Path Tracking Control Using a Wavelet Neural Network for Mobile Robot with Extended Kalman Filter

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Abstract: In this paper, we present a wavelet neural network (WNN) approach to the solution of the path tracking problem for mobile robots that possess complexity, nonlinearity and noise. First, we discuss a WNN based control system where the control signals are directly obtained by minimizing the difference between the reference track and the pose of a mobile robot. This compact network structure is helpful to determine the number of hidden nodes and the initial value of weights. Then, the data with various noises provided by odometric and external sensors are here fused together by means of an Extended Kalman Filter (EKF) approach for the pose estimation problem of mobile robots. This control process is a dynamic on-line process that uses the wavelet neural network trained via the gradient-descent method with estimates from EKF. Finally, we verify the effectiveness and feasibility of the proposed control system through simulations.

Keywords: Path tracking control, Wavelet neural network, Extended Kalman Filter, Noises

1. INTRODUCTION

The localization and path tracking problems for mobile robots have been given great attention by automatic control researchers in the recent literatures [1]-[4]. Motion control of mobile robots is a typical nonlinear tracking control issue and has been discussed with different control schemes such as PI, GPC based EKF and so on. In order to provide the control schemes with some degree of robustness, Neural Networks (NN) based controllers have been also proposed in the past [3]-[5]. Neural networks have become an attractive tool to model the complex nonlinear systems due to its inherent ability to approximate arbitrary continuous functions. On the other hand, an amount of research has been done on applications of Wavelet Neural Networks (WNNs), which combine the capability of artificial neural networks in learning from processes and the capability of wavelet decomposition, for identification and control of dynamic systems [6]-[9]. The WNNs can further result in a convex cost index to which simple iterative solutions such as the gradient descent rules are justifiable and are not in danger of being trapped in local minima when choosing the orthogonal wavelets as the activation functions in the nodes. In this paper, we present a WNN approach to the solution of the tracking problem for mobile robots that possess complexity, nonlinearity and noise. This network structure is helpful to determine the number of the hidden nodes and the initial value of weights with compact structure. In our control method, the control signals are directly obtained by minimizing the difference between the reference track and the pose of a mobile robot that is controlled through a WNN. And for the absolute localization, the data with various noises provided by odometric and external sensors are here fused together by means of an Extended Kalman Filter approach [10][11] for the pose estimation problem of mobile robots. This control process is a dynamic on-line process that uses the wavelet neural network trained via the gradient-descent method with estimates from EKF. Through computer simulations, we demonstrate the effectiveness and feasibility of the proposed control method. In Section Ⅱ, the pose estimation based on EKF is described for the accurate path planning with various noises. WNN control structure and design method are described in Section Ⅲ, and computer simulations are given in Section IV. Finally Section Ⅴ presents a brief conclusion.

2. POSE EXTIMATION BASED ON EKF

2. 1 KINEMATIC MODEL OF MOBILE ROBT

The kinematic model of mobile robot used in this paper is composed of two driving wheels and four casters, and is fully described by a three dimensional vector of generalized coordinates X constituted by the coordinates (x, y) of the midpoint between the two driving wheels, and by the orientation angle θ with respect to a fixed frame as shown in Fig. 1. We have the equation for motion dynamics as follows:

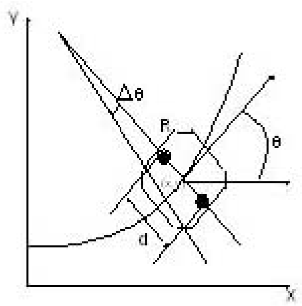

Fig. 1 Mobile robot and coordinate

$$\Delta\theta_k = \frac{R_{posk} - L_{posk}}{d}, \quad R_k = \frac{d}{2}\frac{R_{posk} - L_{posk}}{R_{posk} + L_{posk}}$$
$$u_k^T = (R_{posk}, L_{posk}), x_k^T = (x_k, y_k, \theta_k) \tag{1}$$

where, $u(k)$ is the control variable which is each displacement of right and left wheels.

