Design and Dynamic Analysis of Fish-like Robot:PoTuna

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Abstract: This paper presents the design and the analysis of a "fish-like underwater robot". In order to develop swimming robot like a real fish, extensive hydrodynamic analysis were made followed by the study of biology of the fishes especially its maneuverability and propel styles. Swimming mode is achieved by mimicking fish-swimming of carangiform. This is the swimming mode of the fast motion using its tail and peduncle for propulsion. In order to generate configurations of vortices that gives efficient propulsion yawing and surging with a caudal fin has applied and in order to submerge and maintain the body balance pitching and heaving motion with a pair of pectoral fin is used. We have derived the equation of motion of PoTuna by two methods. In first method, we use the equation of motion of underwater vehicle with the potential flow theory for the power of propulsion. In second method, we apply the method of the equation of motion of UVM(Underwater Vehicle-Manipulator). Then, we compare these results.

Keywords: fish-like underwater robot, potential flow theory, kane's method

1. INTRODUCTION

Fish are very impressive swimmers in many ways, and it is possible that submersible robots swimming like fish might be superior to submersibles using propulsors. For example, fish-like robots might be more quieter, more maneuverable, and possibly more energy efficient.

Nearly 70 years ago, Sir James Gray put forward the so called *Gray's Paradox* which stimulated much research and controversy[1], [2], [3]. The paradox suggests that if the resistance of an actively swimming dolphin is equal to that of a rigid model towed at the same speed, the dolphin's muscles must be capable of generating at least seven times more power than is typical of mammalian muscle[1]. In recent years, Triantafyllou et al.[3], [4], [5] proposed an explanation of Gray's paradox. He proposed that fish are able to utilize the energy that exists in the eddies of an oncoming flow by repositioning the vorticities and the fish's oscillating swimming motion induces flow relaminization which serves to reduce the body drag[13].

While a variety of propulsion schemes have been investigated, most investigations of fish-like swimming have focused on carangiform-like swimming. In carangiform swimming, the front two-thirds of the fish's body moves in a largely rigid way, while the propulsive body movements are confined mainly to the rear third of the body-primarily the tail[12]. Kelly et al.[16], [17] derived the equations of motion of a submerged foil in standard control-affine form via the method of reduced Lagrangian in a geometric framework. Up to now, many experiments and dynamic analyses are concentrated on the forward and turning motion of fish-robot[10],[11],[15]. In this paper, we consider the design of a carangiform fish robot and its 3D dynamic analysis. We have derived the equation of motion of fish-like robot by two methods. We apply the potential flow theory for tail power and the method of UVM(Underwater Vehicle-Manipulator). Then, we compare these results. The rest of the paper is organized as follow.

Fig. 1. The Picture of fish robot-PoTuna

Section 2 describes the design of fish-like robot(PoTuna). Section 3 presents the dynamic model of the fish-like robot. Section 4 contains some concluding remarks.

2. Hardware Apparatus

We try to develop a high speed and submergible fish robot. For high speed, we mimic tuna with carangiform mode and lunate tail. Tuna's body is so rigid and well-streamlined that it's suitable for high speed. We use the lift force of a pair of pectoral fin for submerging and the ventral fin for turning. We try to develop the unit-type fish robot, which consists of several components as a power unit and a control unit. The fish robot named PoTuna has about 1 m of body length and 25 kg of body weight. Figures 1 and 2 are a picture and a schematic of PoTuna.

It consists of a tail unit, a battery unit, a R/C unit, a pectoral unit. Tail unit has a RC motor and peduncle/tail mechanism, this is underactuated system with one motor for two link. Battery unit is used 7.2V, 3300mAh Ni-Mh. R/C unit is included a RF receiver and a 8051 controller. Pectoral unit has two servos and mechanisms for turning and up-down motion.

The body shape of fish robot affects strongly to propulsive

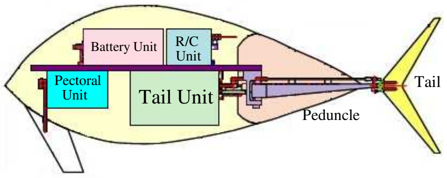

Fig. 2. The Structure of fish robot-PoTuna

performance like swimming speed. Of course, the body is wished to have little drag in the swimming. However, we cannot decide the body shape with only hydrodynamic considerations. Because, it must be well-balanced to the center of gravity and the buoyant position, and it is limited from the location of every mechanical components. The design points for the body are as follows.

- The body shape is matched to the location of mechanical components,
- Well-balanced design between the total volume (displacement) and the total weight is required,
- Well-balanced shape between the center of gravity and the buoyant point is required,
- A few drag force is required,
- A few perturbation of water flow is required around a tail fin,
- A simple shape is better for easy building.

3. System Modeling

In this section, we will derive the equation of motion for PoTuna using two method.

3.1. PoTuna Modeling using the Potential flow theory

In first method, we consider that PoTuna's body is rigid and the force by tail oscillation is a kind of external force as drag, thrust, and so forth. We assume that the fluid flow is inviscid, incompressible and irrotational and also that the fluid is at rest at infinity. Under these assumptions, the hydrodynamic forces and torques that act on the foil are determined using potential flow theory[6],[7]. The presence and effects of free and central vortices are ignored for the purpose of motion planning and controller design, and are treated as disturbances. Figure 3 shows the coordinate frame of fish robot using potential flow theory.

