Analysis of Internal Loading for Triple Manipulator Robotics

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Abstract: Multiple robotics systems with several sub-chains have a characteristic that grasp a common object with internal loading not to loose the grip. The investigation for the internal loading of a triple manipulator has been few as compared to a dual manipulator. In this paper, type of the internal loading for a triple manipulator system is investigated through analysis of the null space of the system. Several types of the internal loading are shown for general planar and spatial type triple robots, which rigidly grasp the common object. The general scheme is applied to analysis of the internal loading for the three-fingered and three-legged robots having a point contact with the grasped object.

Keywords: Multiple robotics, Internal Loading, Contact Point

1. INTRODUCTION

Analysis of internal loading at the multiple robotic systems has been a hot research area. Walker[1] proposed a weighted pseudo-inverse solution that remove an internal loading in particular solution. Albert[2] proposed an weighting matrix in order to minimize unwanted moment at the grasping space. Kumar[8] analyzed an internal loading as interaction force at the walking vehicle and the multi-fingered robot, and proposed no interaction force condition. Lipkin[6] analyzed geometry of the wrench at the optimal distribution and defined physical meaning of internal loading using the concept of wrench and twist. Nakamura[5] defined an internal loading using virtual work principle and optimized an internal loading using the condition of the static friction constraint. Cheng[9] addressed configuration of two closed chains at the multiple robotics system and force balance at the contact point. Uchiyama[4] showed the type of the internal loading for the two-arm. Nahon[7] used a weighting matrix in order to unify the units of force and moment at the algorithm minimizing internal loading. He also minimized internal loading using quadratic form and two constraint conditions. Choi[3] proposed minimized constraint condition by the quadratic form, optimized force distribution using a minimized internal loading, but does not describe type of the exact internal loading. Zuo[12] addressed the difference between internal force and interaction force. So, internal force consists of interaction and parallel force at the contact point. Kerr[13] described an internal force as grasping force at the multi-fingered hands and proposed optimal selection of internal grasp force using linear programming. Yoshikawa[11, 14] defined grasping force that is an internal force satisfying the friction constraint and manipulating force, and also proposed the virtual truss as a model of the object grasped at n contact points. Li[15] proposed the algorithm for three-finger force-closure grasp.

So far, the definition of the internal loading has not been classified and generalized. In this paper, we analyze internal loading using reduced row echelon method at the triple manipulator and show the shape of the internal loading basis. We explain the concept of internal force through planar and spatial type triple manipulator systems. Three-fingered and three-legged systems are illustrated as special cases.

2. INTERNAL LOADING AT THE TRIPLE MANIPULATOR

2.1 Concept of Internal Loading

The invisible force and moment are exerted on the grasped object by the multiple robotic arms. An internal loading is defined as the forces and moments that do not affect the motion of the end-effector. The relationship between the grasping force and moment at the grasping space and the operational force and moment at the object space can be described as

$$\underline{P} = G\underline{F} \ , \tag{1}$$

where

$$\underline{F}_i = \begin{bmatrix} f_{xi} & f_{yi} & f_{zi} & n_{xi} & n_{yi} & n_{zi} \end{bmatrix}^T , \tag{2}$$

$$\underline{F} = \begin{bmatrix} \underline{F}_1^T & \underline{F}_2^T & \cdots & \underline{F}_n^T \end{bmatrix}^T , \tag{3}$$

and

$$\underline{P} = \begin{bmatrix} f_x & f_y & f_z & n_x & n_y & n_z \end{bmatrix}^T . \tag{4}$$

$\underline{F}_i$ denotes the force and moment of the i-*th* manipulator and $\underline{F}$ represents the force and moment vector of n manipulators. $\underline{P}$ denotes the resultant force and moment at the object space. Then, the general solution of Eq. (1) can be expressed as

$$\underline{F} = G^+\underline{P} + \left(I - G^+G\right)\underline{\varepsilon} \ , \tag{5}$$

where $\underline{\varepsilon}$ denotes an arbitrary $(6n)\times 1$ vector, and

$$G = \begin{bmatrix} I_3 & 0_3 & I_3 & 0_3 & \cdots & I_3 & 0_3 \\ S_1 & I_3 & S_2 & I_3 & \cdots & S_3 & I_3 \end{bmatrix}, \tag{6}$$

