A Self-Organizing Scheme for Swarm Systems

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Abstract: A control system design based on coupled nonlinear oscillators (CNOs) for a self-organized swarm system is presented. In this scheme, agents self-organize to flock and arrange group formations through attractive and repulsive forces among themselves using CNOs. Virtual agents are also used to create richer group formation patterns. The objective of the swarm control in this paper is to follow a moving target with a final group formation in the shortest possible time despite some obstacles. The simulation results have shown that the proposed scheme can effectively construct a self-organized multi-agent swarm system capable of group formation and group immigration despite the emergence of obstacles.

Keywords: Swarm system, cooperative behavior, coupled oscillators, self-organization.

1. INTRODUCTION

The field of cooperative mobile agents or robots offers an incredibly rich application domain, integrating a huge number of distinct fields from the social sciences and engineering. That so many theories have been brought to bear on "cooperative robotics" clearly illustrates the energy and the allure of the field. Yet, cooperative robotics is still an emerging field, open to direction and discovery. It is generally believed that proper organization of swarms of cooperating mobile agents provides significant benefits over single unit approaches for various missions. For specific tasks, cooperating agents do not need to be sophisticated or expensive to compete with their advanced independent counterparts. In addition, the integrated, multi-agent systems facilitate increased mobility, survivability, sensor coverage and information flows.

A swarm is a distributed system with a large number of autonomous robots [1]. In [2], many simple agents occupied one or two dimensional environments and were able to perform tasks such as pattern generation and self-organization. Self-organization in a swarm is the ability to distribute itself "optimally" for a given task, e.g., via geometric pattern formation or structural organization. Mechanisms for self-organization in swarms are studied in [1,3]. Swarm behavior as demonstrated by a flock of birds, a shoal of fishes, and a colony of insects provides a useful method for implementing a distributed network of mobile sensor platforms. Such mobile sensor swarm systems are useful for various search or surveillance activities. Swarm behavior ensures safe separation between swarm members while enforcing a certain level of cohesion. These two properties, when considered in the context of sensors and wireless communications, provide for low redundancy coverage and a robust and reliable system. That is why swarm robot systems are becoming more and more significant in industrial, commercial and scientific applications. In [4-6], the swarm system concept is utilized in flight, underwater vehicles and real robots, respectively. The number of agents currently being used in industrial projects is increasing fast. In addition, examples from possible applications include large-scale displays and distributed sensing [7].

Much research has been done to investigate the multi-agents system with different mediums: air, water and ground, and numerous interesting results have been achieved. A variety of control strategies have been employed, including decentralized control, and event-based optimal control [4,5].

However, the increased cost for each unit and the complication ac-companying the scaling of the number of group members are the two limiting factors to those system-theoretic efforts. On the other hand, in recent years much attention has been attracted on the behavior-based reactive systems. The behavior-based intelligences are motivated by natural species and can show great adaptability and robustness to the time-varying environment with relatively simple algorithms, as well as corresponding low computation costs during real-time operations [8]. Recent research results show that a variety of nonlinear systems can exhibit self-organization, reactive behavior to external stimulus and pattern formation [9,10]. More specifically, the coupled nonlinear oscillators have been extensively studied for their simplicity to implement and exhibit a wide variety of novel and complex spatiotemporal behaviors. In [11], it was reported that by using a nonlinear oscillator scheme, a sequence of basic behavior such as random walking, obstacle avoidance and light following was coordinated in a single robot to achieve more complicated behavior. However, these behavior-based computational organizations lack insightful comprehension to the problems and sometimes even exhibit unpredicted and undesirable performances. Training for selection of proper parameter values in different working environments [11] is time consuming. It seems that neither of those approaches can present a universal solution to the problem of designing a cooperative network of mo-bile agents. These schemes should be combined in a certain trade-off relationship for employment in different levels for different scenarios to create a future hierarchically architectural and multi-strategy adaptive intelligent system consisting of a swarm of in-homogeneous mobile agents.

In this paper, a self-organized control scheme based on the CNOs for a multi-agent swarm system is pro-posed and explored. This approach enables agents to follow a moving target, while moving in a group and avoiding the obstacles that may appear within the formation. While others have previously studied a target-following strategy [12,13], the purpose of this study is specifically to obtain the global behaviors such as group formation and group immigration by using simple local individual rules as well as target-following strategies.