3.1.1 The Dynamic Equation of Fish Body

The motion of a rigid body in a 3D space can be expressed as[14]:

$$\mathbf{M}_{\mathrm{RB}}\ddot{\mathbf{q}} + \mathbf{C}_{\mathrm{RB}}(\dot{\mathbf{q}})\dot{\mathbf{q}} = \boldsymbol{\tau} \tag{1}$$

where $\mathbf{M}_{\mathrm{RB}}$, $\mathbf{C}_{\mathrm{RB}}(\dot{\mathbf{q}})$ are the inertia matrix and the Coriolis/Centripetal matrix of fish body, respectively, η is the position of fish robot with respect to the inertial frame, $\dot{\mathbf{q}}$ is the velocity of fish robot with respect to the body fixed frame(Bth frame) and $\boldsymbol{\tau}$ is given by:

$$\begin{aligned}\boldsymbol{\tau} &= \boldsymbol{\tau}_{\mathrm{REST}} + \boldsymbol{\tau}_{\mathrm{DAMP}} + \boldsymbol{\tau}_{\mathrm{ADD}} + \boldsymbol{\tau}_{\mathrm{FK}} + \boldsymbol{\tau}_{\mathrm{EXT}} \\ \boldsymbol{\tau}_{\mathrm{DAMP}} &= \boldsymbol{\tau}_{\mathrm{PDMP}} + \boldsymbol{\tau}_{\mathrm{WDMP}} + \boldsymbol{\tau}_{\mathrm{SKIN}} + \boldsymbol{\tau}_{\mathrm{VORT}}\end{aligned}$$

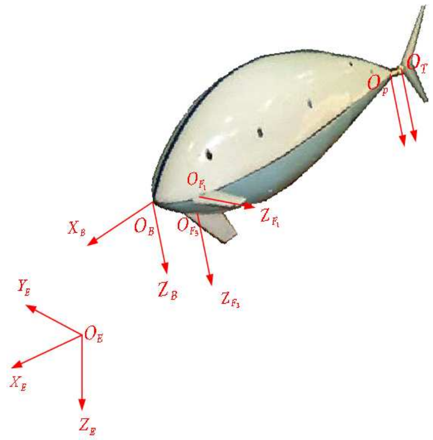

Fig. 3. The Coordinate frame of fish robot-PoTuna using potential flow theory

where $\boldsymbol{\tau}_{\mathrm{REST}}$ denotes the restoring forces and moments, due to weight and buoyancy, $\boldsymbol{\tau}_{\mathrm{DAMP}}$ denotes the forces and moments due to different type of damping, $\boldsymbol{\tau}_{\mathrm{ADD}}$ denotes the added mass forces and moments due to the inertia of the surrounding fluid, $\boldsymbol{\tau}_{\mathrm{FK}}$ denotes the Froude-Kriloff forces, which are due to the inertia matrix of the displaced fluid and $\boldsymbol{\tau}_{\mathrm{EXT}}$ denotes a generic external force which can be considered the force by tail oscillation. The final equations of motion are given:

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\dot{\mathbf{q}} \tag{2}$$

$$(\mathbf{M}_{\mathrm{RB}} + \mathbf{M}_{\mathrm{A}})\ddot{\mathbf{q}}_r = \boldsymbol{\tau}_{\mathrm{COR}} + \boldsymbol{\tau}_{\mathrm{DAMP}} + \boldsymbol{\tau}_{\mathrm{REST}} + \boldsymbol{\tau}_{\mathrm{EXT}} \tag{3}$$

where $\mathbf{J}$ is the transformation matrix of Bth frame with respect to inertial frame, $\ddot{\mathbf{q}}_r$ is the relative acceleration between fish robot and fluid, and other force and moment terms are given by:

$$\begin{aligned}\boldsymbol{\tau}_{\mathrm{REST}} &= -\mathbf{g}(\boldsymbol{\eta}) = \boldsymbol{\tau}_g(\boldsymbol{\eta}) + \boldsymbol{\tau}_b(\boldsymbol{\eta}) \\ \boldsymbol{\tau}_{\mathrm{DAMP}} &= -D(\dot{\mathbf{q}}, \alpha)\dot{\mathbf{q}}_r \\ \boldsymbol{\tau}_{\mathrm{COR}} &= -\mathbf{C}_{\mathrm{RB}}(\dot{\mathbf{q}})\dot{\mathbf{q}} - \mathbf{C}_{\mathrm{A}}(\dot{\mathbf{q}}_r)\dot{\mathbf{q}}_r\end{aligned}$$

where α is the angle of attack.

