# PDFF Controller Design by CDM for Position Control of Traveling-Wave Ultrasonic Motor

S.Nundrakwang\*, D. Isarakorn\*, T. Benjanarasuth\*, J. Ngamwiwit\* and N. Komine\*\*

\*Department of Control Engineering,

Faculty of Engineering and Research Center for Communications and Information Technology

King Mongkut's Institute of Technology Ladkrabang, Ladkrabang, Bangkok, 10520 Thailand

(Tel: +66-2-739-2405, Fax: 66-2-326-4225; E-mail: knjongko@kmitl.ac.th)

\*\*Department of Applied Computer Engineering, School of Information Technology and Electronics, Tokai University

1117 Kitakaname, Hiratsuka-Shi, Kanagawa-Ken, 259-1292, Japan

(Tel: +81-463-58-1211, Fax: 81-463-50-2240; E-mail: komine@keyaki.cc.u-tokai.ac.jp)

**Abstract:** Ultrasonic motors have many excellent performances. A variety of ultrasonic motors has been developed and used as an actuator in motion control systems. However, this motor has nonlinear characteristics. Therefore, it is difficult to achieve the precise position control system incorporating with the ultrasonic motor. This paper describes a position control scheme for traveling-wave type ultrasonic motor using a pseudo-derivative control with feedforward gains (PDFF) controller designed by the coefficient diagram method (CDM). The PDFF control system satisfies both the tracking and regulation performances, which are the most important for the precise position control system. The CDM is shown to be an efficient and simple method to design the parameters of PDFF controller. The effectiveness of the proposed control system is demonstrated by experiments.

Keywords: Position control system, ultrasonic motor, PDFF controller, coefficient diagram method.

#### **1. INTRODUCTION**

The ultrasonic motor is an innovative actuator that has shown a high potential in direct-drive application. During the past two decades, various ultrasonic motors have been developed and investigated in the literature [1-2]. One of the most popular types is the traveling-wave ultrasonic motor. The operating principle of the traveling-wave motor is to produce the traveling wave at the stator by two orthogonal bending modes. The superposition of the two bending modes creates the elliptical motions on the surface of the stator. If the rotor is in contact with the stator by a preload, the rotor can be rotated by friction force. The merits of the ultrasonic motor are high precision, quick response, hard brake with no backlash, high power to weight ratio and negligible electromagnetic interference (EMI). While the ultrasonic motor has high potential in precise position control system, it is difficult to control the position of ultrasonic motor with high performance due to its nonlinear features. Therefore, many control schemes have been proposed to achieve the precise position control of ultrasonic motors, such as fuzzy reasoning control [3], neural network [4-5] and adaptive control [6-7]. These control techniques however have many difficulties to be applied to actual implementation.

Hence, this paper proposes an effective position control scheme using a pseudo-derivative control with feedforward gains (PDFF) controller designed by the coefficient diagram method (CDM) [8] for traveling-wave ultrasonic motor. The PDFF controller must be designed to satisfy both the tracking and regulation performances, which are the most important for the precision position control system [9]. However, it is quite complicated to design and tune all of the parameters. Therefore, the CDM which is an algebraic control design approach is adopted to design the PDFF controller parameters. This method is an efficient tool to design the parameters of PDFF controller based on the stability and the speed of the controlled system in order to meet the desired performance criteria. Stability is designed from the stability index  $\gamma_i$ , and speed is designed from either the equivalent time constant  $\tau$ or the tuning factor  $\alpha$ . In this work, the transfer function of the PDFF control system in term of stability index  $\gamma_{i}$ , equivalent time constant  $\tau$  and tuning factor  $\alpha$  has been developed in general form. The coefficients of the numerator and denominator of the transfer function are related to the

controller parameters algebraically in an explicit form. Consequently, the PDFF controller parameters can be obtained appropriately by assigning the values of stability index  $\gamma_i$ , equivalent time constant  $\tau$  and tuning factor  $\alpha$ .

# 2. CONTROL SYSTEM STRUCTURE



Fig. 1 Structure of PDFF control system.