2. SWARM MODEL DESCRIPTION

2.1. Environment model

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The agent model is based on the premise that in the near future technology will allow the production and deployment of large-scale masses of robots. These robots will be small in size and will likely possess only basic capabilities and mission specific sensors. Direct communication between agents may or may not exist. The environment model is very "object-oriented" in its approach to agent construction. Sensors and behaviors are encapsulated when possible. This approach allows individual components to be added and removed from the model as if the corresponding physical component were being added to or removed from an actual agent. This modular design permits rapid capability reconfiguration during concept exploration.

2.2. Agent model

The model of an autonomous mobile robot is constructed by building upon an autonomous agent object. The basic model of the agent can be thought of as simply a physical shell. In abstract programming terms it may also be thought of as an object with general capabilities. The basic agent possesses only locomotion as an innate capability. Neighbor position information may be used in a group formation manner.

2.3. Behavior architecture

Once a set of individual behaviors has been developed, a framework or architecture must be constructed to initiate behavioral responses and coordinate multiple behaviors. The behavior of the swarm system in the proposed algorithm is largely divided into three parts: group formation, group immigration and collision avoidance. Fig. 1 illustrates the swarm agent's behaviors in order of their priority where , and are designed in Section 3. The priority goes from collision avoidance (highest) to group formation (lowest).

2.4. Swarm classification

Observe that the taxonomy is being developed in the context of networked sensor systems and hence concentrates on communications aspects. The work here is concentrated with link establishment and duration for wireless communications. Results indicate that there are numerous types of swarm formations as shown in Figure 2. The vertical axis represents the scale of the framework whether the entire swarm formation is global or regional. The lateral axis represents the flexibility of formation in a swarm system.

The depth axis represents the degree of coupling between agents and is tightly or loosely coupled in the sense of sharing environment information through some form of communication. Several examples serve to illustrate the classification scheme. A flock of birds flying in V-formation is an example of a swarm in the [global, fixed, loose] class. A colony of ants foraging in widely scattered groups might be categorized as a [regional, flexible, tight] swarm.

In the proposed approach, group formation can be obtained by using simple local individual interactive rules. Thus, the framework is fully scalable for the distributed control that operates independently of the size of the group. Also, it shows the flexibility of the formation. As for the degree of coupling between agents, the tight or loose formation can be obtained in accordance with the coupled oscillator parameters and virtual agents in Section 3. Thus, the proposed approach is classified as a [global, flexible, tight or loose] swarm in the lined region of Figure 2.

3. THE PROPOSED MULTI-AGENT SWARM SYSTEM



Fig. 1. Behavior architecture.



Fig. 2. Swarm classification.

In this section, localized distributed controls are utilized throughout group behaviors such as forma-tion and migration. A self-organizing scheme based on the CNOs for group formation is proposed. Some virtual agents are also used to create patterns of richer group formation.

3.1 Coupled nonlinear oscillators for self-organization

Here, we refer to how cooperative behavior among different agents is actually motivated and achieved. The group formation behavior seeks to establish a specific relationship between adjacent neighbors.

The CNOs are proposed and explored for group formation among different agents. The coupled oscillator having simple interaction potential functions among the agents to maintain the group formation is modeled as

$$U_{i}(k) = \sum_{j \neq i} \begin{cases} c_{a} e^{-\|\mathbf{y}_{jm}(k) - \mathbf{y}_{jm}(k)\|^{2} / l_{a}} \\ -c_{r} e^{-\|\mathbf{y}_{im}(k) - \mathbf{y}_{jm}(k)\|^{2} / l_{r}} \end{cases},$$
(1)

where c_a , c_r , l_a , and l_r are the strengths and correlation distances of the attractive and repulsive forces, respectively. \mathbf{y}_{im} is the position of an *i* th agent, and \mathbf{y}_{jm} is the position of each agent except the *j* th agent. In this scheme mobile agents self-organize through attractive and repulsive forces among themselves. Fig. 3 shows the attractive and repulsive force between two agents where

$$r = \left\| \mathbf{y}_{im}(k) - \mathbf{y}_{jm}(k) \right\|, \quad r_c = \sqrt{\frac{l_r l_a}{l_a - l_r} \ln(\frac{c_r l_a}{c_a l_r})}, \text{ and } \nabla U_i(k)$$

and $U_i(k)$ refers to the force and potential, respectively.