3.1.2 The External Force by Tail Oscillation

Consider an elliptical Joukowski foil whose profile is described by a set of points in the complex plane $S \in C = \{z = x + iy\}$ with $x \in R$ in the direction of the chord length, and $y \in R$ in the direction normal to it. S is the image of the unit circle $C = \{\zeta = e^{i\overline{\theta}}, \overline{\theta} \in [0, 2\pi]\}$ under the Joukowski mapping $F := \zeta \rightarrow z$ given by:

$$z = F(\zeta) := \zeta + \zeta_c + \frac{a^2}{\zeta + \zeta_c} \tag{4}$$

where a and ζ_c determine the shape of the foil. The complex potential for the flow around the Joukowski foil is given by:

$$\begin{aligned}w(z) =& U(t)w_1(\zeta) + V(t)w_2(\zeta) + \Omega(t)w_3(\zeta) \\ &+ \gamma_c(t)w_4(\zeta) + \sum_k \gamma_k w_5(\zeta; \zeta_k(t))\end{aligned} \tag{5}$$

Here, $\zeta = F^{-1}(z)$, and $U(t), V(t)$ are the absolute velocities in the directions parallel and perpendicular to the foil's chord, and $\Omega(t)$ is the angular velocity of the foil respectively. γ_c is the strength of the central vortex, and γ_k and $F(\zeta_k(t))$ with $k = 1, \cdots, n_k$ are the strengths and locations of the n_k free vortices. Once the complex potential has been defined, we can derive force per unit span X and Y along the x and y using Milne-Thomson's method[6].

$$\begin{aligned} X + iY = &- \dot{U}(m_{11} + im_{21}) - \dot{V}(m_{12} + im_{22}) \\ &- \dot{\Omega}(m_{16} + im_{26}) + \Omega U(m_{21} - im_{11}) \\ &+ \Omega V(m_{22} - im_{12}) + \Omega^2(m_{26} - im_{16}) \\ &+ i(W + i\Omega\zeta_c)\Gamma + i\frac{d}{dt}[2\pi Z_1 - \Gamma(2a - \zeta_c)] \\ &- 2\pi i Z_2 - 2\pi\Omega Z_1 \end{aligned} \tag{6}$$

where the added mass coefficient

$$\begin{aligned} m_{11} &= \pi r_c^2 - 2\pi a^2 + \pi r_c^2 \frac{a^4}{(r_c^2 - \delta^2)^2} \\ m_{12} &= 0 \\ m_{21} &= 0 \\ m_{22} &= \pi r_c^2 + 2\pi a^2 + \pi r_c^2 \frac{a^4}{(r_c^2 - \delta^2)^2} \\ m_{16} &= \mathrm{Im}\bigg[- \pi r_c^2 \zeta_c - 2\pi a^2 \bar{\zeta}_c + 2\pi \frac{a^4 \zeta_c}{r_c^2 - \delta^2} \\ &\quad + \pi r_c^2 \frac{a^6 \bar{\zeta}_c}{(r_c^2 - \delta^2)^3} \bigg] \\ m_{26} &= \mathrm{Re}\bigg[\pi r_c^2 \zeta_c + 2\pi a^2 \bar{\zeta}_c - 2\pi \frac{a^4 \zeta_c}{r_c^2 - \delta^2} \\ &\quad - \pi r_c^2 \frac{a^6 \bar{\zeta}_c}{(r_c^2 - \delta^2)^3} \bigg] \end{aligned}$$

$$\begin{aligned} Z_1 &= \sum_k \gamma_k \left(\frac{r_c^2}{\bar{\zeta}_k} + \frac{a^2}{\zeta_k + \zeta_c} \right) \\ Z_2 &= \sum_k \gamma_k \frac{dz_k}{dt} \end{aligned}$$

The moment is given:

$$\begin{aligned} M = &- \dot{U}m_{61} - \dot{V}m_{62} - \Omega m_{66} - U^2 m_{21} + V^2 m_{12} \\ &+ UV(m_{11} - m_{22}) - U\Omega m_{26} + V\Omega m_{16} \\ &+ \frac{d}{dt}\left[2\pi S_1 - \Gamma\left(a^2 - \delta^2 + a\zeta_c + \frac{a^3\zeta_c}{\delta^2 - r_c^2} \right) \right] \\ &- 2\pi S_2 + \mathrm{Re}[\bar{W}(2\pi Z_3 - \Gamma\zeta_c)] \end{aligned} \tag{7}$$

where

$$\begin{aligned} m_{61} &= m_{16} \\ m_{62} &= m_{26} \\ m_{66} &= \pi\bigg[2a^4 + 2a^2\delta^2 + 2a^4 \frac{\delta^4 + a^2\delta^2 - 2r_c^2\delta^2}{(r_c^2 - \delta^2)^2} \\ &\quad + a^8 r_c^2 \frac{2\delta^2 - r_c^2}{(r_c^2 - \delta^2)^4} - \frac{r_c^2 a^4}{r_c^2 - \delta^2} + r_c^2 \delta^2 \\ &\quad + a^4 r_c^2 \frac{a^4 + r_c^4 - \delta^4}{(r_c^2 - \delta^2)^3} \bigg] \\ S_1 &= \sum_k \gamma_k \mathrm{Re}\bigg[\frac{r_c^2 \zeta_c}{\zeta_k} + \frac{a^2 r_c^2}{\zeta_k(\zeta_k + \zeta_c)} \\ &\quad + \frac{a^4 \zeta_c}{(\delta^2 - r_c^2)(\bar{\zeta}_k + \bar{\zeta}_c)} + \frac{a^2 \bar{\zeta}_c}{\zeta_k + \zeta_c} \bigg] \\ S_2 &= 2\pi \sum_k \gamma_k \mathrm{Re}\left(\frac{d\bar{z}_k}{dt} z_k \right) \\ S_3 &= 2\pi \sum_k \gamma_k \left(\frac{r_c^2}{\bar{\zeta}_k} + \frac{a^2}{\zeta_k + \zeta_c} \right) \end{aligned}$$

