Receding Horizon Finite Memory Controls for Output Feedback Controls of Discrete-Time State Space Models

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Abstract: In this paper, a new type of output feedback control, called a receding horizon finite memory control (RHFMC), is proposed for stochastic discrete-time state space systems. Constraints such as linearity and finite memory structure with respect to an input and an output, and unbiasedness from the optimal state feedback control are required in advance. The proposed RHFMC is chosen to minimize an optimal criterion with these constraints. The RHFMC is obtained in an explicit closed form using the output and input information on the recent time interval. It is shown that the RHFMC consists of a receding horizon control and an FIR filter. The stability of the RHFMC is investigated for stochastic systems.

Keywords: Finite memory control; Unbiasedness; Receding horizon; Output feedback.

1. Introduction

For mathematical analysis, plants or processes are often represented as infinite impulse response (IIR) types, which can also be described over state space. Controls are often represented in the form of state feedback controls when all states are available, or output feedback controls when only partial states, known as outputs, are available. In case of output feedback controls, filters are often introduced to obtain the state information from inputs and measured outputs. These filters are also conventionally of IIR types. A typical output feedback control for stochastic linear systems is the LQG control where the Kalman filter of the IIR type is used to estimate all the states and the LQ control in the form of state feedback controls is calculated from the estimated state.

However, in fields of discrete-time signal processing, the finite impulse response (FIR) type is much preferable to the IIR type despite of heavy calculation of FIR filters. The guaranteed stability, the robustness to numerical error and temporary uncertainties, and perfect signal reconstruction such as a linear phase properties are well known good properties of the FIR structure. When signal models are represented as general state space models with systems and measurement noises, the FIR filters were proposed for estimation of the states. The recursive limited memory filter [1] and the optimal FIR filter [2] were given with some limitations. Recently the unbiased FIR filter [3] were obtained by directly minimizing performance index of minimum variance subject to the unbiasedness constraint. Good properties such as deadbeatness for systems without noises have been obtained for these filters. So it will be meaningful to investigate whether we can adopt the FIR structure even in the output feedback control for state space system models.

Output feedback controls (u_k) at a current time k with finite memory structure can be represented using measurements (y_i) and inputs (u_i) during a finite time, i.e., a horizon $[k - N_f \quad k]$, as

$$u_k \quad = \quad \sum_{i=k-N_f}^{k-1} H_{k-i} y_i + \sum_{i=k-N_f}^{k-1} L_{k-i} u_i \tag{1}$$

for some gains H_i and L_i. Note that even though the control (1) uses the finite measurements and inputs on the recent time interval as FIR filters, this is not of the FIR form. So this kind of the control will be called finite memory controls (FMC) rather than FIR controls.

In this paper, the output feedback control u_k at a current time k with the finite memory structure will be obtained from a usual receding horizon linear quadratic gaussian (LQG) criterion

$$\begin{aligned} & \mathbf{E}\Big[\sum_{j=0}^{N_c-1} \big[x_{k+j|k}^T Q_c x_{k+j|k} + u_{k+j|k}^T R_c u_{k+j|k} \big] \\ + \quad & x_{k+N_c|k}^T F x_{k+N_c|k} \Big]. \end{aligned} \tag{2}$$

where k in the right side of vertical bar means the current time.

These output feedback controls with the finite memory structure for the cost criterion (2) can be called receding horizon finite memory controls (RHFMC). To the best of authors' knowledge, for discrete-time state space models, there is no general result for output feedback controls with the finite memory like the FIR filter using finite measurements and inputs. The discrete-time system is more useful to apply to digital computers.

The receding horizon control has several advantages and thus widely applied to industrial problems. [4], [5], [6]. We will require a constraint that the proposed RHFMC should be unbiased from the optimal state feedback control that is obtained when full information with respect to the state is available. This unbiasedness constraint has a physical meaning that it allows the proposed RHFMC to track the optimal state feedback control on average. Since any output feedback control can not be better in view of performance than the optimal

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state feedback control, it is desirable that RHFMC should be unbiased from the optimal state feedback control. It will be shown in this paper that, surprisingly, even with this requirement the optimal solution exists. The stability will be checked for the proposed RHFMC.

