A Covariance Matrix Estimation Method for Position Uncertainty of the Wheeled Mobile Robot

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Abstract :A covariance matrix is a tool that expresses odometry uncertainty of the wheeled mobile robot. The covariance matrix is a key factor in various localization algorithms such as Kalman filter, topological matching and so on. However it is not easy to acquire an accurate covariance matrix because we do not know the real states of the robot. Up to the authors knowledge, there seems to be no established result on the covariance matrix estimation for the odometry. In this paper, we propose a new method which can estimate the covariance matrix from empirical data. It is based on the PC-method and shows a good estimation ability. The experimental results validate the performance of the proposed method.

Keywords : Odometry calibration, generalized Voronoi graph, covariance matrix, mobile robot, relative localization.

1 INTRODUCTION

The position acquired from odometry has uncertainties due to wheel slippages, kinematic inconsistency, etc. These uncertainties have been assumed as Gaussian distribution and expressed by a covariance matrix.

The covariance matrix, however, is hard to estimate because it is not easy to know the real position. Thus previous researchers have set the covariance matrix as a multiple of a constant and travelled distance[1, 2].

In this paper, we propose an empirical method that estimates the covariance matrix. Our method is based on the PC-method[3] which uses two lengthy path data for odometry calibration. If we completely compensate for the systematic errors[4] using the PC-method, then corrected paths will be fully overlapped. However, there remain residual random errors which is induced by the non-systematic error sources[4].

The proposed method can estimate variances of the random errors. These variances enables us to estimate the covariance matrix of the odometry.

For the validation, we ran the robot along 10 different paths and experiments were performed 10 times for each path. The validation process were done by comparing the end point and an estimated end point from the covariance matrix.

These extensive experiments validate that acquired covariance matrix can represent the real end position with 4%-18% offsets.

This paper is organized as follows: In section 2 the PC-method is briefly introduced. The way of the covariance matrix estimation is explained in section 3. The experiments are shown in section 4, 5 for the synchro and the differential drive robots. Conclusion follows in section 6.

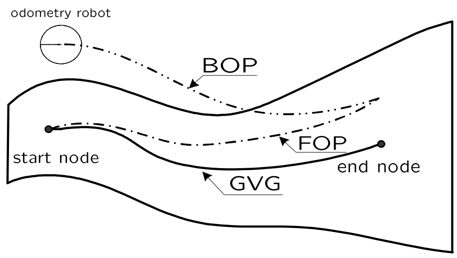


Fig. 1: Schematic explanation of the PC-method

2 The PC-method

The PC-method is based on the idea that sensor based navigation through the GVG bounds absolute error but odometry does not. The GVG is a set of points whose edges and nodes are equidistant to two or more than two objects in configuration space, respectively. Thus if we navigate the GVG twice, once forward and the other backward, we have two different odometry paths which are actually the same GVG in the real world.

The procedures of the PC-method is given below with a schematic explanation in Fig. 1

1. Navigate the GVG from a start node to an end node. The start and the end node can be chosen arbitrarily as long as the length between the two nodes is long enough. Let us denote odometry data from this navigation as a FOP(Forward Odometry Path).
2. Turn the robot 180° and navigate from the end node to the start node. We denote this odometry data as a BOP(Backward Odometry Path).
3. Calculate a Corrected FOP(CFOP) and a Corrected BOP(CBOP) by using given error model and initial error parameters guess.
4. Find out the error parameters that minimize error between the CFOP and the CBOP.

There are two things to remark. One is that the PC-method utilizes *two lengthy path data*, as opposed to *the end point errors* as in the UMBmark, for accurate error parameters estimation. The other is that, we happen to use the GVG for sensor based navigation. However, other sensor based navigation algorithm such as wall following can be used instead of the GVG.

3 An empirical method for the covariance matrix estimation.

Even after precise error compensation, there still remain error residuals which are mainly induced by non-systematic error sources. The covariance matrix expresses these error residuals as uncertainty. However, exact knowledge of this matrix is difficult to achieve because no information on the ground truth can be provided. Thus researchers simply have assumed the error residual uncertainty to be a multiple of travel distance and a constant whose value is intuitively selected[1, 2]. Recently, an estimation method was proposed in [5]. This approach, however, cannot be applied to the odometry because it is designed for scan matching, and it takes too long for calculation.