The general PDFF control system structure shown in Fig. 1 consists of a plant, a feedforward controller, a feedback controller and an integral controller. The transfer function from R(s) to C(s) is given as

$$\frac{C(s)}{R(s)} = \frac{G_p(s) \left[ K_{pf} + \frac{K_i}{s} + K_{df}s + K_{af}s^2 + \dots \right]}{1 + G_p(s) \left[ K_p + \frac{K_i}{s} + K_ds + K_as^2 + \dots \right]}.$$
 (1)

In order to use CDM to design the controller parameters properly, the controlled system consisting of the CDM standard block diagram of SISO system with the feedforward and feedback controllers is proposed (see Fig. 2).  $A_p(s)$  and  $B_p(s)$  are the polynomials of the plant  $G_p(s)$ ,  $A_c(s)$ ,  $B_c(s)$ and  $B_a(s)$  are the polynomials of the CDM controller,  $B_{ff}(s)$ is the polynomial of the feedforward controller and  $B_{fb}(s)$  is the polynomial of the feedback controller. After rearranging the plant and the feedback controller shown in Fig. 2,  $A_p^*(s)$  and  $B_p^*(s)$  which are the polynomials of the modified plant  $G_p^*(s)$  can be obtained and is shown in Fig.3.



Fig. 2 CDM standard block diagram of SISO system with the feedback controller.



Fig. 3 Rearranged CDM standard block diagram.

From the block diagram of Fig. 3 it follows:

$$\frac{C(s)}{R(s)} = \frac{B_p^*(s)[B_a(s) + B_{ff}(s)A_c(s)]}{A_c(s)A_p^*(s) + B_c(s)B_p^*(s)}$$
(2)

and

$$\frac{C(s)}{D(s)} = \frac{A_c(s)B_p^*(s)}{A_c(s)A_p^*(s) + B_c(s)B_p^*(s)}.$$
(3)

It is seen from (2) and (3) that the feedforward controller  $B_{\mathcal{J}}(s)$  has an influence on the transfer function from R(s) to C(s) and can be used to increase the speed of the transient response of the controlled system, while the transfer function from D(s) to C(s) is not affected.

# 3. COEFFICIENT DIAGRAM METHOD

The CDM is an algebraic control design approach. Polynomials are used for system representation. CDM design is based on a stability index  $\gamma_i$  and an equivalent time constant  $\tau$  which will be defined later.

From Fig. 3, the transfer function of the modified plant  $G_{n}^{*}(s)$  in the polynomial forms can be expressed as

$$A_{p}^{*}(s) = p_{k}s^{k} + p_{k-1}s^{k-1} + \dots + p_{0}$$
(4a)

$$B_{p}^{*}(s) = q_{m}s^{m} + q_{m-1}s^{m-1} + \dots + q_{0}$$
(4b)

and the controller in the polynomial forms are given by

$$A_{c}(s) = l_{\lambda}s^{\lambda} + l_{\lambda-1}s^{\lambda-1} + \dots + l_{0}$$
(5a)

$$B_{c}(s) = k_{\lambda}s^{\lambda} + k_{\lambda-1}s^{\lambda-1} + \dots + k_{0}$$
(5b)

$$B_a(s) = k_0 \tag{5c}$$

where  $\lambda < k$  and m < k.  $B_a(s)$  is a pre-filter and is set to be  $k_0$  in order to obtain the step response with zero steady-state error. Hence, the characteristic polynomial of the closed-loop system is

$$P(s) = A_{c}(s)A_{p}^{*}(s) + B_{c}(s)B_{p}^{*}(s)$$
$$= \sum_{i=0}^{n} a_{i}s^{i}$$
(6)

where  $a_0, a_1, ..., a_n$  are the coefficients of the characteristic polynomial. The stability index  $\gamma_i$ , the equivalent time constant  $\tau$  and stability limit  $\gamma_i^*$  are defined as follows:

$$\gamma_i = \frac{a_i^2}{a_{i+1}a_{i-1}}$$
(7)

$$\tau = \frac{a_1}{a_0} \tag{8}$$

$$\gamma_i^* = \frac{1}{\gamma_{i+1}} + \frac{1}{\gamma_{i-1}}; \quad \gamma_0, \gamma_n = \infty$$
(9)

