Estimation of vehicle cornering stiffness via GPS/INS

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Abstract: This paper demonstrates a unique method for measuring vehicle states such as body sideslip angle and tire sideslip angle using Global Positioning System(GPS) velocity information in conjunction with other sensors. A method for integrating Inertial Navigation System (INS) sensors with GPS measurements to provide higher update rate estimates of the vehicle states is presented, and the method can be used to estimate the tire cornering stiffness. The experimental results for the GPS velocity-based sideslip angle measurement. From the experimental results, it can be concluded that the proposed method has an advantage for future implementation in a vehicle safety system.

Keywords: GPS, INS, Cornering stiffness, Side slip angle, Vehicle, parameter estimation

1. INTRODUCTION

In the vehicle dynamics, emergency of driving is friction coefficient between tire and road goes to physical limit. In this situation slip angle of vehicle is increase and sensitive of yaw moment of steering angle is decrease. It means that as steering angle is increase, yaw moment is not changed. So reliability of vehicle is the beyond the control. Objectivity of vehicle dynamic control (VDC) system is to control the vehicle in lateral direction as driver's will. To do this, separating brake to left and right side tire that makes yaw moment to recover the stability of vehicle is need. In previous time vehicle yaw moment is occurred by driver's steering but in VDC system vehicle detects the emergency, braking in each tire is controlled by VDC algorithm not related driver's steering and that makes yaw moment. There are 3 technologies to develop VDC system

1) detector vehicle stability and control algorithm
2) estimator of vehicle and road state
3) design the actuator to distribute the braking.

Especially to design estimator of vehicle and road state it is necessary to estimate sideslip angle and cornering stiffness and essential physical parameter to make VDC system [1][2]. Sideslip angle can be estimate to use yaw angular velocity estimator[3], non-linear estimator and inertial measurement sensor[4][5]. But when vehicle approaches to neutral steering that vehicle maintain constant circular rotation and front wheel and real wheel of vehicle is taken lateral force, it is impossible to measure sideslip angle from yaw angular velocity and steering angle. Cornering stiffness shows that relation between tire and road of vehicle. It is not easy to measure by direct sensor measurement directly. When the vehicle cruises at high speed, the tire also rotates at high speed. It is hard to mount a precision sensor to vehicle. To solve this problem, there are methods to estimate important variables indirectly. Masmoudi and Hedrick estimated the axis of vehicle torque by using sliding mode estimator[6], Ray estimated tire force by using Kalman filter and using that force estimated the road friction coefficient[7]. And observer is used to estimate state variable. Huh and Stein define the gain to get well-condition by scaling the system matrix and designs the observer that robust to noise [8]. But these methods have many restrict conditions so it is hard to apply to real situation.

Therefore, in this paper, we use the new method to measure the sideslip and cornering stiffness. From GPS/INS navigation system we can get the position and velocity of vehicle. A method for integrating sensors with GPS measurements to provide higher update rate estimates of the vehicle states is presented can be used to estimate the cornering stiffness of tire. The experimental results for the GPS/INS velocity-based sideslip angle measurement.

2. STATE MEASUREMENT

The bicycle model shown in Figure 1 is 2 DOF vehicle model in a plane with lateral velocity and yaw rate as the states. The lateral and yaw motions are described in Equation (1)

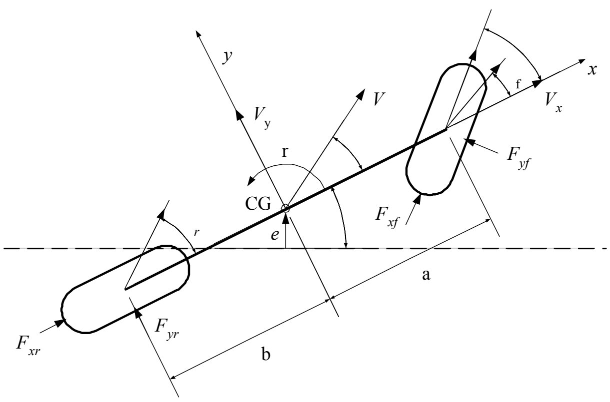

Fig. 1 Simple Bicycle Model of Vehicle

$$\begin{bmatrix}\dot V_y\\ \dot r\end{bmatrix} = \begin{bmatrix} \frac{-C_{\alpha f}-C_{\alpha r}}{mV_x} & -V_x+\left(\frac{C_{\alpha r}b-C_{\alpha f}a}{mV_x}\right)\\ \frac{C_{\alpha r}b-C_{\alpha f}a}{I_zV_x} & \frac{-C_{\alpha f}a^2-C_{\alpha r}b^2}{I_zV_x}\end{bmatrix}\begin{bmatrix}V_y\\ r\end{bmatrix}+\begin{bmatrix}\frac{C_{\alpha f}}{m}\\ \frac{C_{\alpha f}a}{I_z}\end{bmatrix}\delta \quad (1)$$