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ \theta_{k+1} \end{bmatrix} = \begin{bmatrix} x_k - (\frac{1}{4d}) \times A \times B \times \sin\theta_k + (\frac{1}{2}) \times A \times \cos\theta_k \\ y_k + (\frac{1}{4d}) \times A \times B \times \cos\theta_k + (\frac{1}{2}) \times A \times \sin\theta_k \\ \theta_k + (\frac{1}{d}) \times B \end{bmatrix} \tag{2}$$

where, $A = R_{posk} + L_{posk}$, $B = R_{posk} - L_{posk}$

2. 2 POSE ESTIMATION OF MOBILE ROBOT

Typical internal sensors are optical incremental encoders which are fixed to the axis of the driving wheels or to the steering axis of the vehicles. At each sampling instant the position is estimated on the basis of the encoder increments along the sampling interval. A drawback of this method is that the errors of each measure are summed up. For absolute localization, a proper set of sensors measuring must be provided. In this paper, vision sensor and incremental encoders are generally fused for the localization problem. The robot coordinates in a global coordinate can be described by nonlinear function (2), and as follows:

$$\begin{aligned} x_k &= f(x_{k-1}, u_{k-1}, w_{k-1}), \quad & w_k &\sim N(0, Q) \\ z_k &= h(x_k, v_k), & v_k &\sim N(0, R) \end{aligned} \tag{3}$$

where, the random variables w_k and v_k represent the process and measurement noise, respectively. In this paper, we assumed that z_k is directly acquired from vision system with measurement noise. A discrete extended Kalman filter can then be designed as follows:

$$\begin{aligned} \hat{x}_k^- &= f(\hat{x}_{k-1}, u_k, 0) \\ P_k^- &= A_k P_{k-1} A_k^T + Q_{k-1} \\ K_k &= \hat{P}_k^- H_k^T (H_k \hat{P}_k^- H_k^T + R_k)^{-1} \\ \hat{x}_k &= \hat{x}_k^- + K_k (z_k - h(\hat{x}_k^-, 0)) \\ P_k &= (I - K_k H_k) P_k^- \end{aligned} \tag{4}$$

where,

$$A_k = \left.\frac{\partial f}{\partial x_k}\right|_{(x_k = \hat{x}_k, w_k = 0)}$$

$$= \left.\begin{bmatrix} 1 & 0 & (\frac{-1}{4d}) \times A \times B \times \cos\theta_k - (\frac{1}{2}) \times A \times \sin\theta_k \\ 0 & 1 & (\frac{-1}{4d}) \times A \times B \times \sin\theta_k + (\frac{1}{2}) \times A \times \cos\theta_k \\ 0 & 0 & 1 \end{bmatrix}\right|_{(x_k = \hat{x}_k, w_k = 0)}$$

$$H_k = \frac{\partial h}{\partial x_k} |_{(x_k = \hat{x}_k, v_k = 0)}$$

3. WAVELET NEURAL NETWORK CONTROLLER

3.1 WNN STRUCTURE

In our WNN structure, N_i input, multidimensional wavelets, and two-output structure are considered as shown in Fig. 2, where, N_i inputs are composed of errors and past errors between reference trajectory and controlled trajectory, and output R_{posk} and L_{posk} are control variables. Each control variable is as follows:

$$\begin{aligned} R_{posk} &= \Psi(\mathbf{E}, \gamma) = \sum_{j=1}^{N_w} c_{1j} \Phi_{1j}(\mathbf{E}) + a_{10} + \sum_{k=1}^{N_i} a_{1k} e_k \\ L_{posk} &= \Psi(\mathbf{E}, \gamma) = \sum_{j=1}^{N_w} c_{21j} \Phi_{2j}(\mathbf{E}) + a_{20} + \sum_{k=1}^{N_i} a_{2k} e_k \end{aligned} \tag{5}$$

where, $\Phi_{1,2j}(\mathbf{E}) = \prod_{k=1}^{N_i} \phi(z_{1,2jk}), \;\; with \;\; z_{1,2jk} = \dfrac{e_k - m_{1,2jk}}{d_{1,2jk}}$