where i = 1,...,n-1. In general, the equivalent time constant  $\tau$  and the standard stability index  $\gamma_i$  are chosen as:

$$\tau = \frac{t_s}{2.5} \sim \frac{t_s}{3} \tag{10}$$

$$\gamma_i = [2.5, 2, 2, ..., 2] ; i = 1, ..., n - 1$$
 (11)

where  $t_s$  is the user specified settling time. In the actual design, the stability index  $\gamma_1 = 2.5$ ,  $\gamma_2 = \gamma_3 = 2$  are strongly recommended. However, for  $\gamma_{n-1} \sim \gamma_4$ , the condition can be relaxed as

$$\gamma_i > 1.5 \gamma_i^* \; ; \; n-1 \ge i \ge 4 \; .$$
 (12)

Sometimes the larger value of stability index  $\gamma_i$  is selected in order to improve robustness related parameter change. The standard stability index values stated in (11) can be used to design the controller if the following condition is satisfied

$$p_k/p_{k-1} > \tau/(\gamma_{n-1}\gamma_{n-2}...\gamma_1),$$
 (13)

where  $p_k$  and  $p_{k-1}$  are the coefficients of the plant at  $k^{th}$  and  $(k-1)^{th}$ . If the above condition is not met,  $\gamma_{n-1}$  is first increased then  $\gamma_{n-2}$  and so on, until (13) is satisfied. From (7) to (9), the coefficient  $a_i$  can be written by

$$a_{i} = a_{0} \tau^{i} \frac{1}{\gamma_{i-1} \gamma_{i-2}^{2} \cdots \gamma_{2}^{i-2} \gamma_{1}^{i-1}}.$$
 (14)

Then the characteristic polynomial can be expressed in term of  $a_0$ ,  $\tau$  and  $\gamma_i$  as

$$P(s) = a_0 \left[ \left\{ \sum_{i=2}^{n} \left( \prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^{j}} \right) (\tau s)^i \right\} + \tau s + 1 \right].$$
(15)

From the sufficient condition for stability by Lipatov, the stability is guaranteed when all  $\gamma_i$ 's are larger than 1.5. In addition, if all  $\gamma_i$ 's are greater than 4, all the roots are negative real.

# 4. CONTROLLER DESIGN

A designing method of the PDFF controller by CDM to meet desired performance criteria is discussed in this section. The parameters of PDFF controller are designed based on the stability and the speed of the controlled system. Stability is designed from the standard stability index  $\gamma_i$ , and speed is designed from the equivalent time constant  $\tau$  and the tuning factor  $\alpha$ . The stability index  $\gamma_i$ , the equivalent time constant  $\tau$ , and the tuning factor  $\alpha$  are defined based on the closedloop transfer function.

In order to derive the closed-loop transfer function of PDFF control system in term of  $\gamma_i$ ,  $\tau$  and  $\alpha$ , the closed-loop transfer function given by equation (2) may be expressed by

$$\frac{C(s)}{R(s)} = \frac{B(s)}{A(s)}$$
$$= \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0},$$
(16)

where  $m \le n$ , and *a*'s and *b*'s are constants. The denominator polynomial A(s) is the characteristic polynomial of the closed-loop system, and its coefficients can be found from

$$A(s) = a_0 \left[ \left\{ \sum_{i=2}^n \left( \prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^j} \right) (\tau s)^i \right\} + \tau s + 1 \right].$$
(17)

This equation is the same form of equation (15).

The CDM is mainly used to design the characteristic polynomial of the closed-loop system. However, this method can be extended to design the coefficients of the numerator polynomial B(s) as well [10]. Thus, the relationship among the coefficients of the numerator polynomial B(s) can be written by

$$b_{i} = b_{0} (\alpha \tau)^{i} \frac{1}{\gamma_{i-1} \cdots \gamma_{2}^{i-2} \gamma_{1}^{i-1}}$$
$$= b_{0} (\alpha \tau)^{i} \prod_{j=1}^{i-1} \frac{1}{(\gamma_{i-j})^{j}}, \qquad (18)$$

where  $\alpha$  is the tuning factor. The equivalent time constant  $\tau$  is scaled by the tuning factor  $\alpha$  so that the response speed can be adjusted. The value of tuning factor  $\alpha$  is defined as  $0 < \alpha \le 1$ . Substituting each coefficient  $b_i$  into the numerator polynomial B(s), which is expressed in term of  $\gamma_i$ ,  $\tau$  and  $\alpha$  as

$$B(s) = b_0 \left[ \left\{ \sum_{i=2}^{m} \left( \prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^{j}} \right) (\alpha \tau s)^i \right\} + \alpha \tau s + 1 \right].$$
(19)