and $W = U + iV$ is the linear velocity in the $z = ix + y$ plane, and Ω is the angular velocity, and we assume that we don't consider free and central vortices, that is, γ_c, γ_k and Γ equal 0. Then the forces and torques per unit span and mass by tail oscillation is given by:

$$\begin{aligned} X &= -\dot{U}m_{11} - \dot{\Omega}m_{16} + \Omega V m_{22} + \Omega^2 m_{26} \\ Y &= -\dot{V}m_{22} - \dot{\Omega}m_{26} + \Omega U m_{11} - \Omega^2 m_{16} \\ M &= -\dot{U}m_{61} - \dot{V}m_{62} - \Omega m_{66} + UV(m_{11} - m_{22}) \\ &\quad - U\Omega m_{26} + V\Omega m_{16} \end{aligned}$$

Then

$$\boldsymbol{\tau}_{\mathrm{EXT}} = \left[\sum_{k=1}^{n} \rho X \quad \sum_{k=1}^{n} \rho Y \quad 0 \quad 0 \quad 0 \quad \sum_{k=1}^{n} \rho M \right]^T \tag{8}$$

where ρ is the density of water.

3.2. PoTuna Modeling using Kane's method

In this subsection, we will develop a dynamic model for a PoTuna using Kane's dynamic equation. We will select the $N = 8$ generalized velocities of the system as:

$$\dot{\mathbf{q}} = \left[v_x \quad v_y \quad v_z \quad \omega_x \quad \omega_y \quad \omega_z \quad \dot{\theta}_1 \quad \dot{\theta}_2 \right]^T \tag{9}$$

where (v_x, v_y, v_z) is the linear velocity of the fish robot with respect to the inertial frame expressed in the Bth coordinate frame, $(\omega_x, \omega_y, \omega_z)$ is the angular velocity of the fish robot with respect to the inertial frame expressed in the Bth coordinate frame, and $(\dot{\theta}_1, \dot{\theta}_2)$ which are the joint velocities of the peduncle and tail, respectively. Figure 4 and Figure 5 show the 3 dimensional coordinate frame and 2 dimensional coordinate frame of fish robot using Kane's method[9], respectively.

3.2.1 Kinematic Analysis

The kinematic task is more tractable and systematic if we exploit the use of Denavit-Hartenberg(D-H) coordinate frames. Figure 1 illustrates the coordinates that we have selected for our system. The position vector of the C.M. of the peduncle

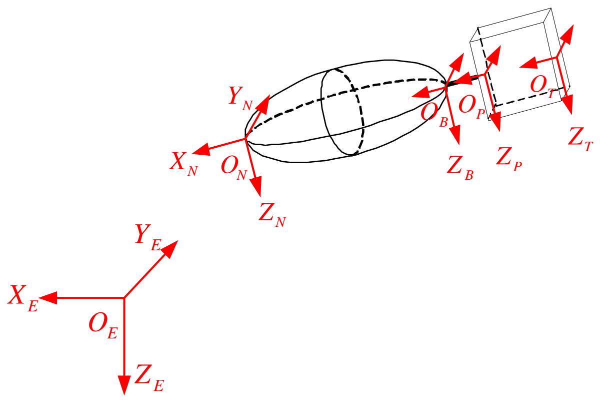

Fig. 4. The 3 dimensional Coordinate frame of fish robot-PoTuna using Kane's method

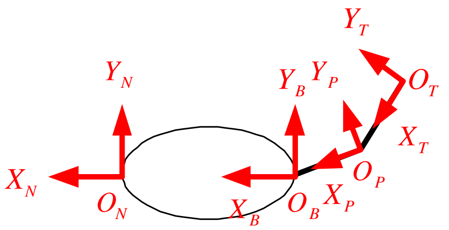

Fig. 5. The 2 dimensional Coordinate frame of fish robot-PoTuna using Kane's method

with respect to the C.M. of the fish body expressed in the B coordinate frame is given by:

$$\mathbf{p}_P^B = \begin{bmatrix} c_{xB} \\ c_{yB} \\ c_{zB} \\ 1 \end{bmatrix} \tilde{+} \mathbf{A}_B^P \mathbf{c}_P^P = \mathbf{c}_B^B \tilde{+} \mathbf{A}_B^P \mathbf{c}_P^P \quad (10)$$

where $\tilde{+}$ denote the addition of the physical coordinates of the two homogeneous vectors on the right hand side, $\mathbf{A}_B^P$ is the homogeneous transformation from coordinate frame P to coordinate frame B, and $\mathbf{c}_P^P$ is the position vector to the C.M. of link P expressed in frame P. The position vector of the C.M. of the link T with respect to the C.M. the vehicle is given similarly by:

$$\mathbf{p}_T^B = \mathbf{c}_B^B \tilde{+} \mathbf{A}_B^P \mathbf{A}_P^T \mathbf{c}_T^T \quad (11)$$