Theorem 1: For $l_a > l_r$ and $\frac{c_r l_a}{c_a l_r} \frac{l_r l_a}{l_a - l_r} > 1$ in (1), each

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agent can maintain its distance from the other agents by the repulsive and attractive forces.

Proof: To begin, suppose the interaction of potential functions between two agents.

$$U_{i}(k) = c_{a}e^{(-\|\mathbf{y}_{im}(k) - \mathbf{y}_{in}(k)\|^{2}/l_{a})} - c_{r}e^{(-\|\mathbf{y}_{im}(k) - \mathbf{y}_{in}(k)\|^{2}/l_{r})},$$
(2)

where \mathbf{y}_{in} is the position of another agent except \mathbf{y}_{in} .

• the case of a repulsive force

In Fig. 4, we suppose that $\mathbf{y}_{im}(k)$ and $\mathbf{y}_{in}(k)$ are located in (x_1, y_1) of a top-left plane and (x_2, y_2) of (0,0), respectively. If we assume $r < r_c$, (x_1, y_1) of $\mathbf{y}_{im}(k)$ gives a repulsive force to $\nabla U_{ix}(k) < 0$ and $\nabla U_{iy}(k) > 0$. From (2) we obtain

$$\nabla U_{i}(k) = -2 \frac{c_{a}}{l_{a}} e^{(-\|\mathbf{y}_{im}(k) - \mathbf{y}_{in}(k)\|^{2} / l_{a})(\mathbf{y}_{im}(k) - \mathbf{y}_{in}(k))}$$

$$+ 2 \frac{c_{r}}{l_{r}} e^{(-\|\mathbf{y}_{im}(k) - \mathbf{y}_{in}(k)\|^{2} / l_{r})(\mathbf{y}_{im}(k) - \mathbf{y}_{in}(k))}$$
(3)

Considering x and y separately gives

$$\nabla U_{ix}(k) = -2 \frac{c_a}{l_a} e^{-(x_1 - x_2)^2 / l_a} (x_1 - x_2) + 2 \frac{c_r}{l_r} e^{-(x_1 - x_2)^2 / l_r} (x_1 - x_2) < 0$$
(4)
$$\nabla U_{iy}(k) = -2 \frac{c_a}{l_a} e^{-(y_1 - y_2)^2 / l_a} (y_1 - y_2) + 2 \frac{c_r}{l_r} e^{-(y_1 - y_2)^2 / l_r} (y_1 - y_2) > 0,$$

$$\begin{aligned} x_1 - x_2 &< 0 \quad \text{and} \quad y_1 - y_2 > 0 \quad \text{give} \\ \frac{c_r}{l_r} e^{-(x_1 - x_2)^2/l_r} &> \frac{c_a}{l_a} e^{-(x_1 - x_2)^2/l_a} \quad \text{and} \\ \frac{c_r}{l_r} e^{-(y_1 - y_2)^2/l_r} &> \frac{c_a}{l_a} e^{-(y_1 - y_2)^2/l_a} \quad \text{, respectively. Using} \end{aligned}$$

natural logarithm gives
$$\frac{l_r l_a}{l_a - l_r} \ln(\frac{c_r l_a}{c_a l_r}) > (x_1 - x_2)^2$$

 $\frac{l_r l_a}{l_a - l_r} \ln(\frac{c_r l_a}{c_a l_r}) > (y_1 - y_2)^2 , \text{ respectively. Combining both}$

equations gives

$$\sqrt{\frac{l_r l_a}{l_a - l_r} \ln(\frac{c_r l_a}{c_a l_r})} > \left\| \mathbf{y}_{im}(k) - \mathbf{y}_{in}(k) \right\|$$
(5)

i.e. $r_c > r$. Thus, when $r < r_c$, (x_1, y_1) of $\mathbf{y}_{im}(k)$ gets the repulsive force between two agents of (x_1, y_1) and (x_2, y_2) . Using the same procedure, we can prove theorem 1 whether (x_1, y_1) of $\mathbf{y}_{im}(k)$ is located on the top-right, bottom-left or bottom-right plane.

• the case of an attractive force





Fig. 4. The repulsive and attractive forces between two agents.

