Stability Analysis and Design of a Nonlinear Neuromuscular Control System of a Myoelectric Prosthetic Hand

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Abstract: A neuromuscular control system of a myoelectric prosthetic hand (PH) constitutes a nonlinear system with a dead zone whose magnitude is equal to its joint angle when the PH just grasps an object. This is because the neuromuscular control system remains an open-loop system until the PH grasps the object but it constitutes a feedback control system after the PH griped the object in which a torque induced in the fingers of the PH is fed back. To improve the transient performance of the control system, it is desirable to make the feed-forward gain as large as possible, so long as the stability of the system is not impaired. It is also desired that the control system remains stable even when the PH lifts a heavy or rigid object, because this makes the closed loop gain large and leads to the closed system unstable. According to the theory of stability analysis of nonlinear systems, we can only know the sufficient conditions that the system should be stable. Thus the nonlinear theory on stability is insufficient to be used to design the neuromuscular control system for improving its transient responses. This paper shows that the nonlinear system with a dead zone can be approximated to a linear feedback system and that well-known methods of analysis and design on linear control systems can be applicable. It is also shown through various simulation results that errors induced by approximation are practically negligible and thus the design methods are quite accurate.

Keywords: prosthetic hand, myoelectric control, dead zone, nonlinear, stability

1. INTRODUCTION

A new type of myoelectrically controlled biomimetic prosthetic hand (PH), which has almost the same dynamics as that of the neuromuscular control system of the human finger muscles, has been developed [1].

The neuromuscular control system of the myoelectric PH constitutes a nonlinear system with a dead zone whose magnitude is equal to its joint angle when the PH just grasps an object, as will be described in the followings. To improve the transient performance of the control system, it is desirable to make the feed-forward gain as large as possible, so long as the stability of the system is not impaired.

According to the well-known theory of stability analysis of nonlinear systems, however, we can only know the sufficient condition that the system should be asymptotically stable. Thus the nonlinear theory on stability is insufficient to be used to design the neuromuscular control system for improving its transient responses.

This paper shows that the nonlinear system with a dead zone can be approximated to a linear feedback system and that well-known methods of analysis and design on linear control systems can be applicable. It is also shown through various simulation results that errors induced by approximation are practically negligible and thus the design methods are quite accurate.

2. OUTLINE OF INVESTIGATED NONLINEAR CONTROL SYSTEM AND ITS STABILITY CRITERION

2.1 Outline of the nonlinear system investigated

In this research, the dynamics of the PH grasping an object is discussed.

Figure 1 shows a schematic configuration of a investigated nonlinear control system, that is, a system of myoelectrically controlled biomimetic PH. In Fig.1, control signal, whose Laplace transform is denoted by $\Theta_i(s)$, can be generated by using EMG (electromyogram) signal of flexor and extensor muscle, and used for driving the myoelectric PH. The controlled object, whose transfer function is denoted by $G_o(s)$, is composed of a servo system to drive PH, and $G_c(s)$ denotes the transfer function of the controller. The controlled variable is an joint angle $\theta(t)$ of the PH, whose Laplace transform is denoted by $\Theta(s)$, and is defined such that the closing direction is taken as positive. The magnitude $\bar{\theta}_d$ in Fig.1 represents the angle just the PH grips the object. Gain P represents the torque induced in the fingers of the PH, and is fed back in the myoelectrically controlled system.

As shown in Fig.1, the control system includes a dead zone and constitutes a nonlinear control system.

2.2 Stability criterion of the nonlinear system investigated

For the investigated nonlinear system, we can get important information on its stability from the third theorem of Popov [2, 3].

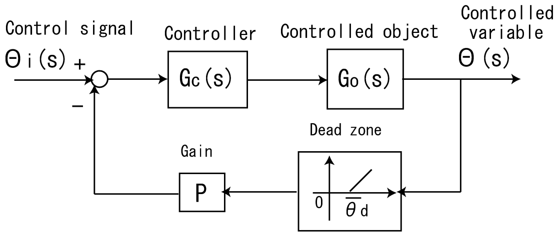


Fig. 1. Schematic configuration of a nonlinear control system investigated.

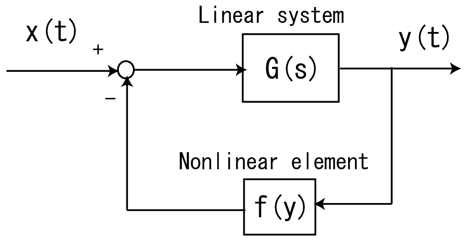


Fig. 2. Configuration of the nonlinear system applicable Popov's theory.

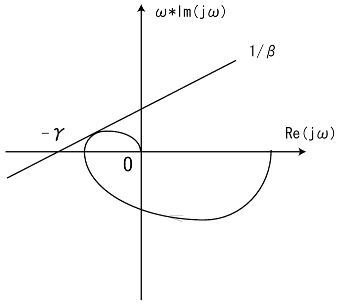


Fig. 3. Popov locus.

Third theorem of Popov :

In a nonlinear system shown in Fig.2, $G(s)$ is a completely controllable and observable linear system, and a nonlinear element $f(y)$ has the characteristics of f(0)=0, and

$$0 \leq \frac{f(y)}{y} \leq \frac{1}{\gamma}, \quad (1)$$

where γ is a positive real value.

Then, the sufficient condition that the nonlinear system shown in Fig.2 should be asymptotically stable, is existence of $\beta > 0$, satisfying the following relationship

$$\omega I_m(j\omega) \leq \frac{1}{\beta}(R_e(j\omega) + \gamma), \quad (2)$$

where $I_m(j\omega)$ and $R_e(j\omega)$ are imaginary and real part of the frequency transfer function of $G(j\omega)$, respectively, that is,

$$G(j\omega) = R_e(j\omega) + jI_m(j\omega). \quad (3)$$

This sufficient condition can also be described as follows: The Popov locus of the nonlinear system, with x-coordinate $R_e(j\omega)$ and y-coordinate $\omega I_m(j\omega)$, is below the straight line with the gradient $1/\beta$ passing the coordinate (-γ, 0), as shown in Fig.3.

The nonlinear system shown in Fig.1 can be easily converted to the system shown in Fig.2. Namely, $G(s) = G_c(s)G_o(s)$, and the nonlinear element f(y) is constituted by the dead zone and the gain P.

Since the nonlinear element of the investigated system has the characteristics

$$\frac{f(y)}{y} = \frac{P(y - \bar{\theta}_d)}{y} = P(1 - \bar{\theta}_d/y) \leq P, \qquad for \;\; y \geq \bar{\theta}_d, \quad (4)$$

it can be seen from Eqs.(1) and (4) that γ can be approximated to be $1/P$, that is,

$$P = 1/\gamma. \quad (5)$$

Hence x-intercept of the Popov locus, γ, approaches the origin the closer, or the value of γ becomes the smaller, the value of P satisfying the stability condition can take the larger value.

3. APPROXIMATE DESIGN METHOD PROPOSED

From the third theorem of Popov, we can only get the sufficient condition, so that there exists a possibility it is much different from the necessary and sufficient condition. Furthermore, the stability condition is not enough to design a control system. To overcome these problems, the following approximation design method is proposed.

3.1 Presumption

In order to be applicable of the proposed design method, the following two presumptions are necessary.

(1) The response of the dead zone element is origin symmetry as shown in Fig.4(a).

This assumption is usually valid because only positive part of the response has a realistic meaning.

(2) The value of $\bar{\theta}_d$ is small enough as the well-known describing function method should be applicable.

This assumption is valid by placing the initial angel of the PH just close to the angle of gripping the object.

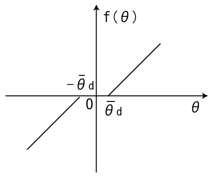
3.2 Transfer function representation of dead zone characteristics

As can be seen from Fig.4(a) and (b), the input and output relationship of the dead zone can be represented as

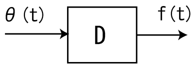
$$f(t) = \begin{cases} 0 & for \; \theta < \bar{\theta}_d \\ \theta(t) - \bar{\theta}_d & for \; \theta \geq \bar{\theta}_d. \end{cases} \quad (6)$$

Hence Laplace transform of the input and output of the dead zone can be obtained as

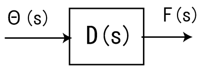
$$F(s) = \begin{cases} 0 & for \; \theta < \bar{\theta}_d \\ \Theta(s) - \bar{\theta}_d/s & for \; \theta \geq \bar{\theta}_d. \end{cases} \quad (7)$$



(a) input and output relationship



(b) Input and output signal



(c) Laplace representation

Fig. 4. Input and output signal of a dead zone element.

Thus, transfer function of the dead zone element, denoted by D(s), can be obtained as

$$D(s) = \frac{F(s)}{\Theta(s)} = \frac{\Theta(s) - \bar{\theta}_d/s}{\Theta(s)}. \tag{8}$$

3.3 Dynamic relationships of input and output of the system

For $\theta_i(t) < \bar{\theta}_d$, the control system remains an open loop control system, hence dynamic relationships of input and output of the system can be represented as

$$\Theta(s) = G_o(s)G_c(s)\Theta_i(s). \tag{9}$$

For $\theta_i(t) > \bar{\theta}_d$, the closed loop transfer function of the control system shown in Fig.1, denoted by W(s), can be obtained by applying the describing function method as follows

$$W(s) = \frac{G_c(s)G_o(s)}{1 + PG_c(s)G_o(s)D(s)}. \tag{10}$$

Substituting Eq.(8) into Eq.(10), output $\Theta(s)$ is given by

$$\Theta(s) = \frac{G_c(s)G_o(s)\Theta_i(s)}{1 + PG_c(s)G_o(s)\frac{\Theta(s) - \bar{\theta}_d/s}{\Theta(s)}}. \tag{11}$$

Thus, we can obtain the following input and output relationship

$$\Theta(s) = \frac{G_c(s)G_o(s)}{1 + PG_c(s)G_o(s)}[\Theta_i(s) + \frac{P\bar{\theta}_d}{s}]. \tag{12}$$

Therefore, we can recognize the importance of improving the dynamic characteristics of

$$W_c(s) \equiv \frac{G_c(s)G_o(s)}{1 + PG_c(s)G_o(s)}. \tag{13}$$

3.4 Stability criterion for myoelectric prosthetic hand

The myoelectric prosthetic hand we are currently developing has the following dynamic characteristics as

$$G_c(s) = k\frac{\tau_2 s + 1}{\tau_1 s + 1} \tag{14}$$

and

$$G_o(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \tag{15}$$

Thus the closed loop transfer function of the control system $W(s)$, given by Eq.(13), can be obtained as

$$W_c(s) = \frac{k\tau_2\omega_n^2 s + k\omega_n^2}{a_0 s^3 + a_1 s^2 + a_2 s + a_3}, \tag{16}$$

where

$$a_0 = \tau_1,$$

$$a_1 = 2\tau_1\zeta\omega_n + 1,$$

$$a_2 = \tau_1\omega_n^2 + 2\zeta\omega_n + kP\tau_2\omega_n^2, \ and$$

$$a_3 = (1 + kP)\omega_n^2. \tag{17}$$

From Routh's criterion, the condition of the system being stable is

$$b_{21} \equiv (a_1 a_2 - a_0 a_3)/a_1 > 0. \tag{18}$$

Therefore, the necessary and sufficient condition of the system being stable can be represented as

$$\frac{\{2\tau_1\omega_n(\tau_1\omega_n + 2\zeta) + 2\}\zeta}{k\{(\tau_1 - \tau_2) - 2\tau_1\tau_2\zeta\omega_n\}\omega_n} > P. \tag{19}$$

We can see from Eq.(19) that when feed-forward gain k becomes large, or the response of the servo system becomes fast (ω_n becomes large), or the damping coefficient ζ becomes small, then the value of P satisfying Eq.(19) becomes small.

When input signal is the step function of magnitude $\bar{\theta}$,

$$\Theta(s) = W_c(s)\frac{\bar{\theta} + P\bar{\theta}_d}{s}. \tag{20}$$

Thus, the final value of $\theta(t)$, denoted by $\bar{\theta}_f$, is obtained as

$$\bar{\theta}_f \equiv \lim_{t\to\infty}\theta(t) = \lim_{s\to 0} s\Theta(s) = \frac{k}{1 + kP}(\bar{\theta} + P\bar{\theta}_d). \tag{21}$$

If $D(s) = 1$, that is, the dead zone does not exist, the final value of the step response is $k\bar{\theta}/(1 + kP)$. We can see, therefore, that the final value of the nonlinear system is different from that of the linear system by $kP/(1+kP) \times \bar{\theta}_d$.

4. EXAMPLES OF NUMERICAL ANALYSIS

This section shows some results obtained from simple numerical examples.

In performing numerical calculations, parameters of $G_c(s)$ and $G_o(s)$ in Eq.(14) and Eq.(15) was assumed as $k = 2, \tau_1 = 0.25, \tau_2 = 0.12, \omega_n = 1$, and $\zeta = 1$ as the reference case. Although the value of k is related with the magnitude of the control signal $\theta(t)$ and the feed back gain P in the actual myoelectric control system, it was assumed to be a constant parameter in this analysis for simplicity of discussion, and the change in the value of P that keeps the nonlinear system stable was investigated. Let denote the maximum value of P obtained by Popov's theorem as P_c, and the proposed method as P_m.

In this section, step responses of the nonlinear system, when its parameters were changed, were also investigated by using Simulink of MATLAB. Figure 5 shows a configuration for system simulation by Simulink. The magnitude of the step input and the dead zone was assumed to be $\bar{\theta}$=25 and $\bar{\theta}_d$=20, respectively. The strting time of the step input was set to 1 second and observation peroid to 20 second.

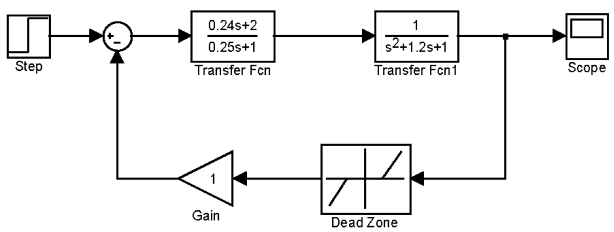


Fig. 5. Configuration for simulation by Simulink.

Example 1 : Reference case

Figure 6 shows Popov locus of the system when parameters takes the reference values. From Fig.6 the sufficient condition the system is stable, was obtained to be $P \leq 8.63$, whereas the maximum value of P obtained by the proposed method is $P \leq 8.70$. In this case, we can see the difference between P_c and P_m is not so large.

Figure 7 shows a step response when P=1. The system response is stable, since P is less than P_c, and the final value of the output $\bar{\theta}_f$ becomes 30.

Example 2 : Case where gain k is increased

Figure 8 shows Popov locus of the system when the gain k of the controller $G_c(s)$ in Eq.(14) was increased from 2 to 10. The value of P_c was obtained to be 1.74 from Fig.8, whereas P_m was 1.74 by the proposed method. In this case, we can see P_c and P_m are practically equal.

Figure 9 shows a step response when $k = 10$, P=1.5 and other parameters' value were set equal to those of reference case. The system response is somewhat oscillatory but the system is stable, since P is less than P_c, and $\bar{\theta}_f$ becomes 34.4.

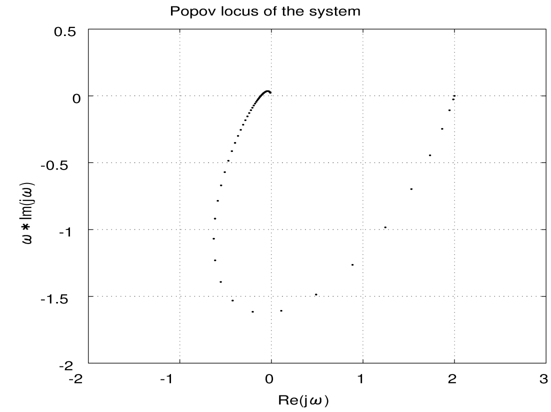


Fig. 6. Popov locus of the reference system ($P_c = 8.63$).

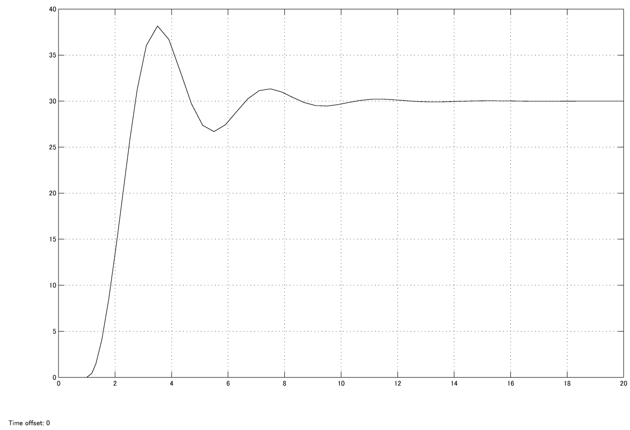


Fig. 7. Step response when P=1 ($P < P_c = 8.63$).

Example 3 : Case where τ_2 was changed

Figure 10 shows Popov locus of the system when τ_2 of the controller $G_c(s)$ in Eq.(14) was changed from 0.12 to 0.1923. P_c was obtained to be 6.55 from fig.10, whereas P_m was obtained to be 81750 by the proposed method. In this case, we can see P_c and P_m are quite different.

Figure 11 shows a step response when P=80000 and other parameters' value were set equal to those of reference case. Although the value of P is much bigger than P_c, the system response in Fig.11 seems to be stable, and $\bar{\theta}_f$ becomes 20.0.

In the above examples, the magnitude of the dead zone is not small, being different from the presumption described in Sec. 3.1, but the results derived from the proposed method were all valid. It should also be noted that Eq.(21), the equation for estimating the final value of the system, was valid to all cases. Hence it can be expected that the proposed method is useful in designing the present nonlinear system.

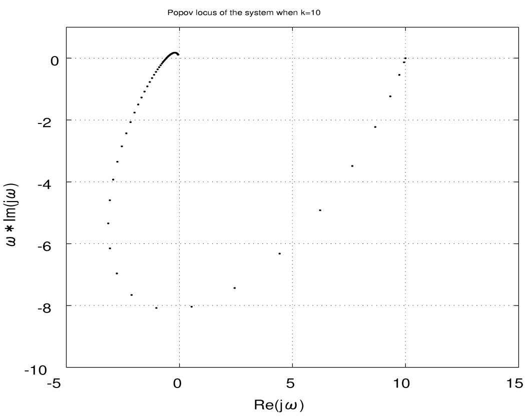


Fig. 8. Popov locus of the system when K=10 ($P_c = 1.74$).

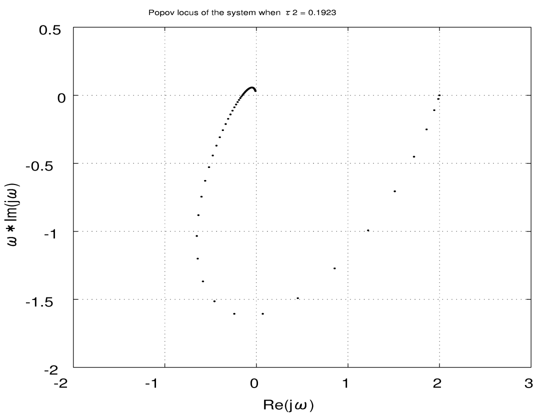


Fig. 10. Popov locus of the system when τ_2=0.1923 ($P_c = 6.55$).

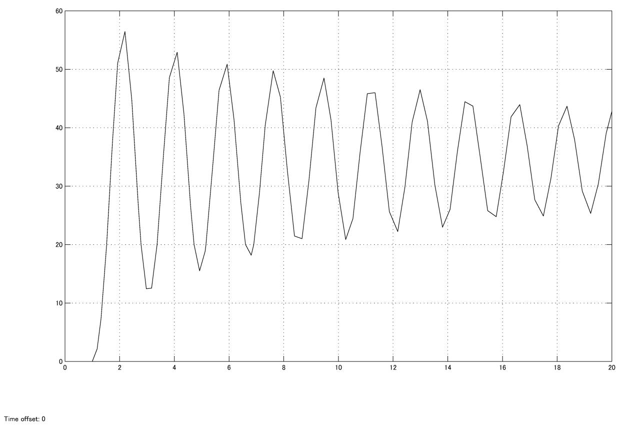


Fig. 9. