A Krein Space Approach for Robust Extended Kalman Filtering on Mobile Robots in the Presence of Uncertainties

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Abstract: In mobile robot navigation, one of the key problems is the pose estimation of the mobile robot. Although the odometry can be used to describe the motions of the mobile robots quite simple and accurately, the validities of the models are limited by a number of error sources contaminating the encoder outputs so that applying the conventional extended Kalman filter to these nominal model does not yield the satisfactory performance. As a remedy for this problem, we consider the uncertain nonlinear kinematic model of the mobile robot that contains the norm bounded uncertainties and also propose a new robust extended Kalman filter based on the Krein space approach. The proposed robust filter has the same recursive structure as the conventional extended Kalman filter and can hence be readily designed to effectively account for the uncertainties. The computer simulations will be given to verify the robustness against the parameter variation as well as the reliable performance of the proposed robust filter.

Keywords: robust extended Kalman filter, Krein space, uncertain nonlinear kinematic model, norm bounded uncertainty, mobile robot,

1. INTRODUCTION

The navigation of a mobile robot to the desired goal requires three major functions: a path finding, a control algorithm and a pose estimation [1]. Hence, the accurate position and orientation (heading) estimation for a mobile robot constitutes an essential component of an automatic guidance system, the problems related to which have been extensively studied by many researchers in this field [1-5]. By far, the extended Kalman filter has been the most common way of obtaining this estimate since a Kalman filter algorithm is undoubtedly optimal in the least mean square sense and, in addition, the model of the mobile robot is usually expressed by some nonlinear equations. It is well known that the extended Kalman filter uses a nonlinear system model along with measurements from internal and external sensors to maintain an estimate of the robot's pose and of a corresponding covariance matrix describing the uncertainty of the pose estimate. So, in order for the estimate to remain optimal, it is required that the model of the mobile robot is perfect. But, a problem always arises from the fact that there are inevitable errors associated with the structure of the mobile robot itself and the robot's motion [2-4], and therefore, the perfect model of the mobile robot is rarely available. If this is the case, most of the Kalman filters run in a somewhat unstable and/or undesirable manner[6-8]. Larsen et al. [2,3] tried to reduce the effects of an inaccurate model by increasing the filter's process noise covariance matrix, which is equivalent to adding fictitious process noise in the model to simulate the uncertainties.

In this paper, we treat directly the parameter uncertainties possibly contained in the nonlinear model of the mobile robot, which are described by the energy bounded constraint, i.e., sum quadratic constraint (SQC) [9] and, accordingly, propose a new method of designing a robust extended Kalman filter via an indefinite inner product space [10], i.e., Krein space approach: For an approximated model obtained from the linearization of an uncertain nonlinear model representing a mobile robot, we first construct an appropriate form of indefinite quadratic cost function and then, by inspection, introduce the corresponding Krein space state space model.

Finally, to design the Krein space robust extended Kalman filter (KREKF), we have only to apply the Krein space Kalman filter algorithm [11-13] to the Krein space model just introduced.

2. MODELS FOR MOBILE ROBOTS

If the mobile robot is equipped with two driving wheels, each of which is mounted with an odometric sensor (encoder), a very feasible and common way of designing the pose estimator is by using these encoder readings as the system model [2-4]. An example of such a mobile robot with additional passive wheels (four castor wheel) mounted at each of robot's corners is shown on figure 1.

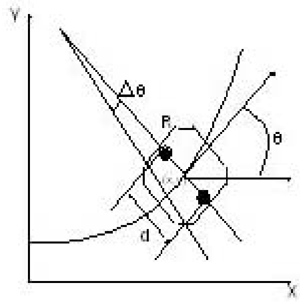

Fig. 1 A mobile robot with a dual drive and encoder system

During one sample period, the encoders will measure angular increments corresponding to the distances d_r and d_l traveled by the right and the left wheel respectively. According to Fig. 1, d_r and d_l can also be transformed to a directional deviation and a radius of the circular movement:

$$\Delta\theta_k = \frac{d_r - d_l}{d} \tag{1}$$

$$R_k = \frac{d}{2}\frac{d_r - d_l}{d_r + d_l} \tag{2}$$

where d is the wheelbase, i.e., the distance between the wheels. Three coordinates (x_k, y_k, θ_k) in a global coordinate frame constitute the state vector for the mobile robot and are observed by some additional absolute measurements z_k. The nominal model of the mobile robot can then be described by

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ \theta_{k+1} \end{bmatrix} = \begin{bmatrix} x_k - \frac{1}{4d} AB\sin\theta_k + \frac{1}{2} A\cos\theta_k \\ y_k + \frac{1}{4d} AB\cos\theta_k + \frac{1}{2} A\sin\theta_k \\ \theta_k + \frac{1}{d} B \end{bmatrix} + w_k \quad (3)$$

$$z_k = h_k(x_k, y_k, \theta_k) + v_k \quad (4)$$

where w_k and v_k represents the white Gaussian process and measurement noises, respectively, and $h_k(\cdot)$ depends on the type of sensor used for the measurement.