If we assume $r > r_c$, (x_1, y_1) of $\mathbf{y}_{im}(k)$ gets an attractive force to $\nabla U_{ix}(k) > 0$ and $\nabla U_{iy}(k) < 0$. Considering *x* and *y* separately gives

$$\nabla U_{ix}(k) = -2 \frac{c_a}{l_a} e^{-(x_1 - x_2)^2 / l_a} (x_1 - x_2) + 2 \frac{c_r}{l_r} e^{-(x_1 - x_2)^2 / l_r} (x_1 - x_2) > 0, \qquad (6)$$
$$\nabla U_{iy}(k) = -2 \frac{c_a}{l_a} e^{-(y_1 - y_2)^2 / l_a} (y_1 - y_2) + 2 \frac{c_r}{l_r} e^{-(y_1 - y_2)^2 / l_r} (y_1 - y_2) < 0$$

$$x_{1} - x_{2} < 0 \text{ and } y_{1} - y_{2} > 0 \text{ give}$$

$$\frac{c_{r}}{l_{r}} e^{-(x_{1} - x_{2})^{2}/l_{r}} < \frac{c_{a}}{l_{a}} e^{-(x_{1} - x_{2})^{2}/l_{a}} \text{ and}$$

$$\frac{c_{r}}{l_{r}} e^{-(y_{1} - y_{2})^{2}/l_{r}} < \frac{c_{a}}{l_{a}} e^{-(y_{1} - y_{2})^{2}/l_{a}}, \text{ respectively. Using}$$

natural logarithm gives $\frac{l_r l_a}{l_a - l_r} \ln(\frac{c_r l_a}{c_a l_r}) < (x_1 - x_2)^2$,

 $\frac{l_r l_a}{l_a - l_r} \ln(\frac{c_r l_a}{c_a l_r}) < (y_1 - y_2)^2$, respectively. Combining both equations gives

$$\sqrt{\frac{l_r l_a}{l_a - l_r} \ln(\frac{c_r l_a}{c_a l_r})} < \left\| \mathbf{y}_{im}(k) - \mathbf{y}_{in}(k) \right\|$$
(7)

i.e. $r_c < r$. Thus, when $r < r_c$, (x_1, y_1) of $\mathbf{y}_{im}(k)$ gets the attractive force between two agents of (x_1, y_1) and (x_2, y_2) . Using the same procedure, similarly we can prove whether (x_1, y_1) of $\mathbf{y}_{im}(k)$ is located on the top-right, bottom-left or bottom-right plane.

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For
$$l_a > l_r$$
 and $\left(\frac{c_r l_a}{c_a l_r}\right)^{\frac{l_r l_a}{l_a - l_r}} > 1$ in (1), each agent can keep

its distance from the other agents by the repulsive and attractive forces. If one agent is far from another agent on the basis of r_c , the agent is drawn to another agent by attractive force. On the other hand, if the distance between individual agents is too close on the basis of r_c , they keep a certain distance so as not to be too close by repulsive force. Thus, each agent possesses a flocking characteristic to keep the group formation while ensuring safe separation between swarm agents.

Because we have proven that an agent can maintain a certain distance from another agent by the repulsive and attractive forces, the position of an agent in theorem 1 is determined by adding all of the repulsive and attractive forces from the other agents.

From (1) we obtain

$$\nabla U_{i}(k) = \left\{ -2 \frac{c_{a}}{l_{a}} e^{\left(-\left\| \mathbf{y}_{im}(k) - \mathbf{y}_{n}(k) \right\|^{2} / l_{a} \right) \left(\mathbf{y}_{im}(k) - \mathbf{y}_{jm}(k) \right)} \right\}$$

$$+ 2 \frac{c_{r}}{l_{r}} e^{\left(-\left\| \mathbf{y}_{im}(k) - \mathbf{y}_{n}(k) \right\|^{2} / l_{r} \right) \left(\mathbf{y}_{im}(k) - \mathbf{y}_{in}(k) \right)} \right\}$$
(8)