The angular velocity of the fish body with respect to an inertial frame E, expressed in the Bth coordinate system is given by:

$$^E\boldsymbol{\omega}^B = \boldsymbol{\omega}_x \hat{x}_B + \boldsymbol{\omega}_y \hat{y}_B + \boldsymbol{\omega}_z \hat{z}_B \quad (12)$$

where the hat denotes a unit vector and the notation $^E\boldsymbol{\omega}^B$ denotes the angular velocity of frame B with respect to frame E expressed in the Bth coordinate frame. The angular velocity of peduncle and tail with respect to the inertial frame expressed in the Bth coordinate frame is given by:

$$\begin{aligned} ^E\boldsymbol{\omega}^P &= {}^E\boldsymbol{\omega}^B + {}^B\boldsymbol{\omega}^P = {}^E\boldsymbol{\omega}^B + \dot{\theta}_1 \hat{z}_B && (13) \\ ^E\boldsymbol{\omega}^T &= {}^E\boldsymbol{\omega}^B + {}^B\boldsymbol{\omega}^P + \bar{\mathbf{A}}_B^P {}^P\boldsymbol{\omega}^T \\ &= {}^E\boldsymbol{\omega}^B + {}^B\boldsymbol{\omega}^P + \bar{\mathbf{A}}_B^P \dot{\theta}_2 \hat{z}_P && (14) \end{aligned}$$

where $\bar{\mathbf{A}}_B^P$ denotes the rotation submatrix of $\mathbf{A}_B^P$, $^B\boldsymbol{\omega}^P$ is the angular velocity of peduncle with respect to body and $^P\boldsymbol{\omega}^T$ is the angular velocity of tail with respect to peduncle. The linear velocity of the C.M. of the body with respect to the inertial frame, expressed in the Bth coordinate system is given by:

$$\mathbf{v}_B^B = v_x \hat{v}_B + v_y \hat{y}_B + v_z \hat{z}_B \quad (15)$$

The linear velocity of the C.M. of an peduncle or tail with respect to the inertial frame, expressed in the Bth coordinate frame is given by:

$$\mathbf{v}_i^B = \mathbf{v}_B^B + \frac{d\mathbf{p}_i^B}{dt} + {}^E\boldsymbol{\omega}^B \times \mathbf{p}_i^B \quad (16)$$

The angular acceleration about the C.M. of the vehicle with respect to the inertial frame, expressed in the Bth coordinate frame is given by:

$$^E\boldsymbol{\alpha}^B = \dot{\omega}_x \hat{x}_B + \dot{\omega}_y \hat{y}_B + \dot{\omega}_z \hat{z}_B \quad (17)$$

The angular acceleration of an peduncle or tail with respect to the inertial frame expressed in the Bth coordinate frame is found from the following:

$$^E\boldsymbol{\alpha}^i = \frac{d\,^E\boldsymbol{\omega}^i}{dt} + {}^E\boldsymbol{\omega}^B \times {}^E\boldsymbol{\omega}^i \quad (18)$$

The linear acceleration of the C.M. of the vehicle with respect to the inertial frame, expressed in the Bth coordinate frame is given by:

$$\mathbf{a}_B^B = \frac{d\mathbf{v}_B^B}{dt} + {}^E\boldsymbol{\omega}^B \times \mathbf{v}_B^B \quad (19)$$

The linear acceleration of the C.M. of peduncle or tail with respect to the inertial frame expressed in the Bth coordinate frame is found similarly by the following:

$$\mathbf{a}_i^B = \frac{d\mathbf{v}_i^B}{dt} + {}^E\boldsymbol{\omega}^B \times \mathbf{v}_B^i \quad (20)$$

where i denotes the peduncle or the tail.

3.2.2 Inertia Forces

The generalized inertia force of the system requires that we develop expressions for the inertia force and torque of each part in the system. The inertia forces of body, peduncle and tail are given by the following:

$$\mathbf{R}_i^* = -m_i \mathbf{a}_i^B \quad (21)$$

where m_i is the mass of fish body, peduncle or tail. The inertia torques of body, peduncle or tail are given by the following:

$$\mathbf{T}_i^* = -\mathbf{I}_i^B \cdot {}^E\boldsymbol{\alpha}^i - {}^E\boldsymbol{\omega}^i \times \mathbf{I}_i^E \cdot {}^E\boldsymbol{\omega}^i \quad (22)$$

where $\mathbf{I}_i^B$ is the central inertia matrix of fish body, peduncle or tail, expressed in the Bth coordinate frame. The generalized inertia force for the system is now found to obtain the following:

$$\mathbf{F}_r^* = \sum_{i=B,P,T} \left(\frac{\partial\,^E\boldsymbol{\omega}^i}{\partial \dot{q}_r} \cdot \mathbf{T}_i^* + \frac{\partial \mathbf{v}_i^B}{\partial \dot{q}_r} \cdot \mathbf{R}_i^* \right) \quad (23)$$
$$(r = 1, \cdots, N)$$

3.2.3 Gravity Forces

Gravity can be treated as a generalized active force which acts at the center of mass of each part in the system. The forces due to gravity acting on fish body, peduncle and tail are given by:

$$\mathbf{R}_{\text{grav}_i} = m_i \mathbf{g}^B \tag{24}$$

where $\mathbf{g}^B = [g_x, g_y, g_z]^T$ is the gravity vector expressed in the Bth coordinate frame. The generalized active force due to gravity is given by the following:

$$(\mathbf{F}_r)_{\text{gravity}} = \sum_{i=B,P,T} m_i \frac{\partial \mathbf{v}_i^B}{\partial \dot{q}_r} \cdot \mathbf{g}^B \tag{25}$$