$$S_i = \begin{bmatrix} 0 & -r_{z_i} & r_{y_i} \\ r_{z_i} & 0 & -r_{x_i} \\ -r_{y_i} & r_{x_i} & 0 \end{bmatrix}, \tag{7}$$

where $G \in R^{6\times 6m}$ and S_i denote the transformation matrix and a skew symmetric matrix formed with the position vector $\underline{r}_i = \begin{bmatrix} r_{x_i} & r_{y_i} & r_{z_i} \end{bmatrix}^T$ of the end-effector at the *i th* manipulator, respectively. I_3 and 0_3 denote 3 by 3 identity and zero matrix, respectively. The weighted pseudo-inverse solution with row full-rank can be expressed as

$$G^+ = W^{-1}G^T\left(GW^{-1}G^T\right)^{-1} \in R^{6m\times 6} \ , \tag{8}$$

where W denotes a weighting matrix. Doty[10] addressed the weighted pseudo-inverse for more general case. The first term on the right-hand side of Eq. (5) represents a particular solution, and the second term denotes a homogenous solution that creates internal loading without affecting the motion involved in the particular solution. The number of internal loadings for a multiple manipulator system consisting of n arms is $3\times n-3$ in the plane and $6\times n-6$ in the space. In this paper, the geometry of $\left(I-G^{+}G\right)$ will be analyzed.

2.2 Internal loading on planar domain

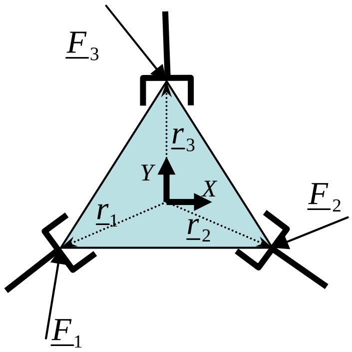


Fig. 1. Triple manipulator in planar domain

When three manipulators rigidly grasp a common object in the planar domain, the position vector of each manipulator is $\underline{r}_1=\left[-1\quad -1/\sqrt{3}\right]^T$, $\underline{r}_2=\left[1\quad -1/\sqrt{3}\right]^T$, $\underline{r}_3=\left[0\quad 2/\sqrt{3}\right]^T$. The transformation matrix can be expressed as

$$G=\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ -r_{y1} & r_{x1} & 1 & -r_{y1} & r_{x1} & 1 & -r_{y1} & r_{x1} & 1 \end{bmatrix} \tag{9}$$

and $\underline{F}_i$ for each manipulator can be expressed as $\left[f_{xi}\quad f_{yi}\quad n_{zi}\right]^T$. Then, the internal loading matrix can be obtained as

$$I-G^{+}G=$$

$$\begin{bmatrix} 0.62 & 0.08 & -0.08 & -0.38 & -0.08 & -0.08 & -0.24 & 0 & -0.08 \\ 0.08 & 0.52 & 0.14 & 0.08 & -0.19 & 0.14 & -0.17 & -0.33 & 0.14 \\ -0.08 & 0.14 & 0.86 & -0.08 & -0.14 & -0.14 & 0.17 & 0 & -0.14 \\ -0.38 & 0.08 & -0.08 & 0.62 & -0.08 & -0.08 & -0.24 & 0 & -0.08 \\ -0.08 & -0.19 & -0.14 & -0.08 & 0.52 & -0.14 & 0.17 & -0.33 & -0.14 \\ -0.08 & 0.14 & -0.14 & -0.08 & -0.14 & 0.86 & 0.17 & 0 & -0.14 \\ -0.24 & -0.17 & 0.16 & 0.24 & 0.17 & 0.17 & 0.48 & 0 & 0.17 \\ 0 & -0.33 & 0 & 0 & -0.33 & 0 & 0 & 0.67 & 0 \\ -0.08 & 0.14 & -0.14 & -0.08 & -0.14 & -0.14 & 0.17 & 0 & 0.86 \end{bmatrix}. \tag{10}$$

This matrix is a 9 by 9 square matrix whose rank is 6. Therefore, the dimension of the internal loading basis is 6, and by using the row-reduced echelon, the 6 vectors consisting of the basis can be obtained as

$$\begin{bmatrix}1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T \tag{11}$$