x, y: frame that based from C.G of vehicle
V : velocity vector of vehicle
F$\boldsymbol{F_{xf}}$, $\boldsymbol{F_{xr}}$: force that act to front and rear wheel to tire's latitude axis
$\boldsymbol{F_{yf}}$, $\boldsymbol{F_{yr}}$: force that act to front and rear wheel to tire's longitudinal axis
r : yaw rate of vehicle
a, b : distance from front and rear wheel to C.G of vehicle
$\boldsymbol{a_f}$, $\boldsymbol{a_r}$: side slip angel of front and rear wheel
b : angle between center line and velocity vector from C.G. of vehicle
d : steering angle of front wheel
y : angle between Earth-centered earth-fixed frame and vehicle frame.

Using that, lateral slip of vehicle can be measured.

$$\beta = \psi_{GPS}^{VEL} - \psi \tag{2}$$

ψ_{GPS}^{VEL} is the angle of velocity vector of vehicle that measured by GPS. Angle between vehicle center line and earth frame is estimated by using yaw gyro. Gyro is initialized when vehicle go a straight line and bias can be estimated and removed. But bias is varied as time. To correct bias, attitude data of GPS is used, and corrects the ψ. Lateral velocity of vehicle can be calculated by following equation.

$$V_y^{GPS} = V^{GPS} \sin(\beta) \tag{3}$$

Slip angle that measured by GPS is slip angle of point that GPS antenna is mounted so it is not actual slip angle of C.G, front and rear wheel of vehicle. To get the velocity of point P by GPS antenna A, relative velocity vector from point A to point P is needed.

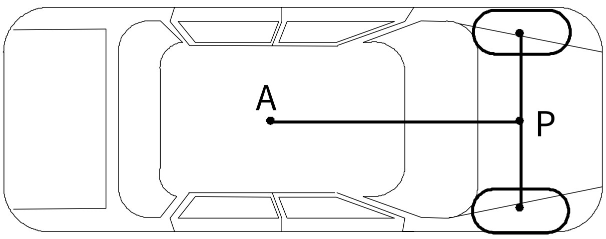

Fig. 2 Position of A and P

$$V_p = V_A + \vec{\omega} \times \vec{r}_{A/P} \tag{4}$$

ω includes roll, pitch and yaw of vehicle. we can get velocity of point P by using measurement of ω,. To get the slip angle of point P, we can use the following equation.

$$\beta_p = \tan^{-1}\left(\frac{V_Y^P}{V_X^P}\right) \tag{5}$$

Vx, Vy is velocity that measured from vehicle base coordinate. Slip angles of tire (α_f and α_r from Fig. 1) are the angle between actual tire direction and moving direction of tire. It is caused by difference of distance between CG axis of car. So using the velocity measurement of CG and distance from CG to car axis, velocity vector of front and rear tire can be estimated by using equation (4) and side slip angle of each tire can be estimated by using (5). In case of rear wheel, direction of tire and vehicle are same so side slip angle estimated above equation can be applied directly. But in case of front wheel, side slip angle contains steering angle so side slip angle obtains to remove steering angle.

$$\begin{aligned} \alpha_f &= \beta_f^{tire} - \delta \\ \alpha_r &= \beta_r^{tire} \end{aligned} \tag{6}$$

Cornering stiffness of front and rear wheel is estimated by using acceleration of GPS measurement through Kalman filtering. A lot of method exists to estimate cornering stiffness. In this paper cornering stiffness can be obtain using following Newton equation in bicycle model.

$$\begin{aligned} \sum F_y &= m\ddot{y} = F_{yf} + F_{yf}\cos(\delta) \\ \sum M_z &= I_z\ddot{\psi} = aF_{yf} - bF_{yr}\cos(\delta) \end{aligned} \tag{7}$$

Force of side direction of front and rear tire is obtained by definition of cornering stiffness.

$$\begin{aligned} F_{yf} &= 2C_{\alpha f}\alpha_f \\ F_{yr} &= 2C_{\alpha r}\alpha_r \end{aligned} \tag{8}$$

Substitute (7) for (8), cornering stiffness can be obtain as following equations.

$$\begin{aligned} C_{\alpha f} &= \frac{bm\ddot{y} + I_z\ddot{\psi}}{2(a+b)\alpha_f} \\ C_{\alpha r} &= \frac{am\ddot{y} - I_z\ddot{\psi}}{2(a+b)\alpha_r\cos(\delta)} \end{aligned} \tag{9}$$

3.Error model for Kalman filter

Algorithm for integrate GPS/INS is followed[9]. State variable for system is defined as following

$$x = [\delta P^T \quad \delta V^T \quad \gamma^T \quad \varepsilon_g^T \quad \varepsilon_a^T]^T \tag{10}$$