3.2.4 Hydrodynamic Forces

The hydrodynamic forces induced by the motion of a rigid body in an underwater environment are very complex and highly nonlinear. The forces may be developed using incompressible fluid flow using Navior-Stokes equation, and rarely lead to a closed form solution. As is often the case, these forces may be treated as lumped approximations for certain applications within certain underlying assumptions. The net effect of added mass, buoyancy, Froude-Kriloff, and drag are often treated as the superposition of each individual force.

The added mass force results from the interaction of fluid in the immediate vicinity of a submerged link which is acceleration on the fluid through a pressure distribution which acts on the link body. The force required to accelerate the surrounding fluid results in an effective inertia which can be modeled with a 6×6 positive definite added mass inertia matrix, $\mathbf{I}_A$. In general the 36 elements of the added mass matrix, $\mathbf{I}_A$, for a body in a real fluid would be distinct and may be determined from experimental testing techniques. It has been shown by McMillan et al.[8] and can be derived from Fossen[14], that the inertia force and torque of a submerged body induced by the added mass phenomena has the following form:

$$\begin{bmatrix} \mathbf{R}^*_{A_i} \\ \mathbf{T}^*_{A_i} \end{bmatrix} = -\mathbf{I}^B_{A_i} \begin{bmatrix} \dot{\mathbf{v}}_i^B \\ {}^E\boldsymbol{\omega}^i \end{bmatrix} - \begin{bmatrix} {}^E\tilde{\boldsymbol{\omega}}^i & 0 \\ \tilde{\mathbf{v}}_i^B & {}^E\tilde{\boldsymbol{\omega}}^i \end{bmatrix} \mathbf{I}^B_{A_i} \begin{bmatrix} \mathbf{v}_i^B \\ {}^E\boldsymbol{\omega}^{B,P,T} \end{bmatrix}$$

where ${}^E\tilde{\boldsymbol{\omega}}^i$ and $\tilde{\mathbf{v}}_i^B$ are skew symmetric matrices, and $\mathbf{I}^B_{A_i}$ is the 6×6 added mass matrix for body, peduncle or tail expressed in the Bth coordinate frame. Substituting these two equations into results

$$\begin{aligned} \begin{bmatrix} \mathbf{R}^*_{A_i} \\ \mathbf{T}^*_{A_i} \end{bmatrix} &= -\mathbf{I}^B_{A_i} \begin{bmatrix} \mathbf{a}_i^B \\ {}^E\boldsymbol{\alpha}^i \end{bmatrix} + \mathbf{I}^B_{A_i} \begin{bmatrix} {}^E\boldsymbol{\omega}^i \times \mathbf{v}_i^B \\ {}^E\boldsymbol{\omega}^i \times {}^E\boldsymbol{\omega}^i \end{bmatrix} \\ &\quad - \begin{bmatrix} {}^E\tilde{\boldsymbol{\omega}}^i & 0 \\ \tilde{\mathbf{v}}_i^B & {}^E\tilde{\boldsymbol{\omega}}^i \end{bmatrix} \mathbf{I}^B_{A_i} \begin{bmatrix} \mathbf{v}_i^B \\ {}^E\boldsymbol{\omega}^i \end{bmatrix} \end{aligned}$$

We can account for the relative acceleration and velocity of the fluid by introducing the following relationship:

$$\begin{aligned} \mathbf{v}_i^r &= \mathbf{v}_i^B - \mathbf{v}_f^B \\ \mathbf{a}_i^r &= \mathbf{a}_i^B - \mathbf{a}_f^B \end{aligned}$$

where, $\mathbf{v}_f^B$ is the velocity of the fluid expressed in the Bth coordinate frame, and $\mathbf{a}_f^B$ is the acceleration of the fluid expressed in the Bth coordinate frame. The final form of the inertia force and torque resulting from added mass is now given by:

$$\begin{aligned} \begin{bmatrix} \mathbf{R}^*_{A_i} \\ \mathbf{T}^*_{A_i} \end{bmatrix} &= -\mathbf{I}^B_{A_i} \begin{bmatrix} \mathbf{a}_i^r \\ {}^E\boldsymbol{\alpha}^i \end{bmatrix} + \mathbf{I}^B_{A_i} \begin{bmatrix} {}^E\boldsymbol{\omega}^i \times \mathbf{v}_i^r \\ {}^E\boldsymbol{\omega}^i \times {}^E\boldsymbol{\omega}^i \end{bmatrix} \\ &\quad - \begin{bmatrix} {}^E\tilde{\boldsymbol{\omega}}^i & 0 \\ \tilde{\mathbf{v}}_i^r & {}^E\tilde{\boldsymbol{\omega}}^i \end{bmatrix} \mathbf{I}^B_{A_i} \begin{bmatrix} \mathbf{v}_i^r \\ {}^E\boldsymbol{\omega}^i \end{bmatrix} \end{aligned}$$

The generalized inertia force due to the added mass for the entire system is then given by the following:

$$(\mathbf{F}^*_r)_{\text{AM}} = \sum_{i=B,P,T} \left(\frac{\partial\, {}^E\boldsymbol{\omega}^i}{\partial \dot{q}_r} \cdot \mathbf{T}^*_{A_i} + \frac{\partial \mathbf{v}_i^B}{\partial \dot{q}_r} \cdot \mathbf{R}^*_{A_i} \right) \tag{26}$$

This is a general formulation for the incorporation of the hydrodynamic force and torque into the dynamic model. No assumptions were necessary on how the coefficients of the added mass matrix are derived.