Note that d_r and d_l, i.e., $A = d_r + d_l$ and $A = d_r + d_l$ are used as the system inputs.

3. STATE ESTIMATION IN KREIN SPACE

In this section, first, a brief review of a theory for linear estimation in Krein space, the Krein space Kalman filter, is given as preliminaries. More detailed specifications and proofs of the theory are found in [12,13]. And second, to handle nominal nonlinear systems using this theory, we derive the Krein space extended Kalman filtering algorithm. Notations used in this paper are as follows: elements in Krein space are denoted by bold faced letters, and elements in Hilbert space are denoted by normal letters.

3.1 Kalman filter in Krein space

Consider the time-variant state-space model over C, the field of complex numbers

$$\begin{cases} x_{i+1} = F_i x_i + G_i w_i \\ y_i = H_i x_i + v_i, \end{cases} \quad i \geq 0 \quad (5)$$

where $F_i \in C^{n\times n}$, $G_i \in C^{n\times m}$, and $H_i \in C^{p\times n}$ are known matrices, w_i and v_i are unknown quantities representing the process noise and measurement noise, respectively. y_i is the measured output which is assumed known for all $i \geq 0$ and the initial condition x_0 is also unknown vector.

In many applications one is confronted with the following deterministic minimization problem: Given $\{y_i\}_{i=0}^N$, minimize over x_0 and $\{w_i\}_{i=0}^N$ the quadratic form

$$J(x_0, w, y) = x_0^* \Pi_0^{-1} x_0 + \sum_{i=0}^N \begin{bmatrix} w_i^* & v_i^* \end{bmatrix} \begin{bmatrix} Q_i & S_i \\ S_i^* & R_i \end{bmatrix}^{-1} \begin{bmatrix} w_i \\ v_i \end{bmatrix} \quad (6)$$

subject to the state-space constraints (5), and where $Q_i \in C^{m\times m}$, $S_i \in C^{m\times p}$, $R_i \in C^{p\times p}$, and $\Pi_0 \in C^{n\times n}$ are (possibly indefinite) given Hermitian matrices. Note that the symbol * denotes the Hermitian transpose operator and we have assumed the invertibility of the center matrices.

It is known that such deterministic problems can be solved via a variety of methods, such as dynamic programming or Lagrange multipliers, however, it surely seems to be easier to use 'the partial equivalence' discussed in [12,13]: compared with the (partially) equivalent Krein space (or stochastic) minimization problem, a Krein space state-space model can be introduced as follows

$$\begin{cases} \mathbf{x}_{i+1} = F_i \mathbf{x}_i + G_i \mathbf{w}_i \\ \mathbf{y}_i = H_i \mathbf{x}_i + \mathbf{v}_i, \end{cases} \quad 0 \leq i \leq N \quad (7)$$

where the initial state $\mathbf{x}_0$, and the process and measurement disturbances, $\{\mathbf{w}_i\}_{i=0}^N$ and $\{\mathbf{v}_i\}_{i=0}^N$, are such that

$$< \begin{bmatrix} \mathbf{w}_i \\ \mathbf{v}_i \\ \mathbf{x}_0 \end{bmatrix}, \begin{bmatrix} \mathbf{w}_j \\ \mathbf{v}_j \\ \mathbf{x}_0 \end{bmatrix} >= \begin{bmatrix} \begin{bmatrix} Q_i & S_i \\ S_i^* & R_i \end{bmatrix}\delta_{ij} & 0 \\ 0 & \Pi_0 \end{bmatrix} \quad (8)$$