Hence, the transfer function of PDFF control system in term of  $\gamma_i$ ,  $\tau$  and  $\alpha$  can be obtained by

$$\frac{C(s)}{R(s)} = \frac{b_0 \left[ \left\{ \sum_{i=2}^m \left( \prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^j} \right) (\alpha \tau s)^i \right\} + \alpha \tau s + 1 \right]}{a_0 \left[ \left\{ \sum_{i=2}^n \left( \prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^j} \right) (\tau s)^i \right\} + \tau s + 1 \right]}.$$
 (20)

This transfer function is a general form for designing the PDFF controller. Then, the parameters of PDFF controller can be designed by following procedures:

1) Derive the closed-loop transfer function (1) which contains the unknown parameters of PDFF controller.

2) Define the settling time  $t_s$  in order to find the equivalent time constant  $\tau$  from Equation (10), and determine the proper values of stability index  $\gamma_i$  and tuning factor  $\alpha$ . Then, substituting these parameters to the closed-loop transfer function (20).

3) The PDFF controller parameters are obtained by equating the transfer function (1) to the transfer function (20).

# 5. MODELING OF ULTRASONIC MOTOR

#### 5.1 Experimental setup



Fig. 4 Experimental system.

In this work, an ultrasonic motor model USR30 is used. The motor is driven by SHINSEI motor driver model D6030. This driver generates the two electrical signals ( $V_a$  and  $V_b$ ) of identical frequency but with a phase difference to feed the ultrasonic motor. The input of driver is the control input voltage  $V_{in}$ , which represents the desired speed of the motor. The incremental rotary encoder detects the angle position with the resolution of 0.045 degree. The rotor position is obtained by using the output pulse signal of the encoder. The experimental system of position control for ultrasonic motor is shown in Fig. 4.

#### 5.2 Mathematical model

The control input voltage  $V_{in}$  can also represent the thrust force generated from the piezoelectric ceramics and transmitted to the elastic body of stator. When the rotor is

attached to the stator with the preload, the thrust force will produce the torque T at the rotor. Therefore, the relation between the control input voltage  $V_{in}$  and the torque T at the rotor can be expressed by

$$T = k_f V_{in} \tag{21}$$

where  $k_f$  denotes the thrust force constant (N - m/V). By the Newton's second law, the torque T is applied to the inertia and friction. Hence

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T = k_f V_{in}$$
<sup>(22)</sup>

where J is the moment of inertia of the motor and load  $(kg - m^2)$ , B is the viscous-friction coefficient of the motor and load (N - m/rad/sec) and  $\theta$  is the angular displacement of the motor shaft (radian). Static and coulomb frictions would also be presented to some degree, but they must be neglected in a linearized analysis.

Assuming that all initial conditions are zero, and taking the Laplace transform of Equation (22), we obtain the following equation:

$$G_{p}(s) = \frac{\theta(s)}{V_{in}(s)} = \frac{1}{J_{n}s^{2} + B_{n}s}$$
(23)

where  $J_n = \frac{J}{k_f}$  and  $B_n = \frac{B}{k_f}$ . The model is second-order system and the system type is one.

system and the system type is one

# 5.3 Parameter identification

By applying the control input voltage  $V_{in}$  to the system of Fig. 4, the output of the system can be obtained as

$$\theta(s) = \frac{1}{s\left(J_n s + B_n\right)} \frac{V_c}{s},\tag{24}$$

where  $V_c$  is the magnitude of the constant control input. Expanding  $\theta(s)$  into partial fractions gives

$$\theta(s) = \frac{V_c}{B_n} \left[ \frac{1}{s^2} - T \frac{1}{s} + \frac{T^2}{Ts+1} \right],$$
(25)

where

$$T = \frac{J_n}{B_n}.$$
 (26)