The PC-method is capable of the covariance matrix estimation for odometry. If we assume that the systematic errors are completely compensated for, then position differences between the corrected forward odometry path(CFOP) and the corrected backward odometry path(CBOP) are accrued by random errors which are caused by the non-systematic error sources.

Define the position difference as $d(k)$ and the random error as $v(k)$ with subscripts x, y from $k = 1$ to $k = n$ as shown in Fig. 2. Note that k begins at the end node and finishes at the start node.

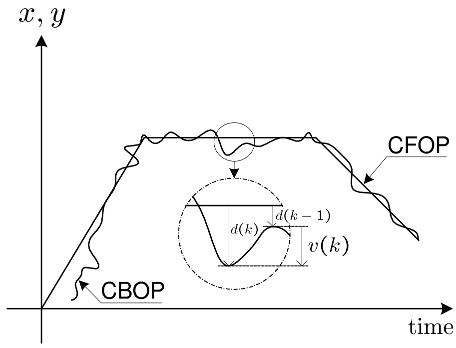


Fig. 2: Notations for the covariance matrix estimation using the PC-method.

The $v(k)$ and variance of $v(k)$ can be calculated as follows:

$$v(k) = d(k) - d(k-1). \tag{1}$$

$$\begin{aligned} \mathrm{var}[v] &= \frac{1}{n}\sum_{i=1}^{n} v^2(k) \\ &= \frac{1}{n}\sum_{i=1}^{n}(d(k) - d(k-1))^2. \end{aligned} \tag{2}$$

The $v(k)$ affects the position as in (3).

$$\begin{aligned} x(k) &= f_x(x(k-1), \theta(k-1)) + v_x(k) \\ &= x(k-1) + \Delta T \cos(\theta(k-1)) + v_x(k), \\ y(k) &= f_y(y(k-1), \theta(k-1)) + v_y(k) \\ &= y(k-1) + \Delta T \sin(\theta(k-1)) + v_y(k). \end{aligned} \tag{3}$$

The variances of $x(k)$ and $y(k)$ can be calculated by using the classical model[6, 7, 8] as

$$\begin{aligned} \sigma_x^2(k) &= \frac{\partial f_x}{\partial x(k-1)}\sigma_x^2(k-1)\frac{\partial f_x}{\partial x(k-1)}^T \\ &\quad + \frac{\partial f_x}{\partial \theta(k-1)}\sigma_\theta^2(k-1)\frac{\partial f_x}{\partial \theta(k-1)}^T + \mathrm{var}[v_x] \\ &= \sigma_x^2(k-1) + \Delta_T^2 \sin^2(\theta(k-1))\sigma_\theta^2(k-1) + \mathrm{var}[v_x], \\ \sigma_y^2(k) &= \frac{\partial f_y}{\partial y(k-1)}\sigma_y^2(k-1)\frac{\partial f_y}{\partial y(k-1)}^T \\ &\quad + \frac{\partial f_y}{\partial \theta(k-1)}\sigma_\theta^2(k-1)\frac{\partial f_y}{\partial \theta(k-1)}^T + \mathrm{var}[v_y] \\ &= \sigma_y^2(k-1) + \Delta T^2 \cos^2(\theta(k-1))\sigma_\theta^2(k-1) + \mathrm{var}[v_y], \end{aligned} \tag{4}$$

where $\sigma_x^2(k)$, $\sigma_y^2(k)$ and $\sigma_\theta^2(k)$ are variances of x, y and θ at k-th step.

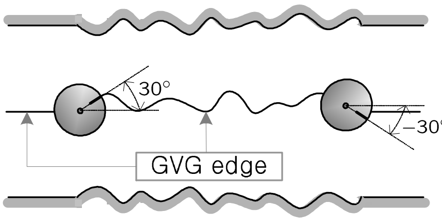


Fig. 3: Large θ variations occurs during the GVG navigation.

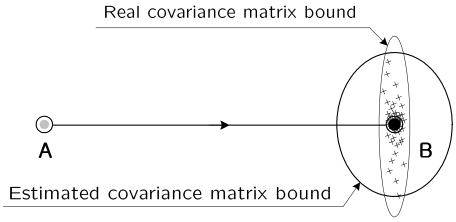


Fig. 4: A robot goes along a straight path from A to B. If the covariances of the position are estimated using the PC-method, then σ_x^2 will be overestimated and σ_y^2 will be underestimated because our model does not consider the effect of the heading angle.