The buoyancy force is proportional to the mass of the fluid displaced by the body, peduncle and tail and acts through the center of buoyancy of the body, peduncle and tail. For a homogeneous symmetric shape, the center of buoyancy and center of mass are equivalent. For our model, we assume that the buoyancy force acts through the center of mass of the body, peduncle or tail and is given by the following:

$$\mathbf{R}_{B_i} = -\rho V_i \mathbf{g}^B \tag{27}$$

where, ρ is the density of the fluid, V_i is the volume of fluid displaced by body, peduncle or tail and $\mathbf{g}^B$ is the gravity vector expressed in the Bth coordinate frame. The generalized active force due to buoyancy for the system is given by the following:

$$(\mathbf{F}_r)_{\text{Buoy}} = -\rho \sum_{i=B,P,T} V_i \frac{\partial \mathbf{v}_i^B}{\partial \dot{q}_r} \cdot \mathbf{g}^B \tag{28}$$

The Froude-Kriloff force is similar to the buoyancy force in that it is proportional to the fluid displaced, but is result of acceleration of the fluid itself. The force due to Froude-Kriloff also acts through the center of buoyancy and is given by:

$$\mathbf{R}_{\text{FK}_i} = \rho V_i \mathbf{a}_f^B \tag{29}$$

The generalized active force due to Froude-Kriloff for the system is given by the following:

$$(\mathbf{F}_r)_{\text{FK}} = \rho \sum_{i=B,P,T} V_i \frac{\partial \mathbf{v}_i^B}{\partial \dot{q}_r} \cdot \mathbf{a}_f^B \tag{30}$$

The fluid damping forces are divided by drag and lift force. The fluid damping forces exerted on a body depends on the square of the relative velocity of the fluid with respect to the body; the geometric shape of the body which is characterized by a drag and lift coefficient and a reference area of the body; and the density of the fluid. The drag forces include pressure drag, skin friction drag, and lift forces. The pressure drag force acts in a direction opposite to the link relative velocity with respect to the fluid and is the primary drag force for slow moving fish robot applications. Skin friction drag which is tangent to the link surface may be neglected for slow moving fish robot applications. Therefore, we will only consider drag forces could be handled for other applications in an analogous manor. The equations for the force and moment on each part due to pressure drag is given by:

$$\begin{aligned}
\mathbf{R}_{\mathrm{Drag}_i} &= -0.5\rho\int_0^L C_D b_i \|\mathbf{v}_i^r(l)^\perp\|\mathbf{v}_i^r(l)^\perp dl \\
\mathbf{T}_{\mathrm{Drag}_i} &= -0.5\rho\int_0^L C_D b_i \|\mathbf{v}_i^r(l)^\perp\|(\bar{\mathbf{A}}_B^i \hat{x}_i \times \mathbf{v}_i^r(l)^\perp) dl
\end{aligned}$$

where $b_i dl$ is the reference area of body, peduncle or tail, b_i is the width of the rectangle that circumscribes the frontal projection of the infinitesimal element of body, peduncle or tail, and dt is the length of the infinitesimal element. C_D is the drag coefficient, and $\mathbf{v}_B^r(l)^\perp$, $\mathbf{v}_P^r(l)^\perp$ and $\mathbf{v}_T^r(l)^\perp$ are the relative velocity of body, peduncle and tail with respect to the fluid normal to the part along the length, l, of the part. The drag coefficient C_D is a function of part geometry and fluid flow angle. It can be represented by:

$$C_D = C_{\mathrm{D,basic}} \sin^2\sigma$$

where $C_{\mathrm{D,basic}}$ is shape parameter, and σ is the angle between the relative velocity of the fluid and the part longitudinal axis. The generalized active force due to the drag force and torque for the system is then given by:

$$(\mathbf{F}_r)_{\mathrm{Drag}} = \sum_{i=B,P,T}\left(\frac{\partial^E \boldsymbol{\omega}^i}{\partial \dot{q}_r}\cdot \mathbf{T}_{\mathrm{Drag}_i} + \frac{\partial \mathbf{v}_i^B}{\partial \dot{q}_r}\cdot R_{\mathrm{Drag}_i}\right) \quad (31)$$