 $\nabla U_i(k)$ is constructed by the influence of the repulsive and attractive forces from the other agents except *i* th agent. We suppose that $\mathbf{y}_{im}(k)$ and $\mathbf{y}_{jm}(k)$ are located in (x_1, y_1) and (x_j, y_j) , respectively. Considering *x* and *y* separately gives

$$\nabla U_{ix}(k) = \sum_{j \neq i} \begin{cases} -2 \frac{c_a}{l_a} e^{-(x_1 - x_j)^2 / l_a} (x_1 - x_j) \\ +2 \frac{c_r}{l_r} e^{-(x_1 - x_j)^2 / l_r} (x_1 - x_j) \end{cases},$$
(9)
$$\nabla U_{iy}(k) = \sum_{j \neq i} \begin{cases} -2 \frac{c_a}{l_a} e^{-(y_1 - y_j)^2 / l_a} (y_1 - y_j) \\ +2 \frac{c_r}{l_r} e^{-(y_1 - y_j)^2 / l_r} (y_1 - y_j) \end{cases}.$$

 $\nabla U_{ix} > 0$ and $\nabla U_{ix} < 0$ cause x_1 to move toward the right and left side on the basis of Cartesian coordinate, respectively. Also, $\nabla U_{iy} > 0$ and $\nabla U_{iy} < 0$ cause y_1 to move toward the top and bottom side on the basis of Cartesian coordinate, respectively.

Remark 1: The system capability of group formation can be fine-tuned to obtain optimal controller parameters (such as c_a , c_r , l_a , and l_r) by using genetic algorithms (GAs) or random neighborhood search (RNS) [16,17].

3.2. Natural patterns of group formation

The group formation could be quantified as the average distance between agents and group center, that is

$$p = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left\| \mathbf{y}_{im} - \mathbf{y}_{center} \right\|^2}$$
(10)

where N is the number of agents and \mathbf{y}_{center} is the center position of a group. Fig. 5 shows the two natural patterns formed by 10 agents after settling down given a set of oscillator parameters. In such a nonlinear system, we can see that there are two stable fixed points in the term of patterns of group formation despite different orientations. Therefore, each pattern is associated with the possibility with respect to random initial group positions.

3.3. Virtual agents

In actual cases, more patterns than the natural ones that the group can form are needed. By employing the virtual agents whose positions are pre-set, we can obtain richer patterns for group formation. Fig. 6 shows the natural pattern and the patterns obtained by using one, two and three virtual agents respectively for a seven-agent group.



Fig. 5. The natural patterns of 10 agents and their possibilities



Fig. 6. The natural pattern and some controlled patterns of 7 agents.

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3.4. Group immigrating and random re-direction

The PD control is used for keeping the group immigrating or following the target.

$$V_{i_immig}(k) = k_p(\mathbf{y}_{im}(k) - \mathbf{y}_{t \operatorname{arg} et}(k)) + k_d(\mathbf{y}_{im}(k-1) - \mathbf{y}_{t \operatorname{arg} et}(k-1))$$
(11)

where $\mathbf{y}_{t \arg et}$ is the moving target.

A random re-direction scheme is employed for the group to avoid obstacles during immigration. The velocity direction is re-adjusted randomly after meeting an obstacle, that is

$$V_{i_obs}(k) = \mathbf{y}_{im}(k) + \alpha \begin{bmatrix} -\cos\theta & \sin\theta\\ \sin\theta & \cos\theta \end{bmatrix} \mathbf{y}_{im}(k)$$
(12)

where $\theta \begin{cases} random[0,2\pi], & \text{if meet an obstacle} \\ 0, & \text{otherwise} \end{cases}$ and α is a

weighting value.

4. SIMULATION

As the well-known collective behavior of ants attacking a larger insect than themselves with cooperation, self-organized swarm agents are designed as agents that follow a moving target while keeping a formation. The task is due to motivations related to the biological inspirations behind cooperative systems.

Figs. 7 and 8 illustrate the different snap shots of an immigration process of ten agents for different selections of the moving target. Each agent is randomly initialized on the basis of (-5, -5) and the moving target is initialized on (3, 3). Design parameters are set to $l_a = 1$, $l_r = 1/30$, $c_a = 1$, $k_p = 2$ and $k_d = 2$. When counting an obstacle, the swarm agents in Figs. 7 and 8 spontaneously separate to avoid the blocking area. After the agents have immigrated to the moving target while keeping their formation, they finally formed a certain kind of group pattern in the neighborhood of the moving target.