As pointed out in [9], (4) is an inconsistent model. In other words, if the propagation of the covariance is done in multiple times for the same path, it will yield different results.

To redeem this problem, we first define a wheel movement W which is an average of rolling distances of wheels as

$$W = \frac{1}{n_w} \sum_{i=1}^{n_w} |s_i|, \tag{5}$$

where n_w and s_i are number of wheels and rolling distance of wheel i. Then we keep updating (4) after a constant wheel movement $W = W_c$.

Now we need to consider the physical meaning of the $v(k)$. In our derivation, $v(k)$ is random errors that are induced during *various θ angles*. Thus $v(k)$ is not purely random error in x, y directions, but a combination of x, y errors and an error induced by $\sigma_\theta(k)$. In other words, the effect of θ errors is implicitly and roughly contained in $v(k)$. So we assume that the effect of σ_θ^2 is contained in $v(k)$. This assumption enables us to write (4) as

$$\begin{aligned} \sigma_x^2(k) &= \sigma_x^2(k-1) + \mathrm{var}[v_x], \\ \sigma_y^2(k) &= \sigma_y^2(k-1) + \mathrm{var}[v_y]. \end{aligned} \tag{6}$$

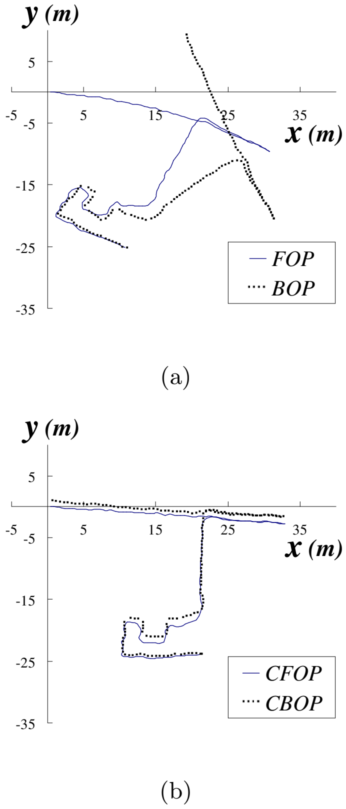


Fig. 5: Forward and backward odometry paths for the error parameters estimation of the synchro-drive robot. (a) is the FOP/BOP and (b) is the CFOP/CBOP.

Finally we assume that the off-diagonal terms of the covariance matrix are zero. Then the covariance matrix is given as

$$\begin{aligned} \Sigma(x(k), y(k)) &= \begin{bmatrix} \sigma_x^2(k) & 0 \\ 0 & \sigma_y^2(k) \end{bmatrix} \\ &= \begin{bmatrix} \frac{W(k)}{W_c}\mathrm{var}[v_x] & 0 \\ 0 & \frac{W(k)}{W_c}\mathrm{var}[v_y] \end{bmatrix} \\ &\quad + \begin{bmatrix} \sigma_x^2(0) & 0 \\ 0 & \sigma_y^2(0) \end{bmatrix}, \end{aligned} \tag{7}$$

where $W(k)$ is a sum of the wheel movements up to *k-th* step and $\sigma_x^2(0)$, $\sigma_y^2(0)$ are initial variances of the position.

Three remarks need to be mentioned. One is that the PC-method only can estimates the σ_x^2 and σ_y^2. The σ_θ^2 is not easy to acquire using the PC-method because there are large variations during the GVG navigation as shown in Fig. 3. The other is that the acquired σ_x^2 and σ_y^2 are not adequate for simple paths because the covariance update model (4) does not consider the effect of the heading angle. For example, if a robot goes along a straight path, then σ_x^2 will be overestimated and σ_y^2 will be underestimated as shown in Fig. 4. Finally the σ_x^2 and σ_y^2 represent the uncertainties under the specific velocity and the floor condition which are used for the FOP and BOP generation. If the robot moves in different speed or other environment, the σ_x^2, σ_y^2 will not yield good results.