The lift force is given by:

$$\begin{aligned}
\mathbf{R}_{\mathrm{Lift}_i} &= \pi\rho S_B(\mathbf{v}_i^B \times \hat{\mathbf{e}}_i^B)\times \mathbf{v}_i^B \\
\mathbf{T}_{\mathrm{Lift}_i} &= \pi\rho S_i \mathbf{p}_i^B \times \left((\mathbf{v}_i^B \times \hat{\mathbf{e}}_i^B)\times \mathbf{v}_i^B\right)
\end{aligned}$$

where $\hat{\mathbf{e}}_i^B$ is unit vector of fish body, peduncle or tail with respect to body frame and S_i is the surface area of fish body, peduncle or tail. The generalized active force due to the lift force and torque for the system is given by:

$$(\mathbf{F}_r)_{\mathrm{Lift}} = \sum_{i=B,P,T}\left(\frac{\partial^E \boldsymbol{\omega}^i}{\partial \dot{q}_r}\cdot \mathbf{T}_{\mathrm{Lift}_i} + \frac{\partial \mathbf{v}_i^B}{\partial \dot{q}_r}\cdot \mathbf{R}_{\mathrm{Lift}_i}\right) \quad (32)$$

Then

$$(\mathbf{F}_r)_{\mathrm{Damping}} = (\mathbf{F}_r)_{\mathrm{Drag}} + (\mathbf{F}_r)_{\mathrm{Lift}} \quad (33)$$

3.2.5 Dynamic Model

Having developed all of the generalized inertia forces and generalized active forces for the fish body, peduncle, and tail, the equations of motion are found to obtain the following dynamic model:

$$\begin{aligned}
(\mathbf{F}_r^*) + (\mathbf{F}_r^*)_{\mathrm{AM}} + (\mathbf{F}_r)_{\mathrm{gravity}} + (\mathbf{F}_r)_{\mathrm{Buoy}} \\
+ (\mathbf{F}_r)_{\mathrm{FK}} + (\mathbf{F}_r)_{\mathrm{Damping}} = 0
\end{aligned} \quad (34)$$

Eq.34 may not be the most convenient form for the equations of motion. Then, we make the following form:

$$\mathbf{M}(\boldsymbol{\xi})\ddot{q} + \mathbf{C}(\boldsymbol{\xi},\dot{\mathbf{q}}) + \mathbf{G}(\boldsymbol{\xi}) + \mathbf{F}_{\mathrm{external}} = 0 \quad (35)$$

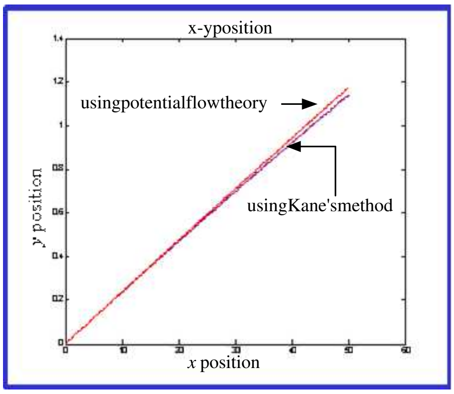

Fig. 6. x-y position plot using Potential flow theory and Kane's method

where ξ is the joint variable vector, $\mathbf{M}(\boldsymbol{\xi})$ is the matrix of inertia term, $\mathbf{C}(\boldsymbol{\xi},\dot{\mathbf{q}})$ is the matrix of Coriolis/Centripetal term and $\mathbf{G}(\boldsymbol{\xi})$ is the vector of gravity effect.

$$\begin{aligned}
(\mathbf{M}(\boldsymbol{\xi})) &= -\frac{\partial((\mathbf{F}_i^*) + (\mathbf{F}_i^*)_{\mathrm{AM}})}{\partial \ddot{q}_j} \\
(\mathbf{C}(\boldsymbol{\xi},\dot{\boldsymbol{q}})) &= -((\mathbf{F}^*) + (\mathbf{F}^*)_{\mathrm{AM}}) - \mathbf{M}(\boldsymbol{\xi})\ddot{\mathbf{q}} \\
\mathbf{G}(\boldsymbol{\xi}) &= -\mathbf{F}_{\mathrm{gravity}} \\
(\mathbf{F})_{\mathrm{external}} &= -(\mathbf{F})_{\mathrm{Buoy}} - (\mathbf{F})_{\mathrm{FK}} - (\mathbf{F})_{\mathrm{Damping}}
\end{aligned}$$

4. Conclusions

The 3D dynamic equations for a fish robot-PoTuna with peduncle and tail have been developed using potential flow theory and Kane's equation. In first method, we use nonlinear dynamic equations of rigid body with external force by tail oscillation obtained by potential flow method. We assume that fish body has many other forces as drag, lift, buoyancy force, gravity and the external force by tail oscillation. The external force is obtained by potential flow theory. These equations have some disadvantages. Because of limitation of control input, we must ensure the constraint. Kane's method provides a straight-forward approach for incorporating external forces into the model. External hydrodynamic forces considered in this model include: buoyancy, damping, added mass, Froude-Kriloff. The other external forces include gravity and joint torques of each part. It may be advantageous in future applications to perform coordinated control of peduncle and tail so that fish's body is reached in a prescribed manner.

Fig 6 shows the result using these two method at two dimensional. Red line is the result of the potential flow method and blue line is the result of Kane's method. We can see almost similar line. We can consider E.O.M of fish robot is the same as E.O.M. of rigid body except for using the tail's forces and moments as thruster's forces and moments. And we consider that the only effect of peduncle transforms the forces and moments of tail to fish's body. In the future, we try to compare these simulation result of dynamics equation with experiment and expand the 3 dimensional simulation.

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