_j\|^2}\{f(t_1) + f(t_2) + \cdots + f(t_k)\}$$

$$+ \frac{v_j^T V}{k\|v_j\|^2}\{\varepsilon(t_1) + \varepsilon(t_2) + \cdots + \varepsilon(t_k)\}$$

Vector $\hat{f}_j(t_k)$ has the probabilistic characteristics as follows

$$\hat{f}_j(t_k) \sim N\left(\frac{v_j^T v_i}{k\|v_j\|^2}\{f(t_1) + f(t_2) + \cdots + f(t_k)\},\ \frac{\sigma}{\sqrt{k}\|v_j\|}\right) \qquad (7)$$

The parameters stated above are defined as follows.

Case 1: When there is no fault ($f(t) = 0$)

False alarm ($\alpha(t_k)$): $\alpha(t_k) = P_{H_0}(\left|\hat{f}_i(t_k)\right| > Th)$

Case 2: When a fault occurs ($f(t) \neq 0$)

We assume that a fault occurs at i-th sensor.

Miss isolation probability($\beta(t_k)$):

$$\beta(t_k) = P_{H_1}(\left|\hat{f}_i(t_k)\right| \le Th)$$

Correct isolation probability($1-\beta(t_k)$):

$$1-\beta(t_k) = P_{H_1}(\left|\hat{f}_i(t_k)\right| > Th)$$

Wrong isolation probability($\gamma(t_k)$):

$$\gamma(t_k) = P_{H_1}(\left|\hat{f}_r(t_k)\right| > Th \text{ and } r = \arg\max_j \left|\hat{f}_j(t_k)\right|, \text{and } r \neq i)$$

Detection time(t_D): Refer to definition 1 in section 5.

H_0 denotes no-fault hypothesis and H_1 fault occurrence hypothesis.

Let's discuss some characteristics of APV method.

Convergence

Suppose the sensor measurement has constant fault $f(t) = b$ and we use sample from t_1 to t_k, then

$$\lim_{k\to\infty} E[(\hat{f}_j(t_k) - \frac{v_j^T v_i}{\|v_j\|^2}b)(\hat{f}_j(t_k) - \frac{v_j^T v_i}{\|v_j\|^2}b)^T] = 0 \qquad ,$$

which means that $\hat{f}_j(t_k)$ converges to $\frac{v_j^T v_i}{\|v_j\|^2}b$ in the sense of mean square convergence. However, we can not use infinity number of measurements because of detection time. So we need to determine optimal number of samples.

False Alarm

If there is no fault, then probability distribution of $\hat{f}_j(t_k)$ is as follows.

$$\hat{f}_j(t_k) \sim N(0,\ \frac{\sigma}{\sqrt{k}\|v_j\|}) \qquad (8)$$

So the probability of false alarm $\alpha(t_k)$ at $t = t_k$ can be calculated as follows.

$$\alpha(t_k) = \frac{2\sqrt{k}\|v_j\|}{\sqrt{2\pi}\sigma}\int_{Th}^{\infty}\exp[-k\|v_j\|^2 \hat{f}_j(t_k)^2 \big/ 2\sigma^2]d\hat{f}_j(t_k) \quad (9)$$

From (9), we know that as the sample number k increases $\hat{f}_j(t_k)$ has less effect from the noise and thus the variance of $\hat{f}_j(t_k)$ decreases, which means that the probability of false alarm of multiple parity vector method is less than that of single parity vector method for the same threshold.

Miss Isolation Probability or Correct Isolation Probability

Suppose the sensor measurement has constant fault $f(t) = b$, then $\lim_{k\to\infty} \hat{f}_i(t_k) = b$ from the above stated convergence characteristic. If $|b| > Th$, then $|\hat{f}_i(t_k)| > Th$ as $k \to \infty$ and thus $\beta(t_k) = 0$.

Wrong Isolation Probability

Suppose that $\|v_1\| = \|v_2\| = \cdots = \|v_n\|$, then the inequality $\frac{|v_j^T v_i|}{\|v_j\|^2} < 1$ holds for $j \neq i$ and thus the following inequality holds from (7):

$$E[|\hat{f}_i(t_k)|] \geq E[|\hat{f}_j(t_k)|], \quad \forall\ j \quad (10)$$

We can not compare the magnitudes of $|\hat{f}_i(t_k)|$ and $|\hat{f}_j(t_k)|$ directly because of measurement noise. However according to (7) the effect of noise becomes less and $|\hat{f}_i(t_k)|$ may be biggest among n values of $|\hat{f}_j(t_k)|$.