Measurement is

$$y = \begin{bmatrix} \delta P_{GPS}^e \\ \delta\theta \end{bmatrix} \tag{11}$$

Then state equation for error model is

$$\dot{x} = Ax + w$$
$$y = Cx + v \tag{12}$$

where,

$$A = \begin{bmatrix} 0 & I_3 & 0 & 0 & 0 \\ G & -\Omega_e & -RF & 0 & R \\ 0 & 0 & -\Omega & I_3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{13}$$

$$C = \begin{bmatrix} I_3 & 0 & -RL_1 & 0 & 0 \\ 0 & 0 & I_3 & 0 & 0 \end{bmatrix} \tag{14}$$

$$w = \begin{bmatrix} 0 & (Rw_a)^T & (w_g)^T & 0 & 0 \end{bmatrix}^T$$

$$v = [-v_p^T \quad -v_\theta^T]^T$$

Error of accelerometer and gyro is $0.05\ m/s^2$ and $0.1 \times \pi / 180$ rad. Position and attitude error of GPS is 0.02m and $0.5 \times \pi / 180$ rad. Covariance matrix of process and measurement noise is as followed

$$E[ww^T] = R$$
$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & (0.05)^2 I_3 & 0 & 0 & 0 \\ 0 & 0 & \left(0.1 \times \frac{\pi}{180}\right)^2 I_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{15}$$

$$E[vv^T] = Q$$
$$= \begin{bmatrix} (0.02)^2 I_3 & 0 \\ 0 & \left(0.5 \times \frac{\pi}{180}\right)^2 I_3 \end{bmatrix} \tag{16}$$

4. Experiment

Cornering stiffness and side slip angle are measured by vehicle experiment. IMU, GPS receiver and PDL for RTK mounted to vehicle. IMU data and GPS data is received using serial interface. To get 3 difference serial data, serial expand card is used. Data of each sensor is recorded in Pentium PC. After recording, result of travel is acquired though post-processing. Post processing is used to get various travel data. This experiment takes place in Pusan National Unversity playground. System cosists of GPS/INS navigation system, steering angle measurement and PC that recording the whole data.. Experimental vehicle is KIA Sportage. Specification of this vehicle is as followed. Yaw moment and position of C.G. is used approximated value. .

Table 1. Specification of vehicle

Wheelbase(m)	2.65
a (distance from C.G. to front wheel, m)	1.4
b (distance from C.G. to rear wheel, m)	1.25
Curb weight (kg)	1465
Total yaw inertia(kg.m²)	2931

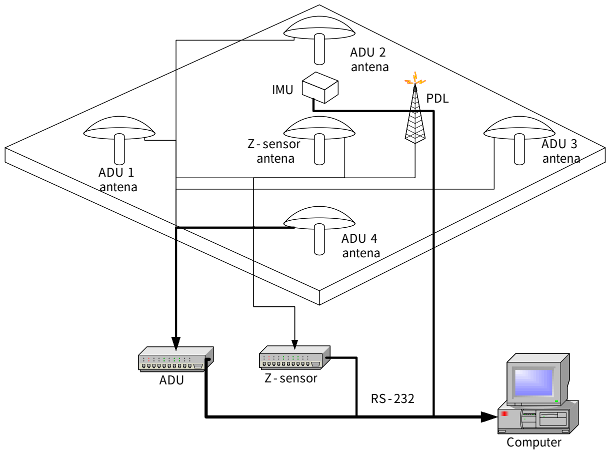

Fig. 3 Diagram of Measurement system at vehicle

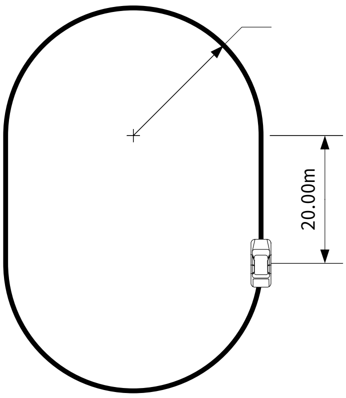

Fig. 4 Layout of test track

Two GPS are used. One is Z-Sensor of Ashtech. It is used to get the precise position. It has 1cm horizontal error and provide 1Hz measurement. The other is ADU2. It is used to get the attitude. It has 4 antennas and uses 12 channels. PDL of PACIFIC CREST is used to send and receive data between base and remote. Maximum transmitting speed is 19,200 Baud Rate. DMU-6X of Crossbow is used to get accelerations of 3-axis accelerometers and angular rate of 3 axis gryos. Maximum data transmitting rate is 38400 Baud Rate.

To measure steering angle of vehicle, potensiometer is equipped to steering wheel. Figure 3 shows diagram of measurement system at vehicle. Test vehicle follows the track that drawn in test field. Tracks shape is shown as follow Figure 5 shows the side slip angle that calculated by equation (5).

Direction of traveling vehicle is counter clock wise direction so expected side slip angle direction is one side. But Figure 5 shows that side slip produces both sides. It observed when vehicle turns a corner and go through straight line. Speed of front wheel and rear wheel is calculated from equation (4) that substituted speed that measured center of gravity of vehicle. Side slip angle is estimated from substituting them to equation (5) and (6) Figure 6 shows side slip angle of front and rear wheel

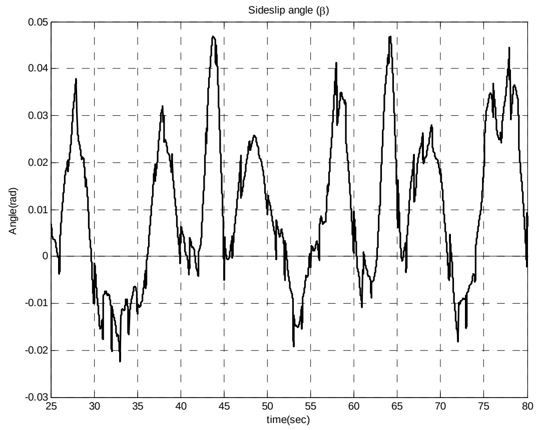

Fig. 5 Calculated sideslip angle

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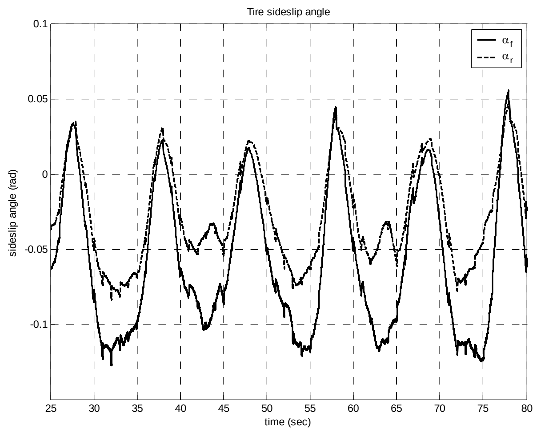

Fig. 6 Tire sideslip angle

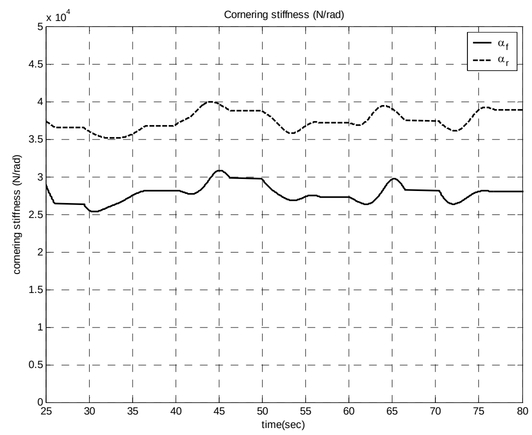

Fig. 7 Cornering stiffness of front and rear

Front wheel is more side slipping than rear wheel. Cornering stiffness of front and rear wheel is acquired that substitute side slip angle to equation (9). Figure 7 shows the cornering stiffness of test vehicle. Mean cornering stiffness of front wheel and rear wheel is estimated 28300N/rad and 38400N/rad. General cornering stiffness on asphalt pavement is 60000N/rad. Because this experiment is on unpaved and wet road, Cornering stiffness of this experiment is lower than on an asphalt pavement. Cornering stiffness of front wheel is lower than rear wheel. Vehicle turn makes more side slipping for front wheel than rear wheel.

5. CONCLUSION

In this paper, it is shown that cornering stiffness that parameter of bicycle model can be estimated using GPS and INS integration. To integrate GPS and INS, Kalman filter is used. Using integrated GPS/INS system, position, velocity and attitude of vehicle is measured. Using these parameters, side slip angle and cornering stiffness is estimated. Side slip angle is estimated using yaw angle that measured gyro in IMU. It is more delicate than other estimation method. To estimate side slip angle, no parameter of vehicle is used, there is no error from uncertain parameter.

GPS/INS integration is post-process, and using post-processed parameter side slip angle is estimated. But to estimate state of vehicle in rear time, it is needed to develop GPS/INS integration system and algorithm in real time.

Vibration in vehicle is an important problem. In this paper, Vibration in vehicle is contained in sensor measurements. After FFT analysis about sensor measurements, vibration makes white noise in whole frequency. In this paper, smoothing is used. To develop real time system, low frequency filter is used to IMU. But that makes delay of signal and it can make problem synchronize with GPS signal. So removing sensor noise vibration from vehicle is important problem

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