4 Experiment I: the synchro-drive robot

The PC-method is applied to the synchro-drive robot to get the error parameters and the covariance matrix. We ran the robot through a lengthy GVG, whose length is $94.4m$ (Fig. 5(a)), to get the forward odometry path (FOP) and the backward odometry path (BOP). Then the error parameters, that minimize the error between the corrected FOP (CFOP) and the corrected BOP (CBOP), are found automatically by the steepest descent method. Fig. 5(b) displays the CFOP and the CBOP after error correction and the error parameters are

$$\left[E_1\ E_2\ E_3\right] = \left[1.29e^{-4}\ 2.30e^{-4}\ 1.571\right]. \quad (8)$$

From the CFOP and the CBOP, the variances of the random errors for a constant wheel movement $W_c = 3.9e^{-3}m$ is calculated as

$$\begin{aligned} var[v_x] &= 1.3e^{-5}m^2, \\ var[v_y] &= 1.7e^{-5}m^2. \end{aligned} \quad (9)$$

To verify the covariance matrix, we ran the robot 100 times along 4 different paths(25 times per each path). The lengths of the paths are $20m$, $40m$, $50m$ and $80m$ as shown in Fig. 6.

The velocity of the robot was set as $0.13m/s$ which is the same speed that is used for the FOP and the BOP generation. Also, we ran the robot at the same floor which is used for the FOP and the BOP generation.

Now, define $e_{x,f}$ and $e_{y,f}$ as

$$\begin{aligned} e_{x,f} &= x_f - \hat{x}_f, \\ e_{y,f} &= y_f - \hat{y}_f, \end{aligned} \quad (10)$$

where (x_f, y_f) and $(\hat{x}_f, \hat{y}_f)$ are a real final point and a final point estimated by the corrected odometry. For the comparison, we normalized $e_{x,f}$ and $e_{y,f}$ over $2\sigma_x$ and $2\sigma_y$ and plotted 100 results in Fig. 7.

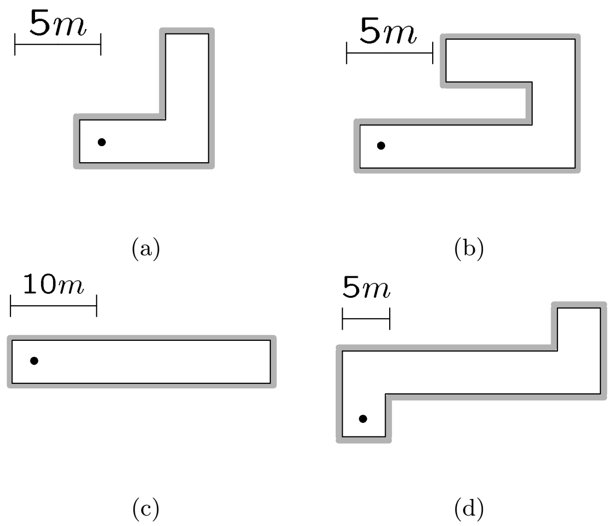


Fig. 6: Four paths are navigated by the robot to verify the covariance matrix. The lengths of the paths are (a) $20m$, (b) $40m$, (c) $50m$ and (d) $80m$. The robot started at the dotted point and ended up the same point.

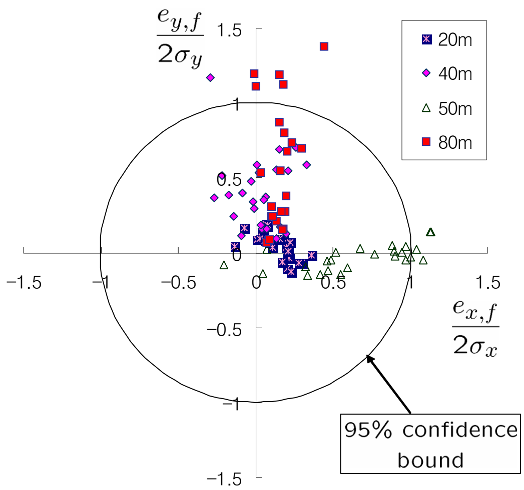


Fig. 7: Data plot of $\frac{e_{x,f}}{2\sigma_x}$ versus $\frac{e_{y,f}}{2\sigma_y}$ for the synchro-drive robot.

If we model the position uncertainty as the Gaussian distribution, then the unit circle in Fig. 7 is 95% confidence bound. In other words, the probability of $\frac{e_{x,f}}{2\sigma_x}$ and $\frac{e_{y,f}}{2\sigma_y}$ to be located within the unit circle is 95%.