Now if we define $\mathbf{y} := [\mathbf{y}_0^* \ \ldots \ \mathbf{y}_N^*]^*$ and denote the linear subspace spanned by the elements $\{\mathbf{y}_0, \mathbf{y}_1, \ldots, \mathbf{y}_N\}$ as $L\{\mathbf{y}\}$, instead of finding the stationary point, i.e., the minimum point of $J(x_0, w, y)$ over $\{x_0, w\}$, we can alternatively find the projection of $\{\mathbf{x}_0, \mathbf{w}\}$ onto $L\{\mathbf{y}\}$ in the Krein space model (7). Furthermore, the projection should be calculated recursively to effectively manage the successive measurement $\mathbf{y}_i$ for all i. The standard method of recursive estimation is to introduce the innovations

$$\mathbf{e}_i = \mathbf{y}_i - \hat{\mathbf{y}}_i, \quad 0 \leq i \leq N \quad (9)$$

where $\hat{\mathbf{y}}_i = \hat{\mathbf{y}}_{i|i-1}$ denotes the projection of $\hat{\mathbf{y}}_i$ onto $L\{\mathbf{y}_0, \mathbf{y}_1, \ldots, \mathbf{y}_{i-1}\}$. Thanks to the orthogonality of the innovations, the calculation of recursive projections can be simplified which leads to the following lemma.

Lemma 3.1.1 Krein space Kalman filter [12,13]

Consider the Krein space state-space model (7) with the condition (8) when $S_i = 0$. Assume that $R_y = [<\mathbf{y}_i, \mathbf{y}_j>]$ is strongly regular, i.e., nonsingular for all i. Then the innovations can be computed via the formula

$$\mathbf{e}_i = \mathbf{y}_i - H_i \hat{\mathbf{x}}_i, \quad 0 \leq i \leq N \quad (10)$$

and the measurement and time update formulas are given by

$$K_{f,i} = P_{i|i-1} H_i^* R_{e,i}^{-1} \ \ (R_{e,i} =< \mathbf{e}_i, \mathbf{e}_i >= H_i P_{i|i-1} H_i^* + R_i) \quad (11)$$

$$\hat{\mathbf{x}}_{i|i} = \hat{\mathbf{x}}_{i|i-1} + K_{f,i} \mathbf{e}_i \quad (12)$$

$$P_{i|i} = P_{i|i-1} - K_{f,i} H_i P_{i|i-1} \quad (13)$$

and

$$\hat{\mathbf{x}}_{i+1|i} = F_i \hat{\mathbf{x}}_{i|i} \quad (14)$$

$$P_{i+1|i} = F_i P_{i|i} F_i^* + G_i Q_i G_i^* \quad (15)$$

, respectively.

The only difference from the conventional Kalman filter expressions is that the matrices P_i and $R_{e,i}$ (and, by assumption, Π_0, Q_i and R_i) may now be indefinite.

It is also noted that the Krein space Kalman filter are used only to compute the stationary point, i.e., the projection of $\mathbf{x}_i$ onto $L\{\mathbf{y}_0, \mathbf{y}_1, \ldots, \mathbf{y}_i\}$. In Hilbert space, projections extremize (minimize) certain quadratic forms, however, in Krein space, it can be in general only asserted that projections stationarize such quadratic forms; further conditions need to be met for the stationary points to be extrema (minima). Among the several conditions presented in [12,13], the one that uses only quantities already present in the Kalman filter recursion, viz. $R_{e,i}$ and R_i is given below.

Lemma 3.1.2 Inertia conditions for a minimum

If $\Pi_0 > 0$ and $\oplus_{i=0}^N Q_i > 0$, then the (unique) stationary points of the quadratic forms (6), for $i = 0,1,\ldots,N$, will each be a unique minimum if, and only if, the matrices

$$R_{e,i} \text{ and } R_i$$

have the same inertia (the number of positive, negative and zero eigenvalues) for all $i = 0,1,\ldots,N$. Note that $\oplus$ represents the matrix direct sum operation.

3.2 Krein space extended Kalman filter

It is shortly mentioned that the extended Kalman filter equations for nominal nonlinear systems can be easily derived through the slight modifications and extension of the results in the previous subsection.

Consider the Krein space discrete-time nominal nonlinear system

$$\begin{cases} \mathbf{x}_{i+1} = f_i(\mathbf{x}_i) + g_i(\mathbf{x}_i)\mathbf{w}_i \\ \mathbf{y}_i = h_i(\mathbf{x}_i) + \mathbf{v}_i, \end{cases} \quad 0 \le i \le N \quad (16)$$

where the nonlinear functions $f_i(\mathbf{x}_i)$, $g_i(\mathbf{x}_i)$, and $h_i(\mathbf{x}_i)$ are assumed to be sufficiently smooth. The initial state $\mathbf{x}_0$, and the process and measurement noises, $\{\mathbf{w}_i\}_{i=0}^N$ and $\{\mathbf{v}_i\}_{i=0}^N$, are assumed to satisfy the following Gramian

$$< \begin{bmatrix} \mathbf{w}_i \\ \mathbf{v}_i \\ \mathbf{x}_0 \end{bmatrix}, \begin{bmatrix} \mathbf{w}_j \\ \mathbf{v}_j \\ \mathbf{x}_0 \end{bmatrix} >= \begin{bmatrix} \begin{bmatrix} Q_i & 0 \\ 0 & R_i \end{bmatrix}\delta_{ij} & 0 \\ 0 & \Pi_0 \end{bmatrix} \quad (17)$$

As in the conventional extended Kalman filter [14], the above nonlinear system can be approximated, by Taylor series expansion, as

$$\begin{cases} \mathbf{x}_{i+1} = F_i\mathbf{x}_i + G_i\mathbf{w}_i + \mathbf{p}_i \\ \mathbf{y}_i = H_i\mathbf{x}_i + \mathbf{v}_i + \mathbf{q}_i, \end{cases} \quad (18)$$

where

$$F_i = \left.\frac{\partial f_i(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_{i|i}}, G_i = g_i(\hat{\mathbf{x}}_{i|i}), H_i = \left.\frac{\partial h_i(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_{i|i-1}}$$

and

$$\mathbf{p}_i = f_i(\hat{\mathbf{x}}_{i|i}) - F_i\hat{\mathbf{x}}_{i|i}, \quad \mathbf{q}_i = h_i(\hat{\mathbf{x}}_{i|i-1}) - H_i\hat{\mathbf{x}}_{i|i-1}$$

, all of which are calculated on line

Since the extended Kalman filtering problem is defined by Kalman filtering problem applied to the linearized system (18), it can also be interpreted as finding a filter that minimizes a scalar quadratic form

$$J(x_0, w, y) = x_0^*\Pi_0^{-1}x_0 + \sum_{i=0}^{N} w_i^*Q_i^{-1}w_i + \sum_{i=0}^{N} v_i^*R_i^{-1}v_i \quad (19)$$

subject to (16) or (18).

Lemma 3.2.1 Krein space extended Kalman filter

Given the Krein space discrete-time nominal nonlinear system (16) with the condition (17), the extended Kalman filter, which stationarizes a scalar quadratic cost function (19) , is obtained by

(Measurement update)

$$L_i = P_{i|i-1}H_i^*R_{e,i}^{-1} \quad (R_{e,i} = H_iP_{i|i-1}H_i^* + R_i) \quad (20)$$

$$\hat{\mathbf{x}}_{i|i} = \hat{\mathbf{x}}_{i|i-1} + L_i(\mathbf{y}_i - h_i(\hat{\mathbf{x}}_{i|i-1})) \quad (21)$$

$$P_{i|i} = P_{i|i-1} - L_iH_iP_{i|i-1} \quad (22)$$

(Time update)

$$\hat{\mathbf{x}}_{i+1|i} = f_i(\hat{\mathbf{x}}_{i|i}) \quad (23)$$

$$P_{i+1|i} = F_iP_{i|i}F_i^* + G_iQ_iG_i^* \quad (24)$$

Note that the condition for a stationary point to be a minimum is the same as in lemma 3.1.2.

4. KREIN SPACE ROBUST EXTENDED KALMAN FILTER DESIGN

Now, we are ready to present the main results of this paper. In this section, the discrete-time uncertain nonlinear systems with the sum quadratic constraint (SQC) are of concern. In order to reliably estimate the states of such systems, a new filtering algorithm, rather than the conventional extended Kalman filter, needs to be developed which takes the uncertainties contained in systems into account. It will be shown that the desired filter can be successfully designed by applying the Krein space extended Kalman filter mentioned in the previous section to the given uncertain nonlinear system

4.1 Problem formulation

Consider the time-varying uncertain nonlinear discrete-time system

$$\begin{cases} x_{i+1} = f_i(x_i) + \tilde{g}_i(x_i)\tilde{w}_i \\ y_i = h_i(x_i) + \tilde{v}_i \\ s_i = k_i(x_i) \end{cases} \quad 0 \le i \le N \quad (25)$$

where $f_i(\cdot)$, $\tilde{g}_i(\cdot)$, $h_i(\cdot)$, and $k_i(\cdot)$ are the smooth nonlinear functions and $\tilde{w}_i$, $\tilde{v}_i$ are the uncertainty inputs assumed as white Gaussian noises. y_i is the measured output and s_i is the uncertainty output.