So, if the sample number is large enough, then the wrong isolation probability decreases much.

Detecinon Time

Detection time is defined in Definition 1 in section 5. Detection time is not a parameter in single parity vector method since the method uses only current measurement. However, detection time is necessary in averaged parity vector method since $\hat{f}_i(t_k)$ depends on the number of measurements which contain fault signal among k samples. As k increases, detection time increases also. Thus we need to choose the optimal sample number for APV algorithm.

4. THRESHOLD DETERMINATION

In this section we propose the optimal threshold for fault isolation of inertial sensor systems based on navigation performance. It is well known that the more redundant sensors we use the less the estimation error covariance of triad solution($x(t)$) becomes. If there is a faulty sensor, the estimation error covariance will be increased. We can determine optimal threshold using the above properties.

From (1) we can obtain least square estimator $\hat{x}(t)$ for $x(t)$ as (11).

$$\hat{x}(t) = (H^T H)^{-1} H^T m(t) \quad (11)$$

We are supposed to analyze the estimation performance of (11) when a fault $f(t)$ occurs, and determine the threshold in this section. Suppose that fault $f(t)$ occurs at i-th sensor and consider two cases described below and analyze the performance of the two, one including the faulty sensor and the other excluding it.

Case 1: Calculate the navigation performance using n sensors including the faulty one

Case 2: Calculate the navigation performance using n-1 sensors except the faulty one

4.1 The case including the i-th faulty sensor

Suppose that $x(t)$, $f(t)$ and $\varepsilon(t)$ have no correlation and denote $\hat{x}_{+i}(t)$ be the least square estimator including i-th faulty sensor in (1). Then $\hat{x}_{+i}(t)$ can be described as follows.

$$\begin{aligned}\hat{x}_{+i}(t) &= (H^T H)^{-1} H^T m(t) \\ &= x(t) + (H^T H)^{-1} H^T (V_{Fi} f(t) + \varepsilon(t))\end{aligned} \quad (12)$$

The error covariance $C_{+i}(t)$ of the estimator $\hat{x}_{+i}(t)$ can be obtained as (13)

$$\begin{aligned}C_{+i}(t) &\equiv E[(\hat{x}_{+i}(t) - x(t))(\hat{x}_{+i}(t) - x(t))^T] \\ &= f(t)^2 (H^T H)^{-1} H^T V_{Fi} V_{Fi}^T H (H^T H)^{-1} + \sigma^2 (H^T H)^{-1}\end{aligned} \quad (13)$$

Since $H^T V_{Fi} = h_i$, error covariance $C_{+i}(t)$ can be described as follows.

$$C_{+i}(t) = f(t)^2 (H^T H)^{-1} h_i h_i^T (H^T H)^{-1} + \sigma^2 (H^T H)^{-1} \quad (14)$$

4-2. The case excluding the i-th faulty sensor

Define $\hat{x}_{-i}$ be the least square estimator of x using n-1 normal sensors with the faulty sensor excluded, then $\hat{x}_{-i}$ can be described as (15)

$$\begin{aligned}\hat{x}_{-i}(t) &= (H^T W_i H)^{-1} H^T W_i m(t) \\ &= x(t) + (H^T W_i H)^{-1} H^T W_i\ \varepsilon(t)\end{aligned} \quad (15)$$

where W_i is a n×n diagonal matrix with (i,i) component 0 and the other components 1.

In this case the error covariance $C_{-i}(t)$ of $\hat{x}_{-i}$ can be given as (16)

$$\begin{aligned}C_{-i}(t) &\equiv E[(\hat{x}_{-i}(t) - x(t))(\hat{x}_{-i}(t) - x(t))^T] \\ &= \sigma^2 (H^T W_i H)^{-1}\end{aligned} \quad (16)$$

The following theorem shows how to obtain the optimal threshold.

Theorem 1. For the two error covariances (14) and (16), the following two statements are equivalent.

i) $|f(t)| \leq \frac{\sigma}{\|v_i\|}$

ii) $C_{+i}(t) - C_{-i}(t) \leq 0$

where σ and v_i are standard deviation of sensor noise and i-th column of V matrix.