Taking the inverse Laplace transform of equation (25), yields

$$\theta(t) = \frac{V_c}{B_n} \left[ t - T + T e^{-t/T} \right], \quad \text{for } t \ge 0$$
(27)

The response curve of equation (27) to the constant control input voltage  $V_c$  is shown in Fig. 5. As *t* approaches to infinity, the equation for the steady-state response  $\theta_{ss}$  can be written by



Fig. 5 Response to a constant control input voltage  $V_c$ .

By considering equation (28) and Fig. 5, it is found that 1) The straight line  $\theta_{ss}$  extrapolated back to the time axis

and intersects this axis at t = T. Thus, T can be found first.

2) From the slope of the steady-state response  $\theta_{ss}$   $(V_c/B_n)$ ,  $B_n$  can be found.

3) From equation (26),  $J_n$  can be found.

Therefore, the parameters of ultrasonic motor can be obtained from the experiment data.

## 6. EXPERIMENTAL RESULTS

The effectiveness of the PDFF controller developed for the position control of traveling-wave ultrasonic motor will be evaluated in this section. In order to design the PDFF controller, the model parameters of the ultrasonic motor must be known first. Applying the control input voltage 2 volts, the open-loop response of the ultrasonic motor (see Fig. 6) can be obtained.



Fig. 6 Open-loop response.

From the open-loop response curve, the slope of the steadystate response is 18.86, the straight line of steady-state response extrapolated back to the time axis and intersects this axis at 0.02 second. Thus, the model parameters  $J_n$  and  $B_n$ are 0.00212 and 0.10604 respectively. Since the plant is approximated to be a second-order system, the structures of feedforward and feedback controllers shown in Fig. 1 become as proportional-derivative control actions.  $K_d$  and  $K_p$  are the derivative and proportional gains of feedback controller respectively,  $K_i$  is the integral gain, and  $K_{df}$  and  $K_{pf}$  are the derivative and proportional gains of feedforward controller respectively. The PDFF control algorithm is first used to study the system responses of the proposed position control scheme, and then to study the disturbance rejection capability. Finally, the effect of tuning factor to the system responses is also investigated.

#### 6.1 System responses of position control

The PDFF controller parameters are designed with three sets of stability index values to study the system responses to a step input. The equivalent time constant  $\tau$  is set to be 0.4 second, for the settling time  $t_s = 1$  second, and the tuning factor  $\alpha$  is 0.55. The PDFF controller parameters are shown in Table 1. The system responses for the 90-degree step input are shown in Fig. 7.

Stability index $\gamma_i$	$K_p$	$K_i$	K <sub>d</sub>	$K_{pf}$	$K_{df}$
$\gamma_1 = 4.5,$ $\gamma_2 = 5.0$	1.3416	3.3539	0.0132	0.7379	0.0361
$\gamma_1 = 5.0,$ $\gamma_2 = 5.0$	1.6562	4.1406	0.0265	0.9109	0.0401
$\gamma_1 = 5.5,$ $\gamma_2 = 5.0$	2.0041	5.0102	0.0397	1.1022	0.0441

Table 1 Parameters of PDFF controller.

The system performances of these responses are also summarized in Table 2. It is found from the results that the three responses to 90-degree step input are almost identical. Furthermore, there are no overshoot and steady-state error.

Table 2 System performance comparison.

Stability index $\gamma_i$	$t_r$ (sec)	$t_s$ (sec)	Overshoot (%)
$\gamma_1 = 4.5, \gamma_2 = 5.0$	0.419	0.765	0.000
$\gamma_1 = 5.0, \gamma_2 = 5.0$	0.439	0.861	0.000
$\gamma_1 = 5.5, \gamma_2 = 5.0$	0.451	0.875	0.000



Fig. 7 Responses to a step input with different stability index values.

#### 6.2 System responses of constant disturbance

In this sub-section, the disturbance rejection capability is studied. The responses, that a step disturbance equivalent to 1 volt is applied to the input terminal of ultrasonic motor while PDFF control system holds the stationary position at 2 seconds, are shown in Fig. 8.