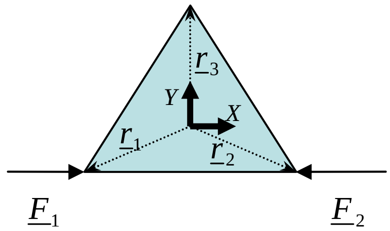
$$\begin{bmatrix}0 & 1 & 0 & 0 & -1 & 2 & 0 & 0 & 0\end{bmatrix}^T \tag{12}$$

$$\begin{bmatrix}0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0\end{bmatrix}^T \tag{13}$$

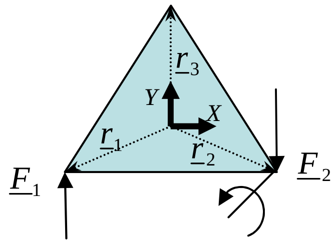
$$\begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & -1.73\end{bmatrix}^T \tag{14}$$

$$\begin{bmatrix}0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & -1\end{bmatrix}^T \tag{15}$$

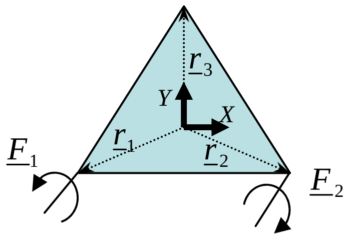
$$\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1\end{bmatrix}^T \tag{16}$$



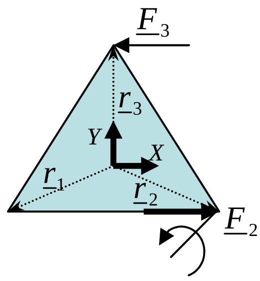
(a) Internal force in the X direction



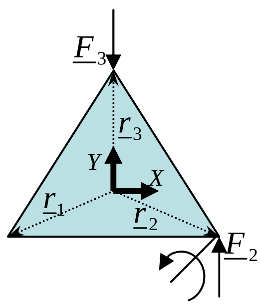
(b) Internal force in the Y direction



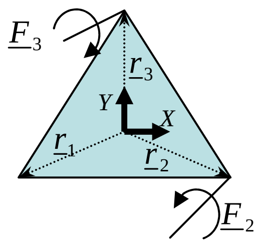
(c) Internal moment in the Z direction



(d) Internal force in the X direction



(e) Internal force in the Y direction



(f) Internal moment in the Z direction

Fig. 2. Shape of internal loading basis on the planar domain

Fig. 2 (b), (d), and (e) show the internal loading that comes from the combination of internal force and moment. If the internal force that is symmetric to the referenced coordinate is exerted on the $\left(\underline{r}_i - \underline{r}_j\right)$ line, then the moment do not occur. However, the internal forces are generally coupled to internal moment, while the internal moments occur independently because the moment is a free vector. Fig.2 (c) and (f) show the case that the internal moments occur independently.

2.3 Internal loading on spatial domain

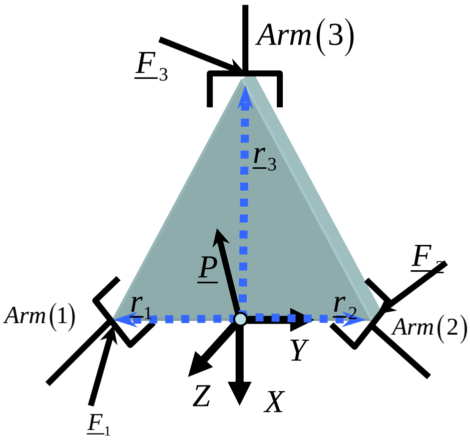


Fig. 3. Triple manipulator in spatial domain

When a robotic system consists of three manipulators at space, the number of independent internal loading will be 12. Analysis of the internal loading for the triple arm has not been addressed so far because of the complexity of the internal loading. When the positions of the end-effectors for the three manipulators are given by

$\underline{r}_1 = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}^T$, $\underline{r}_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$, $\underline{r}_3 = \begin{bmatrix} -\sqrt{3} & 0 & 0 \end{bmatrix}^T$

and G and $\underline{F}$ are given by

$$G = \begin{bmatrix} I_3 & 0 & I_3 & 0 & I_3 & 0 \\ S_1 & I_3 & S_2 & I_3 & S_3 & I_3 \end{bmatrix}, \tag{17}$$

$$\underline{F} = \begin{bmatrix} \underline{F}_1^T & \underline{F}_2^T & \underline{F}_3^T \end{bmatrix}^T, \tag{18}$$