This paper is organized as follows. In Section 2, the RHFMC for discrete-time state space systems is proposed in a form of (1). In Section 3, the separation principle of the RHFMC is discussed and the stability condition is investigated. Finally, conclusions are stated in Section 4.

2. Receding Horizon Finite Memory Controls

Consider a linear discrete-time state space model:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Gw_k, && (3)\\ y_k &= Cx_k + v_k && (4)\end{aligned}$$

where $x_k \in \Re^n$ is the state, $u_k \in \Re^l$ and $y_k \in \Re^q$ are the input and measurement, respectively. At the initial time k_0 of the system, the state x_{k_0} is a random variable with a mean $\bar{x}_{k_0}$ and a covariance P_{k_0}. The system noise $w_k \in \Re^p$ and the measurement noise $v_k \in \Re^q$ are zero-mean white Gaussian and mutually uncorrelated. The covariances of w_k and v_k are denoted by Q_f and R_f, respectively, which are assumed to be positive definite matrices. These noises are uncorrelated with the initial state x_{k_0}.

The system (3)-(4) will be represented in a batch form on the time interval $[k+j-N_f \;\; k+j]$ called the horizon. On the horizon $[k+j-N_f \;\; k+j]$, measurements are expressed in terms of the state x_{k+j} at the time $k+j$ and inputs as follows:

$$\begin{aligned} Y_{k+j-1} &= \bar{C}_{N_f} x_{k+j} + \bar{B}_{N_f} U_{k+j-1} + \bar{G}_{N_f} W_{k+j-1} \\ &+ V_{k+j-1} && (5)\end{aligned}$$

where

$$\begin{aligned} Y_{k+j-1} &\triangleq [y^T_{k+j-N_f} \;\; y^T_{k+j-N_f+1} \;\; \cdots \;\; y^T_{k+j-1}]^T, && (6)\\ U_{k+j-1} &\triangleq [u^T_{k+j-N_f} \;\; u^T_{j-N_f+1} \;\; \cdots \;\; u^T_{k+j-1}]^T, && (7)\\ W_{k+j-1} &\triangleq [w^T_{k+j-N_f} \; w^T_{k+j-N_f+1} \; \cdots \; w^T_{k+j-1}]^T, \\ V_{k+j-1} &\triangleq [v^T_{k+j-N_f} \; v^T_{k+j-N_f+1} \; \cdots \; v^T_{k+j-1}]^T \end{aligned}$$

and $\bar{C}_{N_f}$, $\bar{B}_{N_f}$, $\bar{G}_{N_f}$ are obtained from

$$\bar{C}_i \triangleq \begin{bmatrix} CA^{-i} \\ CA^{-i+1} \\ CA^{-i+2} \\ \vdots \\ CA^{-1} \end{bmatrix} = \begin{bmatrix} \bar{C}_{i-1} \\ C \end{bmatrix} A^{-1}, \quad (8)$$

$$\begin{aligned} \bar{B}_i &\triangleq -\begin{bmatrix} CA^{-1}B & CA^{-2}B & \cdots & CA^{-i}B \\ 0 & CA^{-1}B & \cdots & CA^{-i+1}B \\ 0 & 0 & \cdots & CA^{-i+2}B \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & CA^{-1}B \end{bmatrix} \\ &= \begin{bmatrix} \bar{B}_{i-1} & -\bar{C}_{i-1}A^{-1}B \\ 0 & -CA^{-1}B \end{bmatrix}, && (9)\end{aligned}$$

$$\begin{aligned} \bar{G}_i &\triangleq -\begin{bmatrix} CA^{-1}G & CA^{-2}G & \cdots & CA^{-i}G \\ 0 & CA^{-1}G & \cdots & CA^{-i+1}G \\ 0 & 0 & \cdots & CA^{-i+2}G \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & CA^{-1}G \end{bmatrix} \\ &= \begin{bmatrix} \bar{G}_{i-1} & -\bar{C}_{i-1}A^{-1}G \\ 0 & -CA^{-1}G \end{bmatrix}, && (10)\\ & 1 \le i \le N_f. \end{aligned}$$