$$\phi(z) = -z \exp\left(-\frac{1}{2} z^2\right) : mother\ wavelet$$

$$\gamma = \{a_{1,20}, a_{1,2k}, c_{1,2jk}, d_{1,2jk}\} : WNN\ parameters$$

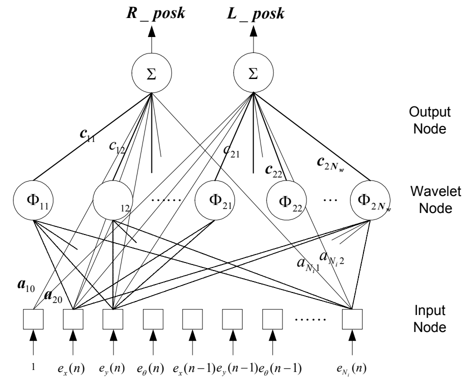

Fig. 2 Wavelet neural network structure

3.2 WNN CONTROLLER

Usually, a WNN structure is used for the modeling of the dynamic systems, in our control system, we design the direct adaptive control system using WNN structure. The purpose of our control system is to minimize the state errors $\mathbf{e}(e_x, e_y, e_\theta)$ between the reference trajectory and the estimated trajectory of a mobile robot. For this purpose, we train the WNN's parameters via the gradient-descent method with estimates from EKF. The overall control system is shown in Fig 3.

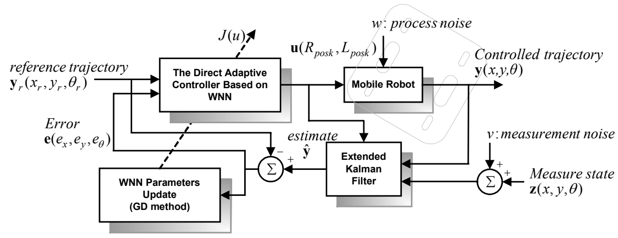

Fig. 3. Wavelet neural network based control system

A WNN controller calculates the control input $u(k) = [R_{posk}(k)\ L_{posk}(k)]'$ by training the inverse dynamics of plant iteratively. Because the WNN parameters cannot be updated directly through the variation rate $J(\gamma, y)$ in the gradient-descent method, we train the parameters of WNN through the transformation of the output error $\mathbf{e}(k)$ between the reference trajectory and the estimated trajectory of a mobile robot.

Training Procedure:

• Define the following cost function so as to train the WNN controller based on direct adaptive control technique.

$$C = \frac{1}{2}\{(x_r - \hat{x})^2 + (y_r - \hat{y})^2 + (\theta_r - \hat{\theta})^2\} \tag{6}$$

where, $e_x = x_r - \hat{x}$, $e_y = y_r - \hat{y}$, $e_\theta = \theta_r - \hat{\theta}$

• Calculate the partial derivative of the cost function with respect to the parameter set of a WNN controller, γ .

$$\begin{aligned} \frac{\partial C}{\partial \gamma} &= -e_x \frac{\partial \hat{x}}{\partial \gamma} - e_y \frac{\partial \hat{y}}{\partial \gamma} - e_\theta \frac{\partial \hat{\theta}}{\partial \gamma} = e_x \frac{\partial \hat{x}}{\partial u}\frac{\partial u}{\partial \gamma} - \cdots \\ &= \mathbf{E}_Y J(u) \frac{\partial u}{\partial \gamma} \end{aligned} \tag{7}$$

where, $J(u) = \frac{\partial \hat{\mathbf{Y}}}{\partial u}$ is the feedforward Jacobian of estimates plant and is as follows:

$$J(u) = \left. \begin{bmatrix} -(\frac{1}{2d}) \times R_{posk} \times \sin\theta_k + (\frac{1}{2}) \times \cos\theta_k & -(\frac{1}{2d}) \times L_{posk} \times \sin\theta_k + (\frac{1}{2}) \times \cos\theta_k \\ (\frac{1}{2d}) \times R_{posk} \times \cos\theta_k + (\frac{1}{2}) \times \sin\theta_k & -(\frac{1}{2d}) \times L_{posk} \times \cos\theta_k + (\frac{1}{2}) \times \sin\theta_k \\ (\frac{1}{d}) & -(\frac{1}{d}) \end{bmatrix} \right|_{x_k = \hat{x}_{k-1}} \tag{8}$$

The partial derivative $\frac{\partial u}{\partial \gamma}$ of the control input u with respect to the parameters of a WNN controller γ can be calculated by using the equations from Eqn. (9) to Eqn. (14).