5. CONCLSION

In this paper, the control design based on the coupled nonlinear oscillators for a swarm system is proposed and studied. Flexible group formation is obtained by using simple local individual interactive rules. Due to the simplicity of the local-range interactive rules, such a system can exhibit strong adaptivity to the time-varying environment including different obstacles and scalability to the variation of the agent number. In addition, by introducing the concept of virtual agents, richer group formation patterns than some natural patterns of the system have been obtained. The proposed swarm system design method provides a new scheme to design large scale cooperative robotics and vehicles. Although self-organization for the swarm system has been studied in the context of the two-dimensional environment, the method could also be extended to the more general three-dimensional space. With the proposed concept and system structure, additional scenarios including intersection of two group robots and immigration in more complicated environments can be further studied.



Fig. 7. Snap shots of group immigration $1(\bullet: \text{ agent}, \circ: \text{moving target}, \Box: \text{ obstacle}).$



Fig. 8. Snap shots of group immigration $2(\bullet: \text{ agent}, \circ: \text{ moving target}, \Box: \text{ obstacle}).$

REFERENCES

- K. Jin, P. Liang and G. Beni, "Stability of synchronized distributed control of discrete swarm structures," *IEEE Int. Conf. on Robotics and Automation*, pp. 1033-1038, 1994.
- [2] G. Beni, "The concept of cellular robotic system," *IEEE Int. Symp. on Intelligent Control*, pp. 57-62, 1998.
- [3] P. Liang and G. Beni, "Robotic Morphogenesis," *IEEE Int. Conf. on Robotics and Automation*, pp. 2175-2180, 1995.
- [4] C. R. Carignan and D. L. Akin, "Autonomous formation flight," *IEEE Control System Magazine*, vol. 20, no. 6, pp. 34-44, 2000.
- [5] D. J. Stilwell and B. E. Bishop, "Platoons of underwater vehicles," *IEEE Control System Magazine*, vol. 20, no. 6, pp. 45-52, 2000.
- [6] A. T. Hayes and P. D. Tabatabaei, "Self-organized flocking with agent failure: off-line optimization and

demonstration with real robots," *Proc. of the IEEE int. Conf. on Robotics & Automation*, pp. 3900-3905, 2002.

- [7] S. Hackwood and J. Wang, "The engineering cellular robotic systems," *IEEE Int. Symp. on Intelligent Control*, pp. 70-75, 1998.
- [8] G. T. Anderson and M. R. Clark, "Navigation of autonomous robots with an intelligent Oscillator Controller," *Proc. of the IEEE/ASME Int. Conf. on Advanced Intelligent Mechatronics*, pp. 1003-1007, 1999.
- [9] A. Mogilner and L. Edelstin-Keshet, "Spatio-angular order in population of self-aligning objects: formation of oriented patches," *Physica D*, vol. 89, pp. 346-367, 1996.
- [10] H. Levine, W. Rappel and Inon Cohen, "Self-organized in systems of self-propelled particles," *Physical Review E*, vol. 63, 017101, 2001.
- [11] M. R. Clark and G. T. Anderson, "Coupled Oscillator Control of Autonomous Mobile Robots," *Autonomous Robots*, no. 9, pp. 189-198, 2000.
- [12] L. E. Parker, "Cooperative robotics for multi-target observation," Intelligent Automation and Soft Computing, special issue on Robotics Research at Oak Ridge National Laboratory, vol. 5, no. 1, pp. 5-19, 1999.
- [13] Barry B. Werger and Maja J. Mataric, "Broadcast of local eligibility for multi-target observation," *Proc. on Distributed Autonomous Robotic Systems*, 2000.
- [14] N. Parekh, S. Parthasarathy and S. Sinha, "Global and local control of spatiotemporal chaos in coupled map lattice," *Physical Review Letters*, pp. 1401-1404, 1998.
- [15] I. Waller and R. Kapral, "Spatial and temporal structure in systems of coupled nonlinear oscillators," *Physical Review A*, vol. 30, no. 4, pp. 2047-2055, 1984.
- [16] D. E. Goldberg, Genetic Algorithms in Search, Optimization, and Machine Learning, Addison Wesley Publishing Company, 1989.
- [17] C. I. Marrison and R. F. Stengel, "Robust control system design using random search and genetic algorithm," *IEEE Trans. on Automat. Contr.*, vol. 42, no. 6, pp. 835-839, 1997.