Note that definitions of (8)-(10) will be used through this paper. It is assumed that A is nonsingular. When continuous-time systems $\dot{x}(t) = \tilde{A}x(t) + \tilde{B}u(t)$ are discretized with the sampling time T, we obtain sampled-data systems, $x_{j+1} = Ax_j + Bu_j$ where $A = e^{\tilde{A}T}$. So the assumption of the nonsingularity of A is not too restrictive to apply in practical view.

The control at the time $k+j$ on the horizon $[k+j-N_f \;\; k+j]$ will be denoted as $u_{k+j|k}$ where $0 \le j \le N_c - 1$. An FMC at the time $k+j$ can be expressed as a linear function of the finite measurements Y_{k+j-1} (6) and inputs U_{k+j-1} (7) on the horizon $[k+j-N_f, k+j]$ as follows:

$$u_{k+j|k} \triangleq H_j Y_{k+j-1} + L_j U_{k+j-1} \quad (11)$$

where H_j and L_j are gain matrices of a linear control. If compared with the form (1), H_j and L_j are denoted by

$$\begin{aligned} H_j &= \begin{bmatrix} H_{N_f,j} & H_{N_f-1,j} & \cdots & H_{1,j} \end{bmatrix} && (12)\\ L_j &= \begin{bmatrix} L_{N_f,j} & L_{N_f-1,j} & \cdots & L_{1,j} \end{bmatrix}. && (13)\end{aligned}$$

It is noted that the control defined in (11) uses the finite measurements and inputs.

If we assume that the full information of the state is available, it is well known that the optimal state feedback control for the optimal criterion (2) can be written

$$u^*_{k+j|k} = -[R_c + B^T K_{N_c-j-1} B]^{-1} B^T K_{N_c-j-1} A x_{k+j} \quad (14)$$

where K_i satisfies

$$K_{i+1} = A^T K_i [I + BR_c^{-1}B^T K_i]^{-1} A + Q_c \quad (15)$$

with the boundary condition

$$K_0 = F. \quad (16)$$

As the control (14), the optimal control is represented in a form of state feedback. It is desirable that the intermediate output feedback FMC control (11) can track the optimal state feedback control (14) on average. Thus, we require a constraint that the expectation of the control (11) must be unbiased from the optimal state feedback control (14) as

$$\mathbf{E}[u_{k+j|k}] = \mathbf{E}[u^*_{k+j|k}] \quad \text{for all states.} \quad (17)$$

The left and the right sides of (17) can be given as

$$\mathbf{E}[u_{k+j|k}] = H_j \bar{C}_{N_f} \mathbf{E}[x_{k+j}] + [H_j \bar{B}_{N_f} + L_j] U_{k+j-1}$$

and

$$\begin{aligned}
&\mathbf{E}[u^*_{k+j|k}] \\
=\ & \mathbf{E}\big[-[R_c+B^TK_{N_c-j-1}B]^{-1}B^TK_{N_c-j-1}Ax_{k+j}\big] \\
=\ & -[R_c+B^TK_{N_c-j-1}B]^{-1}B^TK_{N_c-j-1}A\mathbf{E}\big[x_{k+j}\big].
\end{aligned}$$

Since the unbiasedness condition (17) should be applied to all states and all controls, the following relations can be obtained:

$$H_j\bar{C}_{N_f} = -[R_c+B^TK_{N_c-j-1}B]^{-1}B^TK_{N_c-j-1}A \tag{18}$$

$$H_j\bar{B}_{N_f} = -L_j \tag{19}$$

which will be called the unbiasedness constraint. It is noted that the constraint must hold regardless of the state and the input. This constraint may be too strict, but surprisingly, we were able to obtain the solution. The objective now is to obtain the best gain matrix $H_{B,j}$, subject to the unbiasedness constraints (18)-(19).