The uncertain nonlinear system (25) satisfies the following energy bounded constraint, i.e., SQC:

$$\tilde{x}_0^*\Pi_0^{-1}\tilde{x}_0 + \sum_{i=0}^{N}(\tilde{w}_i^*\tilde{Q}_i^{-1}\tilde{w}_i + \tilde{v}_i^*\tilde{R}_i^{-1}\tilde{v}_i) \le \varepsilon + \sum_{i=0}^{N}\|s_i\|^2 \quad (26)$$

where ε is an arbitrarily small positive real number regarded as a virtual energy bound and $\tilde{x}_0 = x_0 - \hat{x}_0$.

As is usually done in the extended Kalman filter, the nonlinear system functions in (25) can be expanded in Taylor series about $\hat{x}_{i|i}$ and $\hat{x}_{i|i-1}$ to yield the linearized model as follows:

$$\begin{cases} x_{i+1} = F_ix_i + \tilde{G}_i\tilde{w}_i + p_i \\ y_i = H_ix_i + \tilde{v}_i + q_i \\ s_i = K_ix_i + r_i \end{cases} \quad 0 \le i \le N \quad (27)$$

where

$$F_i = \left.\frac{\partial f_i(x)}{\partial x}\right|_{x=\hat{x}_{i|i}}, \tilde{G}_i = \tilde{g}_i(\hat{x}_{i|i})$$

$$H_i = \left.\frac{\partial h_i(x)}{\partial x}\right|_{x=\hat{x}_{i|i-1}}, K_i = \left.\frac{\partial k_i(x)}{\partial x}\right|_{x=\hat{x}_{i|i-1}}$$

and

$$p_i = f_i(\hat{x}_{i|i}) - F_i\hat{x}_{i|i}$$

$$q_i = h_i(\hat{x}_{i|i-1}) - H_i\hat{x}_{i|i-1}$$

$$r_i = k_i(\hat{x}_{i|i-1}) - K_i\hat{x}_{i|i-1}$$

Note that this linearized system is also subject to (26).

If we define an indefinite quadratic form by

$$J(x_0,\widetilde{w},\widetilde{v})=\widetilde{x}_0^*\Pi_0^{-1}\widetilde{x}_0 + \sum_{i=0}^{N}\widetilde{w}_i^*\widetilde{Q}_i^{-1}\widetilde{w}_i + \sum_{i=0}^{N}\widetilde{v}_i^*\widetilde{R}_i^{-1}\widetilde{v}_i - \sum_{i=0}^{N}\|s_i\|^2$$

The SQC (26) can be rewritten as

$$J(x_0,\widetilde{w},\overline{y})=\widetilde{x}_0^*\Pi_0^{-1}\widetilde{x}_0 + \sum_{i=0}^{N}\widetilde{w}_i^*\widetilde{Q}_i^{-1}\widetilde{w}_i + \sum_{i=0}^{N}(\overline{y}_i-\overline{H}_i x_i)^*\overline{R}_i^{-1}(\overline{y}_i-\overline{H}_i x_i)\le\varepsilon \quad (28)$$

where

$$\overline{y}_i=\begin{bmatrix} y_i-q_i \\ -r_i\end{bmatrix},\ \overline{H}_i=\begin{bmatrix} H_i \\ K_i\end{bmatrix},\ \overline{R}_i=\begin{bmatrix}\widetilde{R}_i & 0 \\ 0 & -I\end{bmatrix} \quad (29)$$

It should be, at this point, noted that the conventional extended Kalman filter cannot be directly applied to the uncertain nonlinear system (25) because the covariance matrix $\overline{R}_i$ of the fictitious measurement noise $\overline{y}_i-\overline{H}_i x_i$ is indefinite, and which motivates the use of the Krein space projection method.

The robust state estimation problem can be stated as follows: given the uncertain nonlinear discrete-time system (25) subject to (26), find the state estimates that calculate the stationary point of the indefinite quadratic form (28). This problem will be treated in the next subsection.

Before we proceed further, how the uncertain nonlinear system subject to the norm-bounded uncertainties is related to the system described by (25) and (26).