• Update the WNN parameters. The minimization is performed by the following iterative gradient descent method.

$$\gamma(n+1) = \gamma(n) - \Delta\gamma(n) = \gamma(n) - \eta \frac{\partial C(n)}{\partial \gamma(n)} \tag{9}$$

where, η is the learning rate of a WNN.

From Eqns. (7) and (8), $\frac{\partial u}{\partial \gamma}$ is the gradient of the controller output, u , with respect to parameters set, γ , and the components of this vector are as the follows:

$$-\,parameter : \frac{\partial u(n)}{\partial a_{1,20}} = 1 \tag{10}$$

$$-\,direct\ connection\ parameters\ a_{12,k}\ : \frac{\partial u(n)}{\partial a_{1,2k}} = e_k \tag{11}$$

$$-\,weights\ c_{1,2j}\ : \frac{\partial u(n)}{\partial c_{1,2j}} = \Phi_{1,2j}(\mathbf{E}) \tag{12}$$

$$-\,translatio\,ns\ m_{1,2jk}\ : \frac{\partial u(n)}{\partial m_{1,2jk}} = -\frac{c_{1,2j}}{d_{1,2jk}} \frac{\partial \Phi_{1,2j}(\mathbf{E})}{\partial z_{1,2jk}} \tag{13}$$

$$where, \frac{\partial \Phi_{1,2j}(\mathbf{E})}{\partial z_{1,2jk}} = \phi(z_{1,2j1})\phi(z_{1,2j2})\cdots\dot{\phi}(z_{1,2jk})\cdots\phi(z_{1,2jN_i})$$

$$\dot{\phi}(z_{jk}) = \frac{d\phi(z_{jk})}{dz_{jk}} = (z_{jk}^2 - 1)\exp\left(-\frac{1}{2}z_{jk}^2\right)$$

$$-\,dilations\ d_{1,2jk}\ : \frac{\partial u(n)}{\partial d_{1,2jk}} = -\frac{c_{1,2j}}{d_{1,2jk}} z_{1,2jk} \frac{\partial \Phi_{1,2j}(\mathbf{E})}{\partial z_{1,2jk}} \tag{14}$$

4. SIMULATION RESULTS

In this section, we present simulation results to validate the control performance of proposed WNN controller with the estimator based on EKF for the tracking of a mobile robot. The control objective for our tracking system is to minimizing the error between the reference trajectory and the pose of a mobile robot that is controlled through a WNN controller with noises. The parameters used in this simulation and simulation results are shown in Table 1. This simulation considers the tracking of a trajectory generated by the following displacements:

$$\begin{aligned} R_{posk} &= 1m/s, L_{posk} = 1m/s \quad (0 < t \le 5) \\ R_{posk} &= 2m/s, L_{posk} = 1m/s \quad (5 < t \le 10) \\ R_{posk} &= 1m/s, L_{posk} = 2m/s \quad (10 < t \le 15) \\ R_{posk} &= 1m/s, L_{posk} = 1m/s \quad (15 < t \le 20) \end{aligned}$$

Table 1 Parameters and simulation results

Number of wavelet function		20
Sampling time		0.01
Learning rate		0.1
Departure posture vector		(5,5,0)
Process noise covariance		1.0e-2
Measurement noise covariance		1.0e-3
Control result(MSE)	**x**	0.2900cm
	y	0.6719cm
	θ	0.3936°

Figure 4 shows the tracking control results of WNN controller with EKF for a mobile robot. Also, Figure 5 shows the control errors for tracking of a mobile robot.