$$\begin{aligned}
& H_{B,j} \\
=\ & \min_{H_{B,j}} \mathbf{E}\Big[\sum_{j=0}^{N_c-1}[x^T_{k+j|k}Q_cx_{k+j|k}+u^T_{k+j|k}R_cu_{k+j|k}] \\
+\ & x^T_{k+N_c|k}Fx_{k+N_c|k}\Big] \\
=\ & \min_{H_{B,j}} \mathbf{E}\Big[\sum_{j=0}^{N_c-1} \\
& \{[R_c+B^TK_{N_c-j-1}B]^{-1}B^TK_{N_c-j-1}Ax_{k+j|k}+u_{k+j|k}\}^T \\
& [R_c+B^TK_{N_c-j-1}B] \\
& \{[R_c+B^TK_{N_c-j-1}B]^{-1}B^TK_{N_c-j-1}Ax_{k+j|k}+u_{k+j|k}\} \\
+\ & tr[\sum_{j=0}^{N_c-1}K_jGQG^T]\Big]+\mathbf{E}[x_k^TK_{N_c}x_k].
\end{aligned} \tag{20}$$

Since

$$\begin{aligned}
& u_{k+j|k}+[R_c+B^TK_{N_c-j-1}B]^{-1}B^TK_{N_c-j-1}Ax_{k+j|k} \\
=\ & H_j\bar{G}_{N_f}W_{k+j-1}+H_jV_{k+j-1},
\end{aligned} \tag{21}$$

the following relation for the first term in (20) is obtained

$$\begin{aligned}
& \mathbf{E}\Big[\{[R_c+B^TK_{N_c-j-1}B]^{-1}B^TK_{N_c-j-1}Ax_{k+j|k} \\
+\ & u_{k+j|k}\}^T[R_c+B^TK_{N_c-j-1}B] \\
& \{[R_c+B^TK_{N_c-j-1}B]^{-1}B^TK_{N_c-j-1}Ax_{k+j|k} \\
+\ & u_{k+j|k}\}\Big] \\
=\ & tr(\sqrt{R_c+B^TK_{N_c-j-1}B}H_j\Xi_{N_f}H_j^T\sqrt{R_c+B^TK_{N_c-j-1}B})
\end{aligned}$$

with Ξ_{N_f} given by

$$\begin{aligned}
\Xi_i \ \triangleq\ & \bar{G}_i[\mathrm{diag}(\overbrace{Q_f\ Q_f\ \cdots\ Q_f}^{i})]\bar{G}_i^T+[\mathrm{diag}(\overbrace{R_f\ R_f\ \cdots\ R_f}^{i})] \\
=\ & \begin{bmatrix} \Xi_{i-1} & 0 \\ 0 & R_f \end{bmatrix} \\
+\ & \begin{bmatrix} \bar{C}_{i-1} \\ C \end{bmatrix} A^{-1}GQ_fG^TA^{-T}\begin{bmatrix} \bar{C}_{i-1} \\ C \end{bmatrix}^T.
\end{aligned} \tag{22}$$

The last two terms in (20) are constant for a control gain $H_{B,j}$. All that remains is to minimize the first term in (20) in order to obtain the solution. The objective now is to obtain the optimal gain matrix $H_{B,j}$, subject to the unbiasedness constraint (18)-(19), in such a way that the cost function has minimum variance as follows:

$$\begin{aligned}
& H_{B,j} \\
=\ & \arg\min_{H_{B,j}} tr[\sqrt{R_c+B^TK_{N_c-j-1}B}H_j\Xi_{N_f}H_j^T \\
& \sqrt{R_c+B^TK_{N_c-j-1}B}] \\
=\ & \arg\min_{H}\sum_{l=1}^{n}h^T_{l,j}\Xi_{N_f}h_{l,j}.
\end{aligned} \tag{23}$$