Consider the time-varying uncertain nonlinear discrete-time system

$$\begin{cases} x_{i+1}=f_i(x_i)+E_i\Delta_i k_i(x_i)+g_i(x_i)w_i \\ y_i=h_i(x_i)+v_i\end{cases} \quad 0\le i\le N \quad (30)$$

where Δ_i represents the norm-bounded uncertainty, i.e., $\|\Delta_i\|\le 1$, and x_0, w_i, v_i satisfy the following SQC in the nominal case (i.e., $\Delta_i=0$):

$$\widetilde{x}_0^*\Pi_0^{-1}\widetilde{x}_0+\sum_{i=0}^{N}w_i^*Q_i^{-1}w_i+\sum_{i=0}^{N}v_i^*R_i^{-1}v_i\le\varepsilon \quad (31)$$

If we let $\xi_i=\Delta_i k_i(x_i)$, $s_i=k_i(x_i)$ then

$$\sum_{i=0}^{N}\|\xi_i\|^2\le\sum_{i=0}^{N}\|s_i\|^2 \quad (32)$$

Adding (31) and (32) yields

$$\widetilde{x}_0^*\Pi_0^{-1}\widetilde{x}_0+\sum_{i=0}^{N}\begin{bmatrix} w_i \\ \xi_i\end{bmatrix}^*\begin{bmatrix} Q_i & 0 \\ 0 & I\end{bmatrix}^{-1}\begin{bmatrix} w_i \\ \xi_i\end{bmatrix}+\sum_{i=0}^{N}v_i^*R_i^{-1}v_i\le\varepsilon+\sum_{i=0}^{N}\|s_i\|^2 \quad (33)$$

By inspecting the relation between the system (25) and the SQC (26), the following uncertain nonlinear system can be constructed from the above SQC (33)

$$\begin{cases} x_{i+1}=f_i(x_i)+\begin{bmatrix} g_i(x_i) & E_i\end{bmatrix}\begin{bmatrix} w_i \\ \xi_i\end{bmatrix} \\ y_i=h_i(x_i)+v_i \\ s_i=k_i(x_i)\end{cases} \quad 0\le i\le N \quad (34)$$

Therefore, it can be asserted that the uncertain nonlinear system of the form (25) includes such uncertain nonlinear system subject to the norm-bounded uncertainty as (30), through the several replacements as follows:

$$\widetilde{g}_i(x_i)=\begin{bmatrix} g_i(x_i) & E_i\end{bmatrix},\ \widetilde{Q}_i=\begin{bmatrix} Q_i & 0 \\ 0 & I\end{bmatrix},\ \widetilde{R}_i=R_i \quad (35)$$

$$\widetilde{w}_i=\begin{bmatrix} w_i \\ \xi_i\end{bmatrix},\ \widetilde{v}_i=v_i \quad (36)$$

4.2 Krein space robust extended Kalman filter

To apply the Krein space estimation methodology of Section 3, from the indefinite quadratic cost function (28) identified for the robust extended Kalman filtering problem, we can introduce the following Krein space state space model

$$\begin{cases} \mathbf{x}_{i+1}=F_i\mathbf{x}_i+\widetilde{G}_i\widetilde{\mathbf{w}}_i+\mathbf{p}_i \\ \begin{bmatrix} \mathbf{y}_i \\ \mathbf{0}\end{bmatrix}=\begin{bmatrix} H_i \\ K_i\end{bmatrix}\mathbf{x}_i+\begin{bmatrix}\widetilde{\mathbf{v}}_i \\ -\mathbf{s}_i\end{bmatrix}+\begin{bmatrix}\mathbf{q}_i \\ \mathbf{r}_i\end{bmatrix}\end{cases} \quad 0\le i\le N \quad (37)$$

with

$$\left\langle \begin{bmatrix}\widetilde{\mathbf{w}}_i \\ \begin{bmatrix}\widetilde{\mathbf{v}}_i \\ -\mathbf{s}_i\end{bmatrix} \\ \mathbf{x}_0\end{bmatrix}, \begin{bmatrix}\widetilde{\mathbf{w}}_j \\ \begin{bmatrix}\widetilde{\mathbf{v}}_j \\ -\mathbf{s}_j\end{bmatrix} \\ \mathbf{x}_0\end{bmatrix}\right\rangle = \begin{bmatrix}\begin{bmatrix}\widetilde{Q}_i & 0 \\ 0 & \begin{bmatrix}\widetilde{R}_i & 0 \\ 0 & -I\end{bmatrix}\end{bmatrix}\delta_{ij} & 0 \\ 0 & \Pi_0\end{bmatrix} \quad (38)$$

Now, using lemma 3.2.1, the Krein space robust extended Kalman filter can be easily derived

Theorem 4.2.1

Krein space robust extended Kalman filter (KREKF);

Given the Krein space discrete-time uncertain nonlinear system (37) with the condition (38), the robust extended Kalman filter, which stationarizes a scalar quadratic cost function (28) , can be recursively computed via the following formulas