The experimental results show that the probability is 91% which is very close to the ideal value(95%). Therefore we assert that the covariance matrix estimated by the PC-method is quite accurate.

Note that the expectation values of $e_{x,f}$ and $e_{y,f}$ are not zero. The reason is that there remains systematic errors that are not compensated for during the PC-method correction.

5 Experiment II: the differential drive robot

To validate the covariance matrix for the differential drive robot, the PC-method is applied. The robot was run along a lengthy GVG and the error parameters were automatically found. The FOP/BOP and the CFOP/CBOP are shown in Fig. 8.

From the CFOP and the CBOP, the variances of the random errors for a constant wheel movement $W_c = 6.9e^{-2}m$ are calculated as

$$\begin{aligned} var[v_x] &= 6.6e^{-5}m^2, \\ var[v_y] &= 7.3e^{-5}m^2. \end{aligned} \tag{11}$$

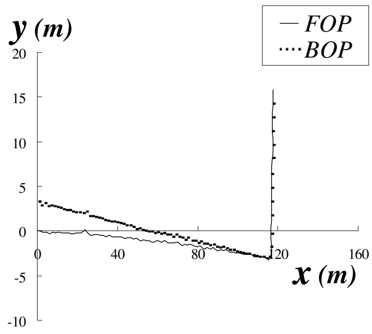
To verify the covariance matrix, we ran the robot 100 times along 4 different paths shown in Fig. 6. The velocity of the robot was set as $0.25m/s$ which is the same speed of the FOP and the BOP generation. For the comparison, we normalized $e_{x,f}$ and $e_{y,f}$ over $2\sigma_x$ and $2\sigma_y$ and plotted 100 results in Fig. 9.

If we model the position uncertainty as the Gaussian distribution, then the unit circle in Fig. 9 is 95% confidence bound. The experimental results show that the probability of the $\frac{e_{x,f}}{2\sigma_x}$, $\frac{e_{y,f}}{2\sigma_y}$ to be located within the unit circle is 77%. There are 18% differences between the experimental result(77%) and the ideal value(95%) and we think that this gap is induced by remaining systematic errors. These errors especially affect to the 40m path(Fig. 6(b)) and the data from that path is off-centered as shown in Fig. 9.

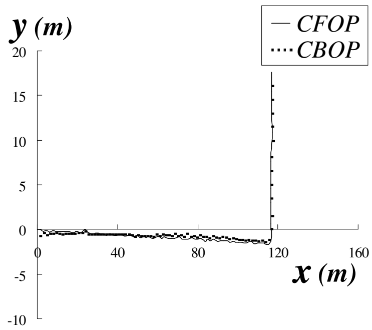
However, the experimental results show that the PC-method can approximately estimate the covariance matrix for the differential drive robot.

6 Conclusion

In this paper we proposed an empirical way of the covariance matrix estimation. This method is based on



(a)



(b)

Fig. 8: Forward and backward odometry paths for the error parameters estimation of the differential drive robot. (a) is the FOP/BOP and (b) is the CFOP/CBOP.

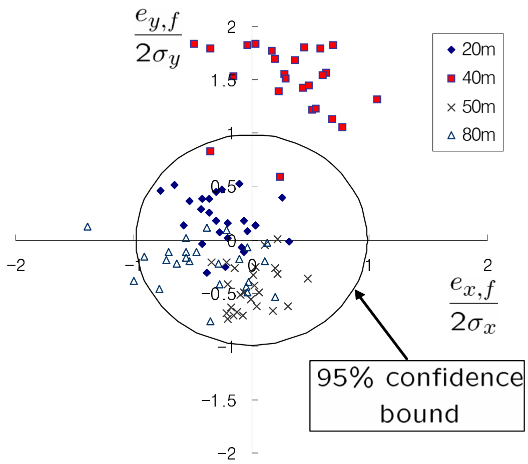


Fig. 9: Data plot of $\frac{e_{x,f}}{2\sigma_x}$ versus $\frac{e_{y,f}}{2\sigma_y}$ of the differential drive robot.

the PC-method which corrects odometry based on two paths.

The experimental results shows that our method can estimate the covariance matrix with 3% and 18% offsets for the synchro and differential drive robot.

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