For convenience, partition the matrix H_j in (11) as

$$\begin{aligned}
& H_j^T\sqrt{R_c+B^TK_{N_c-j-1}B} \\
=\ & \begin{bmatrix} h_{1,j} & h_{2,j} & \cdots & h_{n,j} \end{bmatrix}.
\end{aligned}$$

Since the unbiasedness constraint (18)-(19) is satisfied, the sth unbiasedness constraint is

$$\begin{aligned}
& \bar{C}^T_{N_f}h_{s,j} = -A^TK_{N_c-j-1}B[R_c+B^TK_{N_c-j-1}B]^{-1} \\
& \sqrt{R_c+B^TK_{N_c-j-1}B}e_s, \\
& 1\le s\le n
\end{aligned} \tag{24}$$

where e_s is the sth unit vector such that $e_s = [0,\cdots,0,1,0,\cdots,0]^T$ with the nonzero element in the sth position. For each $h_{s,j}$, the following cost function is established:

$$\begin{aligned}
& J_{s,j}(h_{s,j},\lambda_s) = h^T_{s,j}\Xi_{N_f}h_{s,j}+\lambda^T_{s,j}(\bar{C}^T_{N_f}h_{s,j} \\
+\ & A^TK_{N_c-j-1}B[R_c+B^TK_{N_c-j-1}B]^{-1} \\
& \sqrt{R_c+B^TK_{N_c-j-1}B}e_s)
\end{aligned} \tag{25}$$

where λ_s is the sth vector of Lagrange multipliers, which is associated with the sth unbiasedness constraint (24). Therefore, the objective is now to minimize $J_{s,j}(\cdot,\cdot)$ (25) with respect to $h_{s,j}$ and $\lambda_{s,j}$. To minimize $J_{s,j}(\cdot,\cdot)$, two necessary conditions $\partial J_{s,j}(h_{s,j},\lambda_{s,j})/\partial h_s = 0$ and $\partial J_{s,j}(h_{s,j},\lambda_{s,j})/\partial\lambda_{s,j} = 0$ give $2h_s = -\Xi^{-1}_{N_f}\bar{C}_{N_f}\lambda_s$ and (24), and thus h_s is determined by

$$\begin{aligned}
h_{s,j} =\ & -\Xi^{-1}_{N_f}\bar{C}_{N_f}(\bar{C}^T_{N_f}\Xi^{-1}_{N_f}\bar{C}_{N_f})^{-1}A^TK_{N_c-j-1}B \\
& [R_c+B^TK_{N_c-j-1}B]^{-1} \\
& \sqrt{R_c+B^TK_{N_c-j-1}B}e_s.
\end{aligned} \tag{26}$$

Note that the matrix $\bar{C}^T_{N_f}\Xi^{-1}_{N_f}\bar{C}_{N_f}$ is nonsingular if and only if the matrix $\bar{C}_{N_f}$ is of full rank, since the matrix Ξ_{N_f} is positive definite. The matrix $\bar{C}_{N_f}$ is of full rank if $\{A,C\}$ is observable for $N_f\ge n$. The gain matrix H is reconstructed from h_s (26) as follows:

$$\begin{aligned}
H_j^T =\ & -\Xi^{-1}_{N_f}\bar{C}_{N_f}(\bar{C}^T_{N_f}\Xi^{-1}_{N_f}\bar{C}_{N_f})^{-1}A^TK_{N_c-j-1}B \\
& [R_c+B^TK_{N_c-j-1}B]^{-1}\begin{bmatrix} e_1 & e_2 & \cdots & e_n \end{bmatrix} \\
=\ & -\Xi^{-1}_{N_f}\bar{C}_{N_f}(\bar{C}^T_{N_f}\Xi^{-1}_{N_f}\bar{C}_{N_f})^{-1}A^TK_{N_c-j-1}B \\
& [R_c+B^TK_{N_c-j-1}B]^{-1}
\end{aligned} \tag{27}$$

and then becomes $H_{B,j}$ in (23). If j is replaced by 0, the RHFMC is obtained. Therefore, the RHFMC $u_{k|k}$ with the optimal gain matrix $H_{B,0}$, shortly H_B, is proposed in the following theorem.