(Measurement update)

$$\overline{L}_i=\overline{P}_{i|i-1}\overline{H}_i^*\overline{R}_{e,i}^{-1}\ \ (\overline{R}_{e,i}=\overline{H}_i\overline{P}_{i|i-1}\overline{H}_i^*+\overline{R}_i) \quad (39)$$

$$\hat{\mathbf{x}}_{i|i}=\hat{\mathbf{x}}_{i|i-1}+\overline{L}_i\begin{bmatrix}\mathbf{y}_i-h_i(\hat{\mathbf{x}}_{i|i-1}) \\ -k_i(\hat{\mathbf{x}}_{i|i-1})\end{bmatrix} \quad (40)$$

$$\overline{P}_{i|i}=\overline{P}_{i|i-1}-\overline{L}_i\overline{H}_i\overline{P}_{i|i-1} \quad (41)$$

(Time update)

$$\hat{\mathbf{x}}_{i+1|i}=f_i(\hat{\mathbf{x}}_{i|i}) \quad (42)$$

$$\overline{P}_{i+1|i}=F_i\overline{P}_{i|i}F_i^*+\widetilde{G}_i\widetilde{Q}_i\widetilde{G}_i^* \quad (43)$$

It is not difficult to see that, for the robust extended Kalman filter case, the inertia condition for a stationary point to be a minimum is the same as in lemma 3.1.2 except that Q_i, $R_{e,i}$, and R_i have been replaced by $\widetilde{Q}_i$, $\overline{R}_{e,i}$, and $\overline{R}_i$, respectively.

5. COMPUTER SIMULATIONS

In this section, to verify the performance of the proposed robust filter, we assume that the motion description (3) of the mobile robot is contaminated by the norm-bounded uncertainty, which may be considered as the uncertainty about the effective wheelbase (due to nonpoint wheel contact with floor) and, for the comparison purpose, the conventional extended Kalman filter is also applied.

Let $X_i=[x_i, y_i, \theta_i]^T$ and, then the uncertain nonlinear model of the mobile robot is given by

$$X_{i+1} = X_i + \begin{bmatrix} -\frac{D}{4}AB\sin\theta_i + \frac{1}{2}A\cos\theta_i \\ \frac{D}{4}AB\cos\theta_i + \frac{1}{2}A\sin\theta_i \\ DB \end{bmatrix} + E_i\Delta_i k_i(X_i) + w_i \tag{44a}$$

$$z_i = X_i + v_i \tag{44b}$$

where

$$E_i = \begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.18 \end{bmatrix}, \quad k_i(X_i) = \begin{bmatrix} AB\sin\theta_i \\ AB\cos\theta_i \\ B \end{bmatrix} \tag{45}$$

and the process and measurement noises are zero mean white Gaussian processes with the covariances of

$$Q_i = 0.1\times10^{-6}\times I, \quad R_i = 0.1\times10^{-3}\times I$$

, respectively. The sampling time and the total runtime are chosen to be 0.01s and 30s, respectively, and all the other parameters are defined the same as in Section 2.

When the initial state is given by

$$x_0 = [5.14m \quad 5.43m \quad 30.7^o]^T$$

, the actual trajectory and the estimates of both filters are shown in Fig. 2.

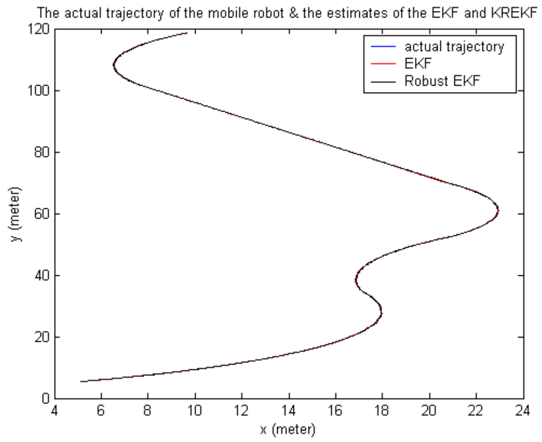

Fig. 2 The actual trajectory of the mobile robot and the estimates of EKF and KREKF

Although, from Fig. 2, the estimates of the EKF and the KREKF seem to be not much different, when we plot the estimation errors for each of the position x, y and the orientation θ, the difference between two filters becomes salient, which are depicted in Fig. 3, Fig. 4, and Fig. 5, respectively.