then the internal loading matrix ($I - G^{+}G$) can be obtained as

$$\begin{bmatrix}
0.52 & -0.08 & 0 & 0 & 0 & -0.14 & -0.19 & -0.08 & 0 & 0 & 0 & -0.14 & -0.33 & 0.17 & 0 & 0 & 0 & -0.14 \\
-0.08 & 0.62 & 0 & 0 & 0 & -0.08 & 0.08 & -0.38 & 0 & 0 & 0 & -0.08 & 0 & -0.24 & 0 & 0 & 0 & -0.08 \\
0 & 0 & 0.4 & 0.2 & 0.12 & 0 & 0 & 0 & -0.2 & 0.2 & 0.12 & 0 & 0 & 0 & -0.2 & 0.2 & 0.12 & 0 \\
0 & 0 & 0.2 & 0.8 & 0 & 0 & 0 & 0 & -0.2 & -0.2 & 0 & 0 & 0 & 0 & 0 & -0.2 & 0 & 0 \\
0 & 0 & 0.12 & 0 & 0.8 & 0 & 0 & 0 & 0.12 & 0 & -0.2 & 0 & 0 & 0 & -0.23 & 0 & -0.2 & 0 \\
-0.14 & -0.08 & 0 & 0 & 0 & 0.86 & 0.14 & -0.08 & 0 & 0 & 0 & -0.14 & 0 & 0.17 & 0 & 0 & 0 & -0.14 \\
-0.19 & 0.08 & 0 & 0 & 0 & 0.14 & 0.53 & 0.08 & 0 & 0 & 0 & 0.14 & -0.33 & -0.17 & 0 & 0 & 0 & 0.14 \\
-0.08 & -0.38 & 0 & 0 & 0 & -0.08 & 0.08 & 0.62 & 0 & 0 & 0 & -0.08 & 0 & -0.24 & 0 & 0 & 0 & -0.08 \\
0 & 0 & -0.2 & -0.2 & 0.12 & 0 & 0 & 0 & 0.4 & -0.2 & 0.12 & 0 & 0 & 0 & -0.2 & -0.2 & 0.12 & 0 \\
0 & 0 & 0.2 & -0.2 & 0 & 0 & 0 & 0 & -0.2 & 0.8 & 0 & 0 & 0 & 0 & 0 & -0.2 & 0 & 0 \\
0 & 0 & 0.12 & 0 & -0.2 & 0 & 0 & 0 & 0.12 & 0 & 0.8 & 0 & 0 & 0 & -0.23 & 0 & -0.2 & 0 \\
-0.14 & -0.08 & 0 & 0 & 0 & -0.14 & 0.14 & 0.08 & 0 & 0 & 0 & 0.86 & 0 & 0.17 & 0 & 0 & 0 & -0.14 \\
-0.33 & 0 & 0 & 0 & 0 & 0 & -0.33 & 0 & 0 & 0 & 0 & 0 & 0.67 & 0 & 0 & 0 & 0 & 0 \\
0.17 & -0.24 & 0 & 0 & 0 & 0.17 & -0.17 & -0.24 & 0 & 0 & 0 & 0.17 & 0 & 0.48 & 0 & 0 & 0 & 0.17 \\
0 & 0 & -0.2 & 0 & -0.23 & 0 & 0 & 0 & -0.2 & 0 & -0.23 & 0 & 0 & 0 & 0.4 & 0 & -0.23 & 0 \\
0 & 0 & 0.2 & -0.2 & 0 & 0 & 0 & 0 & -0.2 & -0.2 & 0 & 0 & 0 & 0 & 0 & 0.8 & 0 & 0 \\
0 & 0 & 0.12 & 0 & -0.2 & 0 & 0 & 0 & 0.12 & 0 & -0.2 & 0 & 0 & 0 & -0.23 & 0 & 0.8 & 0 \\
-0.14 & -0.08 & 0 & 0 & 0 & -0.14 & 0.14 & -0.08 & 0 & 0 & 0 & -0.14 & 0 & 0.17 & 0 & 0 & 0 & 0.86
\end{bmatrix} \tag{19}$$

This matrix is an 18 by 18 square matrix whose rank is 12. Therefore, the basis of internal loading consists of 12 vectors, and by using row-reduced echelon they can be obtained as

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}^T \tag{20}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1.73 \end{bmatrix}^T \tag{21}$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1.73 & 0 \end{bmatrix}^T \tag{22}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}^T \tag{23}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}^T \tag{24}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}^T \tag{25}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T \tag{26}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1.73 \end{bmatrix}^T \tag{27}$$