Theorem 1: *When $\{A, C\}$ is observable and $N_f \geq n$, the RHFMC $u_{k|k}$ on the horizon $[k - N_f, k]$ is given as follows:*

$$u_{k|k} = H_B(Y_{k-1} - \bar{B}_{N_f} U_{k-1}) \quad (28)$$

with the optimal gain matrix H_B determined by

$$\begin{aligned} H_B &= -[R_c + B^T K_{N_c-1} B]^{-1} B^T K_{N_c-1} \\ & \quad A(\bar{C}_{N_f}^T \Xi_{N_f}^{-1} \bar{C}_{N_f})^{-1} \bar{C}_{N_f}^T \Xi_{N_f}^{-1} \end{aligned} \quad (29)$$

where Y_{k-1}, U_{k-1}, $\bar{C}_{N_f}$, $\bar{B}_{N_f}$, and Ξ_{N_f} are given by (6)-(9) and (22), respectively. ∎

The dimension of Ξ_{N_f} in (29) may be large. So the numerical error during inverting the matrix can happened. To avoid the handling of the large matrix, the optimal gain matrix can be obtained from the following recursive equations [3] :

$$H_B = -[R_c + B^T K_{N_c-1} B]^{-1} B^T K_{N_c-1} A \Omega_{N_f}^{-1} \eta_{N_f} \quad (30)$$

where

$$\begin{aligned} \Omega_{i+1} &= [I + A^{-T}(\Omega_i + C^T R_f^{-1} C) \\ & \quad A^{-1} G Q_f G^T]^{-1} A^{-T} (\Omega_i + C^T R_f^{-1} C) A^{-1} \quad (31) \\ \eta_{i+1} &= [I + A^{-T}(\Omega_i + C^T R_f^{-1} C) A^{-1} G Q_f G^T]^{-1} A^{-T} \\ & \quad \begin{bmatrix} \eta_i & C^T R_f^{-1} \end{bmatrix} \quad (32) \end{aligned}$$

with $\Omega_0 = 0$ and $\eta_0 = 0$. Note that recursive equations can be easily derived from the structure of the matrix (22). From Theorem 1, it can be known that the RHFMC $u_{k|k}$ (28) processes the finite measurements and inputs on the horizon $[k - N_f, k]$ linearly and has the properties of unbiasedness from the optimal state feedback control *by design*. Note that the optimal gain matrix H_B (29) requires computation only on the interval $[0, N_f]$ once and is time-invariant for all horizons. This means that the proposed RHFMC is time-invariant. It is a general rule of thumb that, due to the finite memory structure, the proposed RHFMC may also be robust against temporary modeling uncertainties or round-off errors. The separation principle and the stability for the proposed RHFMC will be investigated in the next section

3. Separation Principle and Stability

Before we proceed to investigate the stability of systems without noises, it is shown that the proposed control can be separated as a receding control and an FIR filter.

Theorem 2: The RHFMC (28) can be represented as a receding control and an FIR filter:

$$u_{k|k} = -[R_c + B^T K_{N_c-1} B]^{-1} B^T K_{N_c-1} A \hat{x}_k \quad (33)$$

where the FIR filter $\hat{x}_k$ is given as follows:

$$\hat{x}_k = (\bar{C}_{N_f}^T \Xi_{N_f}^{-1} \bar{C}_{N_f})^{-1} \bar{C}_{N_f}^T \Xi_{N_f}^{-1} [Y_{k-1} - \bar{B}_{N_f} U_{k-1}]. \quad (34)$$

$\hat{x}_k$ in (34) is an actual state estimator.