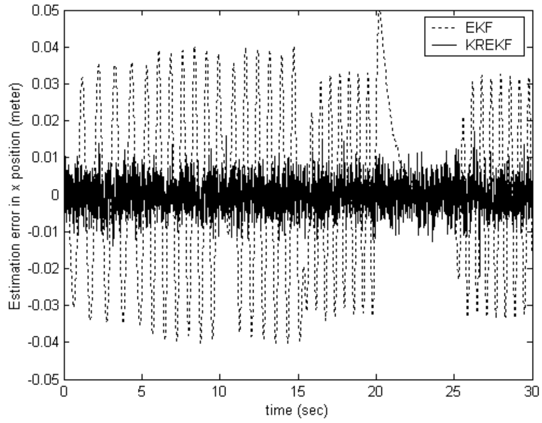

Fig. 3 The x position estimation errors of EKF and KREKF when $\Delta = -1$

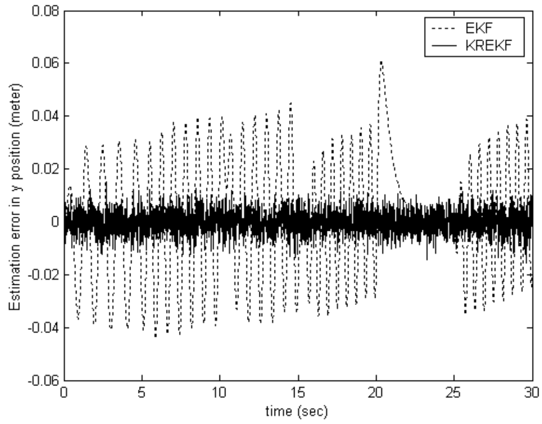

Fig. 4 The y position estimation errors of EKF and KREKF when $\Delta = -1$

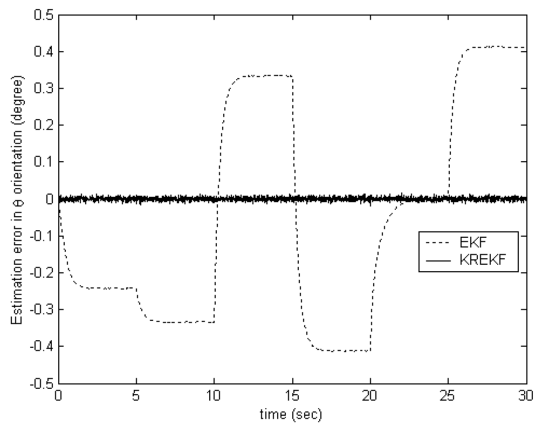

Fig. 5 The θ orientation estimation errors of EKF and KREKF when $\Delta = -1$

Now, for the uncertainty values of $\Delta = -1$, 0 and 1, the estimation error variances of all state are summarized, for both EKF and KREKF, in Table 1 below

Table 1 Estimation error variances for different values of Δ

		x	y	θ
$\Delta = -1$	EKF	5.47×10^{-4}	5.79×10^{-4}	9.15×10^{-2}

	KREKF	1.96×10^{-5}	2.01×10^{-5}	2.09×10^{-5}
$\Delta=0$	EKF	7.14×10^{-7}	6.30×10^{-7}	5.99×10^{-7}
	KREKF	1.90×10^{-5}	1.92×10^{-5}	1.88×10^{-5}
$\Delta=1$	EKF	2.72×10^{-4}	3.86×10^{-4}	8.87×10^{-2}
	KREKF	1.99×10^{-5}	1.96×10^{-5}	2.03×10^{-5}

It is observed that, regardless of whether the uncertainty exists or not, the error variances of all states for KREKF are hardly changed, while those for EKF are rather fluctuated. Specifically, the estimation performance of KREKF, when there exists the uncertainty, is apparently better than that of EKF, and not much deteriorated even with no uncertainty at all.

6. CONCLUSIONS

As an important part of the mobile robot navigation, to formulate the position and orientation estimation problem more realistically, we have directly treated the parameter uncertainties possibly contained in the nonlinear model of the robot, which have been described by SQC, and, accordingly, also proposed a new method of designing a robust extended Kalman filter via Krein space approach. To validate the usefulness of the proposed robust filter, numerical computer simulations have been done. It has been verified that, in spite of the existence of the parameter uncertainties, KREKF exhibits the steady and acceptable performance, which is not the case in the conventional EKF. Consequently, it can be asserted that the proposed KREKF is more robust against the parameter uncertainties and superior, in the overall performance's view, to the conventional EKF.

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