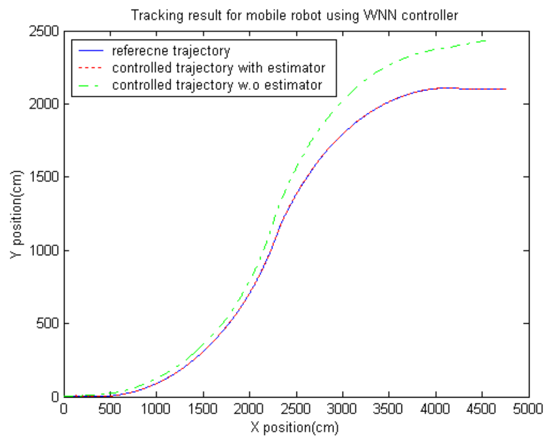

Fig. 4. Direct adaptive control based on WNN controller with EKF for tracking of a mobile robot

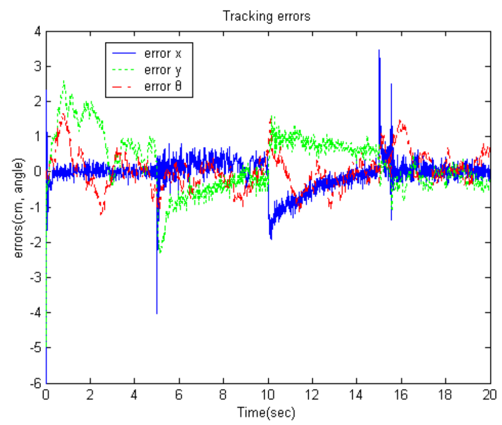

Fig. 5. Position and orientation errors

Fig 6 shows the control inputs of WNN and Fig 7 is feed forward Jacobian of a mobile robot system.

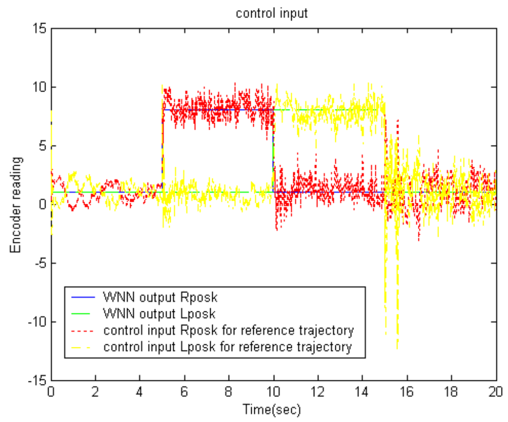

Fig. 6. Control inputs

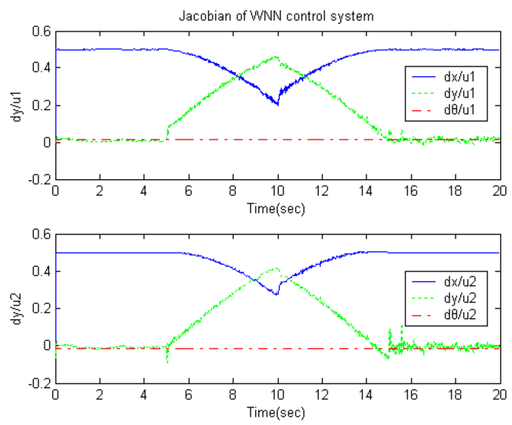

Fig. 7. Feed forward Jacobian

From these results, we confirm that our WNN controller with EKF works well although the tracking error is increased in case that a direction is changed. Also, we noticed that the learning rate and the number of wavelet node have an effect on the control performance. So, there must be the optimization method for these parameters.

5. CONCLUSION

In this paper, a WNN controller based on direct adaptive control scheme was presented for the solution of the tracking problem for mobile robots with various noises. In our control method, the control signals were directly obtained by minimizing the difference between the reference track and the pose of a mobile robot that was controlled through a WNN. And for the absolute localization, the data with various noises provided by odometric and external sensor were here fused together by means of an Extended Kalman Filter approach for the pose estimation problem. The control process was a dynamic online process that used the wavelet neural network trained by the gradient-descent method with estimates from EKF. In this work the WNN's parameters were randomly initialized. Through computer simulations, we verified the effectiveness and feasibility of our WNN control method with EKF although the control errors were increased at the changed directions.

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