Theorem 2 can easily be proved so that the proof is omitted. In [3], it can be found that $\hat{x}_k$ in (34) is an optimal minimum variance state estimator with the FIR structure. It is known that the FIR filter (34) is a quasi-deadbeat filter which has the deadbeat property for the systems without noises. Before the stability for stochastic systems is investigated, it is shown that the stability is guaranteed for deterministic systems that is obtained from (3)-(4) by removing noises, i.e, $x_{k+1} = Ax_k + Bu_k$ and $y_k = Cx_k$.

Theorem 3: [7] If the final weighting matrix F in the cost function satisfies the following inequality:

$$\begin{aligned} & F \geq Q_c + D^T R_c D + (A - BD)^T F (A - BD) \\ & \qquad \text{for some} \quad D \in R^{l \times n}, \end{aligned} \quad (35)$$

the system driven by the proposed RHFMC, is asymptotically stable under the deterministic systems without noises.

If driven by the proposed RHFMC, the system can be represented as

$$\begin{aligned} x_{k+1} &= \Big[A - B[R_c + B^T K_{N_c-1} B]^{-1} B^T K_{N_c-1} A\Big] x_k \\ &- BH\bar{G}_{N_f} W_{k-1} - BHV_{k-1} + Gw_k. \end{aligned} \quad (36)$$

Theorem 3 implies that $A - B[R_c + B^T K_{N_c-1} B]^{-1} B^T K_{N_c-1} A$ is Hurwitz for the terminal weighting matrix F satisfying the inequality (35). Therefore, only if power of noises is finite, the following bound is guaranteed for stochastic systems:

$$\mathbf{E}[x_k x_k^T] < \infty. \quad (37)$$

4. Conclusion

In this paper, a new type of control, RHFMC, is proposed for discrete-time state space models using the input and output information. The proposed RHFMC is obtained by minimizing the optimal criterion, with additional unbiasedness constraints which look difficult to solve. It is very interesting that RHFMC consists of the receding horizon control and the FIR filter. It is shown that there exists a closed form solution to gain matrices even under the strong unbiasedness condition (18)-(19). The RHFMC is unbiased from the optimal state feedback control that can be obtained only if the full state information is available. Due to the finite memory structure, the RHFMC is believed to be robust against temporary modeling uncertainties or numerical errors. It is shown that the stability is guaranteed under the cost monotonicity condition of the receding horizon control. The proposed RHFMC is a new type of control and can be a substitute for the commonly used output feedback controls such as conventional LQG.

In addition, the proposed RHFMC will be very useful for multirate systems, where the FIR structure is usually essential. It is noted that the concept in this paper may be applied to the other type of optimal criterion.

References

[1] A. M. Bruckstein and T. Kailath, "Recursive limited memory filtering and scattering theory," *IEEE Trans. Inform. Theory*, vol. 31, pp. 440–443, 1985.

[2] W. H. Kwon and O. K. Kwon, "FIR filters and recursive forms for continuous time-invariant state-space models," *IEEE Trans. Automat. Contr.*, vol. 32, pp. 352–356, 1987.

[3] Wook Hyun Kwon, Pyung Soo Kim, and Soo Hee Han, "A Receding Horizon Unbiased FIR Filter for Discrete-Time State Space Models," *Automatica*, vol. 38, no. 3, pp. 545–551, 2002.

[4] G. De Nicolao and S. Strada, "On the stability of receding-horizon LQ control with zero-state terminal constraint," *IEEE Trans. Automat. Contr.*, vol. 22, pp. 257–260, 1997.

[5] J. B. Rawlings and K. R. Muske, "The stability of constrained receding horizon control," *IEEE Trans. Automat. Contr.*, vol. 38, pp. 1512–1516, 1993.

[6] L. Magni and R. Sepulchre, "Stability margin of nonlinear receding horizon control via inverse optimality," *Sys. Contr. Lett.*, vol. 32, pp. 241–245, 1997.

[7] W. H. Kwon and K. B. Kim, "On Stabilizing Receding Horizon Controls for Linear Continuous Time-Invariant Systems," *IEEE Trans. Automat. Contr.*, vol. 45, no. 7, pp. 1329–1334, 2000.