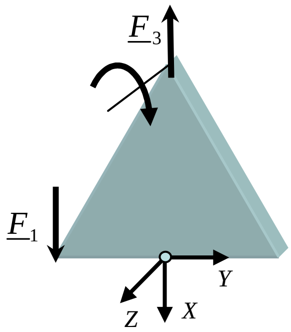
$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1.73 & 0 \end{bmatrix}^T \tag{28}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}^T \tag{29}$$

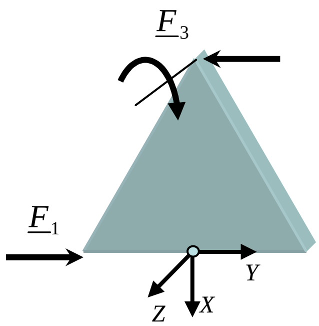
$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}^T \tag{30}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}^T \tag{31}$$

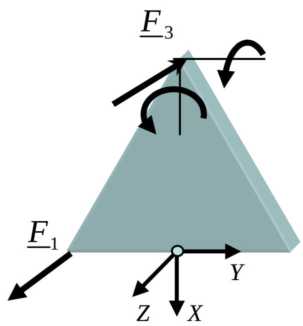
The trends are similar to the planar case. Internal moments also independently happen. However, internal forces are coupled to internal moments, which are called couple moments.



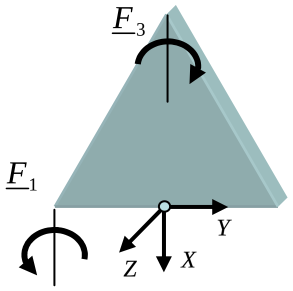
(a) Internal force in the X direction



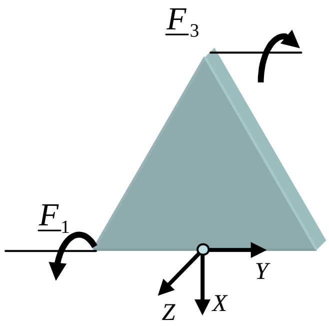
(b) Internal force in the Y direction



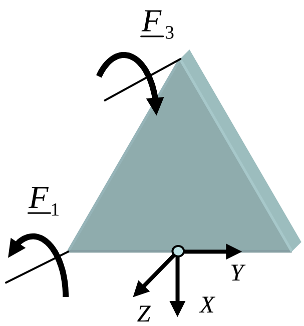
(c) Internal force in the Z direction



(d) Internal moment in the X direction



(e) Internal moment in the Y direction



(f) Internal moment in the Z direction
Fig. 4. Shape of internal loading basis in spatial domain

Fig. 4 shows the shape of the internal loading between the first and the third manipulators. The other internal loading can be also visualized similarly.

3. Internal force analysis at the three-fingered and three-legged systems

We assume that each fingertip makes a point contact, offering friction force to the object. Therefore, Eq. (6) is transformed as

$$G = \begin{bmatrix} I_3 & I_3 & \cdots & I_3 \\ S_1 & S_2 & \cdots & S_n \end{bmatrix} \in R^{6\times 3m} \tag{32}$$

The first term on the right-hand side of Eq. (5) represents the manipulating force, and the second term denotes grasping force or internal force. Kumar[8] also define those terms as an equilibrating force and an interaction force, respectively. The equilibrating forces are the forces required to maintain equilibrium against an external load, and the interaction force must have a zero net resultant. The definition of interaction force is similar to internal force. So, the number of internal forces or interaction force is $3\times n-6$ [8,12] if and only if the contact points do exist noncolinearly. However, interaction forces exist on the $\left(\underline{r}_i-\underline{r}_j\right)$ line, and internal forces can exist in any direction on the plane where the contact points are coplanar.