Fig. 8 Responses to a step disturbance with different stability index values.

It can be observed from the Fig. 8 that the PDFF control system can rapidly correct the position error caused by disturbance. Moreover, the effect of step disturbance is small when the PDFF controller designed by larger value of stability index.

#### 6.3 Effect of tuning factor

The tuning factor  $\alpha$  effects directly to the response speed. Normally, a faster response can be obtained from a larger value of tuning factor  $\alpha$ . However, the fast response may lead to high overshoot. Hence, the effect of tuning factor  $\alpha$  to response speed should be studied in order to find its satisfied value that gives a fast response without overshoot. As the tuning factor directly affects the response speed but not the disturbance rejection capability, only the responses to the step input with various tuning factor values are considered.



Fig. 9 Responses to a step input with various values of tuning factor.

The responses to the 90-degree step input for  $\gamma_1 = 5.5$ ,  $\gamma_2 = 5.0$  and  $\tau = 0.4$  with various values of tuning factor are shown in Fig. 9. The results show that a faster response can be obtained from a larger value of tuning factor, but the response has high overshoot. The satisfied values of tuning factor that give a fastest response without overshoot should be around 0.7.

# 6. CONCLUSIONS

The PDFF controller design by CDM to satisfy both tracking and regulation performances in position control of traveling-wave ultrasonic motor has been introduced. The reasonable mathematical model of the motor has been derived so that CDM can be employed. The transfer function of PDFF control system has been developed in general form by means of CDM. This form can be applied not only to the second-order plant, but also a higher-order plant as well. As the result, the PDFF controller parameters can be obtained easily and properly by assigning the values of stability index, equivalent time constant and tuning factor. Hence, it can be concluded that CDM is successful in the PDFF controller design for position control of traveling-wave ultrasonic motor.

#### REFERENCES

- [1] T.Sashida and T.Kenjo, "An Introduction to Ultrasonic Motors," *Clarendon Press, Oxford*, 1993.
- [2] S.Ueha and Y.Tomikawa, "Ultrasonic Motors: Theory and Applications," Oxford Science Publication, Clarendon Press, 1993.
- [3] Y.Izuno, R.Takeda, and M.Nakaoka, "New Fuzzy Reasoning-Based High-Performance Speed/Position Servo Control Schemes Incorporating Ultrasonic Motor," *IEEE Transactions on Industry Applications*, Vol. 28, 1992, pp. 613-618.
- [4] S.W.Chung, K.T.Chau, and C.C.Chan, "Neuro-Fuzzy Dual-Mode Control of Travelling-Wave Ultrasonic Motors," *IEEE International Conference of Electric Machines and Drives*, 1999, pp. 598-600.
- [5] T.Senjyu, H.Miyazato, S.Yokoda, and K.Uezato, "Position Control of Ultrasonic Motors Using Neural Network," *IEEE Technical Proceedings of Power Electronics Congress*, 1996, pp. 368-373.
- [6] T.Senjyu, H.Miyazato, and K.Uezato, "Quick and Precise Position Control of Ultrasonic Motors Taking Account of Frictional Force Control," *IEEE Industry Applications Conference*, Vol. 1, 1995, pp. 85-89.
- [7] T.Senjyu, H.Miyazato, and K.Uezato, "Quick and Precise Position Control of Ultrasonic Motors with Two Control Inputs," *IEEE Power Electronics Specialists Conference*, Vol. 1, 1995, pp. 415-420.
- [8] S.Manabe, "Coefficient Diagram Method," 14<sup>th</sup> IFAC Symposium on Automatic Control in Aerospace, 1998.
- [9] D.Y.Ohm, "A PDFF Controller for Tracking and Regulation in Motion Control," *Proceedings of 18<sup>th</sup> PCIM Conference, Intelligent Motion*, 1990, pp. 26-36.
- [10]D.Isarakorn, S.Panaudomsup, T.Benjanarasuth, J.Ngamwiwit, and N.Komine, "Application of CDM to PDFF Controller for Motion Control System," *The 4<sup>th</sup> Asian Control Conference*, 2002, pp. 1173-1177.