And $|f(t)| = \frac{\sigma}{\|v_i\|} \Leftrightarrow C_{+i}(t) = C_{-i}(t)$.

proof)

Notice that $H^T W_i H = H^T H - h_i h_i^T$. Then the following equality holds using matrix-inversion formula.

$$(H^T W_i H)^{-1} = (H^T H - h_i h_i^T)^{-1}$$
$$= (H^T H)^{-1} + (H^T H)^{-1} h_i (1 - h_i^T (H^T H)^{-1} h_i)^{-1} h_i^T (H^T H)^{-1}$$

Consider the (n-3)×n matrix V satisfying (17) (see [4] for algorithm)

$$VH = 0, \; VV^T = I_{n-3} \qquad (17)$$

For matrices H and V satisfying (17), equality $V^T V = I - H(H^T H)^{-1} H^T$ holds and thus $1 - h_i (H^T H)^{-1} h_i = \|v_i\|^2$ [4].

So (14) can be written as (18).

$$C_{-i}(t) = \sigma^2 (H^T H)^{-1} + \frac{\sigma^2}{\|v_i\|^2} (H^T H)^{-1} h_i h_i^T (H^T H)^{-1} \qquad (18)$$

The difference of the two covariances (14) and (16)can be calculated as follows.

$$C_{+i}(t) - C_{-i}(t) = (f(t)^2 - \frac{\sigma^2}{\|v_i\|^2})(H^T H)^{-1} h_i h_i^T (H^T H)^{-1} \qquad (19)$$

Since $(H^T H)^{-1} h_i h_i^T (H^T H)^{-1} \geq 0$ we can obtain the equivalence of two statements. $C_{+i}(t) - C_{-i}(t) \leq 0$ $\Leftrightarrow f(t)^2 - \frac{\sigma^2}{\|v_i\|^2} \leq 0$.

Also $|f(t)| = \frac{\sigma}{\|v_i\|}$ $\Leftrightarrow$ $C_{+i}(t) = C_{-i}(t)$ can be obtained easily from (19)■

Theorem 1 means that if the magnitude of the fault $|f(t)|$ is less than or equal to $\frac{\sigma}{\|v_i\|}$, it is better to include the faulty sensor since the error covariance $C_{+i}(t)$ is less than or equal to $C_{-i}(t)$. So we can decompose faults into two categories as follows:

If $|f(t)| < \frac{\sigma}{\|v_i\|}$, then $f(t)$ can be said as a tolerable fault.

If $|f(t)| \geq \frac{\sigma}{\|v_i\|}$, then $f(t)$ can be said as a non-tolerable fault.

Remark 1. According to Theorem 1 we can say that tolerable faults are better not to be isolated and non-tolerable faults should be isolated. So $\frac{\sigma}{\|v_i\|}$ can be used as the threshold for isolation of faulty sensors. We choose $Th = \frac{\sigma}{\|v_i\|}$ be the optimal threshold for FDI in this paper.

5. OPTIMAL NUMBER OF MEASUREMENTS

In this paper we use the average of multiple parity vector. The more samples q we use, the less the false alarm and wrong isolation probability, and the longer the detection time.

Thus there should be some trade-off for determining sample number q considering performance parameters.

To make the analysis simple suppose that the magnitude of fault $f(t) = b$ and the fault of i-th sensor does not occur at the first q sampling period. Let l be the number of samples which include fault signal $f(t)$ among q parity vectors at $t = t_{k+q}$.

In this case the value of $\hat{f}_i(t_{k+l})$ will be given as (20).

$$\hat{f}_i(t_{k+l}) = \hat{f}_i(t_{k+l-1}) + \frac{1}{q\|v_i\|^2} v_i^T (p(t_{k+l}) - p(t_k)) \qquad (20)$$
$$= \frac{l}{q} b + \frac{v_i^T V}{q\|v_i\|^2} \{\varepsilon(t_{k+1}) + \varepsilon(t_{k+2}) + \cdots + \varepsilon(t_{k+l})\}$$

Figure 1 shows the probability density function of $\hat{f}_i(t_k)$ and $\hat{f}_i(t_{k+l})$ when fault $f(t) = b$ occurs from ($k+1$)-th sample. Dotted line shows the density function of no-fault case and solid line shows the density function in case of l faults among q measurements:

$$\hat{f}_i(t_k) \sim N(0, \frac{\sigma}{\sqrt{q}\|v_i\|}) \;, \; \hat{f}_i(t_{k+l}) \sim N(\frac{l}{q} b, \frac{\sigma}{\sqrt{q}\|v_i\|}) \qquad (21)$$

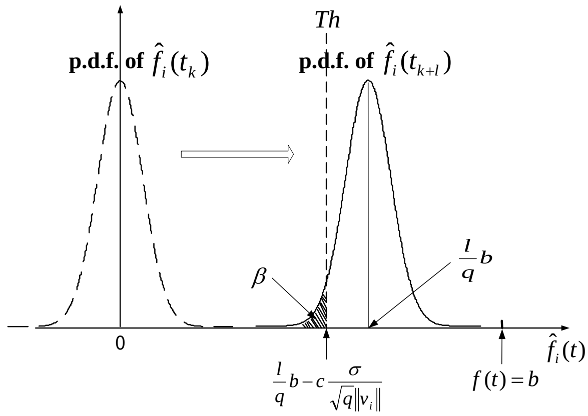


Fig.1 Probability density function of $\hat{f}_i(t_k)$ and $\hat{f}_i(t_{k+l})$

Since for the area where the value $\hat{f}_i(t_{k+l})$ is less than the threshold Th the sensor is not isolated, define this area as miss isolation probability(β). Then correct isolation probability becomes $1-\beta$ and the following equality holds.

$$\frac{lb}{q} - \frac{c\sigma}{\sqrt{q}\|v_i\|} = Th$$

where c is to be determined from miss isolation probability (β).

Definition 1: Suppose fault signal enters from $t = t_{k+1}$ and miss isolation probability(β) is given. When correct isolation probability is greater than $1-\beta$ at $t = t_{k+l}$, define this time as detection time(t_D). Then detection time can be described as follows

$$t_D = l \, t_s \quad (1 \leq l \leq q)$$

where t_s is the sampling interval.■

A FDI method which has good performance is the method which has as much correct isolation probability as users want and has short detection time. We can obtain performance index which satisfies these requirements as (22).

Optimization Problem

Minimize $J(l,b,q) = \dfrac{(1-s)(l-1)^2 + s(b-Th)^2}{(q-l)^2+1}$ (22)

subject to $g(l,b,q) = \dfrac{lb}{q} - \dfrac{c\sigma}{\sqrt{q}\|v_i\|} - Th$.

The minimization problem is to find appropriate values among available l , b , and q satisfying the miss isolation probability β such that for the fault as near as the threshold, detection time is short (i.e., $l \approx 1$) and the sample number is large. In (22), s ($0 < s < 1$) is the parameter that designers are supposed to determine. The closer s gets to 0, it means that we want to make detection time short, and the closer s gets to 1, it means that we want to make the detectable fault smaller.

We can use Lagrangian method to solve the minimization problem. However, it is hard to obtain an explicit solution.

For simplicity, let $l = q$ (i.e., $t_D = qt_s$), then the performance index and constraint in (22) can be described as in (23) and (24).

$$\bar{J}(b,q) = (1-s)\,(q-1)^2 + s(b-Th)^2 \quad (23)$$

$$\bar{g}(b,q) = b - \frac{c\sigma}{\|v_i\|\sqrt{q}} - Th \quad (24)$$

For (23),(24), we can obtain the following third order equation with respect to q by using Lagrange's multiplier method.

$$2(1-s)\|v_i\|^2 q^3 - 2(1-s)\|v_i\|^2 q^2 - sc^2\sigma^2 = 0 \quad (25)$$

Solving (25), we obtain the solution b and q of the minimization problem .

6. SIMULATIONS

In this section we perform Monte Carlo simulations to analyze the performance of the proposed FDI method. We use 6 identical sensors with dodecahedron configuration which is known as optimal[1,3]. We assume that the fault is bias and the measurement noise is white Gaussian with mean 0 and variance $\sigma = 1$.

In this case, measurement matrix H and V satisfying $VH = 0$ and $VV^T = I$ can be obtained as follows:

$$H = \begin{bmatrix} 0.5257 & -0.5257 & 0.8507 & 0.8507 & 0 & 0 \\ 0 & 0 & 0.5257 & -0.5257 & 0.8507 & 0.8507 \\ 0.8507 & 0.8507 & 0 & 0 & 0.5257 & -0.5257 \end{bmatrix}^T$$

$$V = \begin{bmatrix} 0.3717 & 0.3717 & 0 & 0 & -0.6015 & 0.6015 \\ -0.6015 & 0.6015 & 0.3717 & 0.3717 & 0 & 0 \\ 0 & 0 & -0.6015 & 0.6015 & 0.3717 & 0.3717 \end{bmatrix}$$

where $\|v_1\| = \|v_2\| = \cdots = \|v_6\| = 1/\sqrt{2}$.

The threshold stated in Theorem 1 is $Th = \sqrt{2}\sigma$ for the matrix V obtained above.

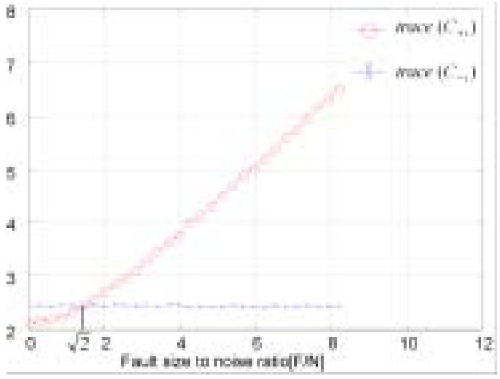
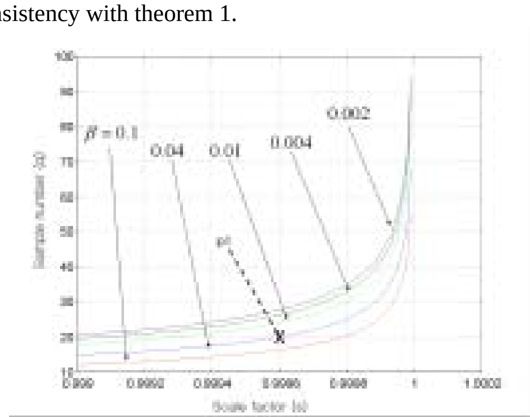
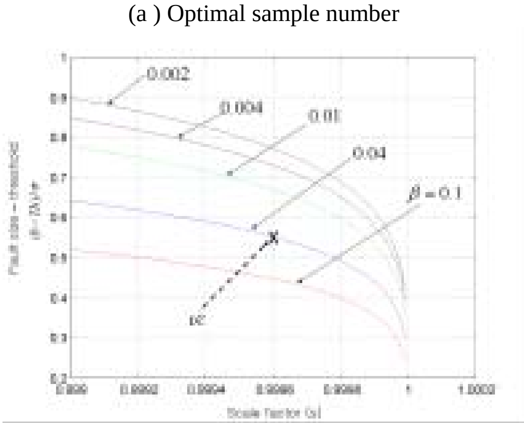


Fig. 2 $trace(C_{+i})$ and $trace(C_{-i})$ w.r.t fault magnitude

Figure 2 is the simulation result of Theorem 1 showing $trace(C_{+i})$ and $trace(C_{-i})$ where x-axis denotes fault size to noise ratio and y-axis denotes the magnitudes of $trace(C_{+i})$ and $trace(C_{-i})$. When fault signal $f(t)$ is greater than $\sqrt{2}\sigma$, the inequality $trace(C_{+i})$ > $trace(C_{-i})$ holds, which shows the consistency with theorem 1.



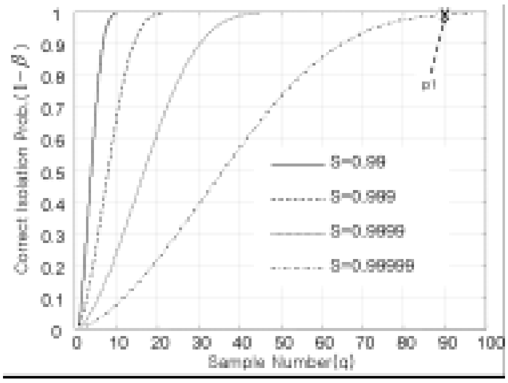
(a) Optimal sample number



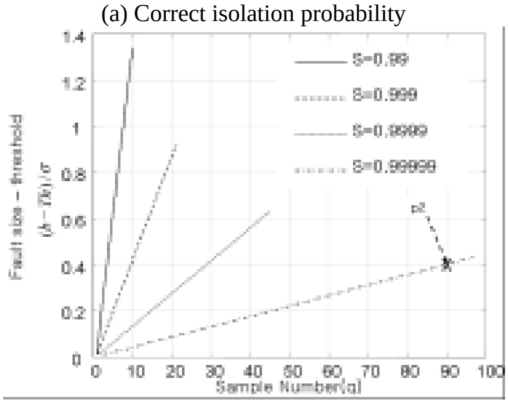
(b) Normalized difference of fault and threshold

Fig.3 Relations between Optimal Sample Number, Difference of Fault and Threshold and Scale Factor ranged over 0.999

Figure 3 shows optimal sample number and normalized difference of fault size and threshold with respect to scale factor s for various wrong isolation probability. For example, let's look at p1 and p2 in Fig. 3, which means that when scale factor s=0.9996 and wrong isolation probability we want is 0.04, the optimal sample number is 20 and we can isolate the fault with magnitude $b = Th + 0.57\sigma$. Figure 3 shows that when scale factor s in performance index is more than or equal to 0.999, q increases very fast.



(a) Correct isolation probability



(b) Normalized difference of fault and threshold

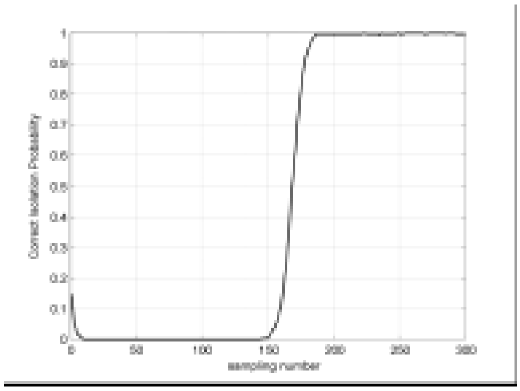
Fig.5 Relations between correct isolation probability, difference of fault and threshold and sample number for scale factor ranged over 0.99.

Figure 5 shows correct isolation probability and normalized difference of fault size and threshold with respect to optimal sample number for various scale factor S. For example, let's look at p1 and p2 in Fig. 5, which means that with scale factor s=0.99999 when we use 90 sample for fault signal of magnitude $b = Th + 0.57\sigma$, correct isolation probability becomes 0.993. Figure 5 shows that to obtain high correct isolation probability we need with a few samples for large fault, but we need many samples for small fault.

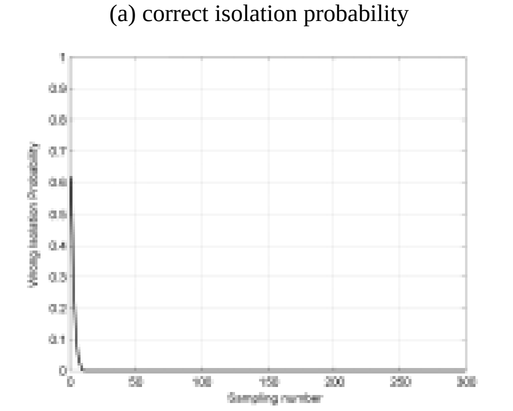
Figure 7 shows correct isolation probability and wrong isolation probability with respect to sample number as a result of 10,000 Monte Carlo simulation. We assume that fault enters from 101-th sample on and q =90, $Th = \sqrt{2}\sigma$. Figure 7-(a) shows that although fault starts at 101-th sample, correct isolation probability increases from 150-th sample and becomes 0.995 at 190-th sample, i.e., detection time is $t_D = 90t_s$. This result is consistent with the result of p1 and p2 points in Fig.5.

In Fig. 7-(b) wrong isolation exists for the first 10 samples despite of no faults. This happens because it is the beginning of the simulation and thus measurement noise has great effect. However, wrong isolation probability becomes around 0 after 10 samples and continues to be around 0 even after 101-th sample where the fault started.

This simulation shows that even for small fault, the proposed APVM has great performance on fault isolation in the cost of detection time.



(a) correct isolation probability



(b) wrong isolation probability

Fig.7 Performance of Averaged Parity Method

7. CONCLUSIONS

We consider a FDI problem for INS redundant inertial sensors and propose a FDI method using averaged parity vectors (APV). We also propose an optimal threshold based on navigation performance, not on false alarm, and propose an optimal sample number.

By using multiple parity vectors we can reduce false alarm and wrong isolation probability, and improve correct isolation probability. Optimal sample number can be determined to obtain as much correct isolation probability as we want and obtain short detection time for small fault just larger than threshold.

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