Yoshikawa[14] defined the internal loading by using three unit vectors given by

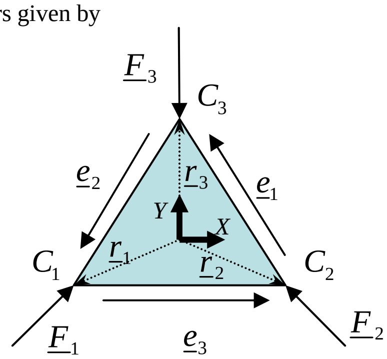


Fig. 5. Internal force

$$\underline{e}_1 = \frac{\underline{r}_3-\underline{r}_2}{\left\|\underline{r}_3-\underline{r}_2\right\|},\ \underline{e}_2 = \frac{\underline{r}_1-\underline{r}_3}{\left\|\underline{r}_1-\underline{r}_3\right\|},\ \underline{e}_3 = \frac{\underline{r}_2-\underline{r}_1}{\left\|\underline{r}_2-\underline{r}_1\right\|}, \tag{33}$$

where $\underline{e}_i$ is the unit vector directing from C_i to C_{i+1}. Then, the internal forces can be constructed by

$$\begin{aligned} \underline{F}_1 &= z_3\underline{e}_3 + z_2\left(-\underline{e}_3\right), \\ \underline{F}_2 &= z_1\underline{e}_1 + z_2\left(-\underline{e}_3\right), \\ \underline{F}_3 &= z_2\underline{e}_2 + z_1\left(-\underline{e}_1\right), \end{aligned} \tag{34}$$

where z_1, z_2, and z_3 are arbitrary real numbers. The matrix form of the internal force can be expressed as

$$\begin{bmatrix} \underline{F}_1 \\ \underline{F}_2 \\ \underline{F}_3 \end{bmatrix} = \begin{bmatrix} 0 & -\underline{e}_2 & \underline{e}_2 \\ \underline{e}_1 & 0 & -\underline{e}_3 \\ -\underline{e}_1 & \underline{e}_2 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}, \tag{35}$$

and three internal forces form the closed triangle.

3.1 Three fingers system

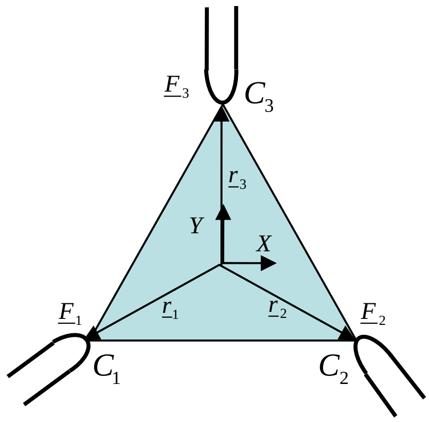


Fig. 6. Three-fingered system

When a three-fingered system contacts a common object, the transformation matrix can be expressed as

$$G = \begin{bmatrix} I_3 & I_3 & I_3 \\ S_1 & S_2 & S_3 \end{bmatrix}. \tag{36}$$

The position vectors of the contact point are

$$\begin{aligned} \underline{r}_1 &= \begin{bmatrix} -1 & -\sqrt{3}/3 & 0 \end{bmatrix}^T, \\ \underline{r}_2 &= \begin{bmatrix} 1 & -\sqrt{3}/3 & 0 \end{bmatrix}^T, \\ \underline{r}_3 &= \begin{bmatrix} 0 & 2\sqrt{3}/3 & 0 \end{bmatrix}^T, \end{aligned} \tag{37}$$

and the internal force matrix can be obtained as

$$I - G^{+}G =$$

$$\begin{bmatrix} 0.58 & 0.14 & 0 & -0.42 & -0.14 & 0 & -0.17 & 0 & 0 \\ 0.14 & 0.42 & 0 & 0.14 & -0.08 & 0 & -0.29 & -0.33 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.42 & 0.14 & 0 & 0.58 & -0.14 & 0 & -0.14 & 0 & 0 \\ -0.14 & -0.08 & 0 & -0.14 & 0.42 & 0 & 0.29 & -0.33 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.17 & -0.29 & 0 & -0.17 & 0.29 & 0 & 0.33 & 0 & 0 \\ 0 & -0.33 & 0 & 0 & -0.33 & 0 & 0 & 0.67 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (38)$$

This matrix is a 9 by 9 square matrix whose rank is 3. Therefore, the basis of internal force is 3. Column 3, 6 and 9 of Eq. (38) has zero value. This is because the plane of the contact points is perpendicular to Z axis of the object coordinate. By using row-reduced echelon, internal basis can be obtained as

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1.73 & 0 & -1 & 1.73 & 0 \end{bmatrix}^T \quad (39)$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & -2 & 0 \end{bmatrix}^T \quad (40)$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & -1.73 & 0 & -1 & 1.73 & 0 \end{bmatrix}^T \quad (41)$$

Eqs. (39) through (41) form a closed force triangle. The interaction force can be obtained using the concept of force equilibrium at the contact point. From Eq. (35), the z vectors corresponding to the internal forces given in Eq. (39) through (41) can be obtained as

$$z = \begin{bmatrix} -2 & 0 & 1 \end{bmatrix}^T, \quad (42)$$

$$z = \begin{bmatrix} -2/\sqrt{3} & -2/\sqrt{3} & 1/\sqrt{3} \end{bmatrix}^T, \quad (43)$$

and

$$z = \begin{bmatrix} -2 & 0 & 0 \end{bmatrix}^T. \quad (44)$$

3.2 Three-legged system

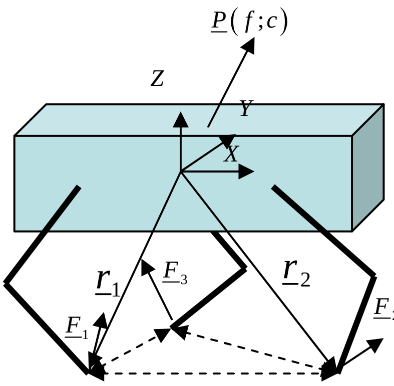


Fig. 7. Three-legged system

The concept of internal force at the three-legged system is the same as that of three-fingered system, but the difference of the concept is the object being grasped. The position vectors of the end-point of legs are

$$\underline{r}_1 = \begin{bmatrix} -2 & -2\sqrt{3}/3 & -3 \end{bmatrix}^T,$$
$$\underline{r}_2 = \begin{bmatrix} 2 & -2\sqrt{3}/3 & -4 \end{bmatrix}^T, \quad (45)$$
$$\underline{r}_3 = \begin{bmatrix} 0 & 4\sqrt{3}/3 & -4 \end{bmatrix}^T.$$

and the internal force matrix can be obtained as

$$I - G^{+}G =$$

$$\begin{bmatrix} 0.54 & 0.13 & -0.15 & -0.39 & -0.13 & 0.12 & -0.15 & 0.01 & 0.04 \\ 0.13 & 0.39 & -0.09 & 0.14 & -0.08 & -0.02 & -0.27 & -0.32 & 0.11 \\ -0.15 & -0.09 & 0.05 & 0.08 & 0.04 & -0.03 & 0.08 & 0.04 & -0.03 \\ -0.39 & 0.14 & 0.08 & 0.56 & -0.15 & -0.12 & -0.17 & 0.01 & 0.04 \\ -0.13 & -0.08 & 0.04 & -0.15 & 0.41 & -0.02 & 0.28 & -0.34 & -0.02 \\ 0.17 & -0.02 & -0.03 & -0.12 & -0.02 & 0.03 & 0 & 0.05 & -0.01 \\ -0.15 & -0.27 & 0.08 & -0.17 & 0.28 & 0 & 0.32 & -0.01 & -0.08 \\ 0.01 & -0.32 & 0.04 & 0.01 & -0.34 & 0.05 & -0.01 & 0.65 & -0.09 \\ 0.04 & 0.11 & -0.03 & 0.04 & -0.02 & -0.01 & -0.08 & -0.09 & 0.03 \end{bmatrix} \quad (46)$$

This matrix is also a 9 by 9 square matrix whose rank is 3. Therefore, the basis of internal force is 3, and by using row-reduced echelon, the internal forces can be obtained as

$$\begin{bmatrix} 1 & 0 & -0.25 & 0 & -1.73 & 0.25 & -1 & 1.73 & 0 \end{bmatrix}^T \quad (47)$$

$$\begin{bmatrix} 0 & 1 & -0.14 & 0 & 1 & -0.14 & 0 & -2 & 0.28 \end{bmatrix}^T \quad (48)$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & -1.73 & 0 & -1 & 1.73 & 0 \end{bmatrix}^T \quad (49)$$

The results of Eqs. (47) through (49) show that the internal force is similar to that of the three-fingered case.

4. CONCLUSION

An internal loading is defined as the forces and moments that do not affect the motion of the end-effecter. The internal loading basis has $6 \times n - 6$ independent vectors in the case of rigidly grasping manipulation, and $3 \times n - 6$ in case of point contact with friction. In this paper, through analysis of the null space of the system, various types of the internal loading for triple manipulator systems are investigated both in planar and spatial domain, and as specific examples, internal forces for both three-fingered and three-legged systems are analyzed.

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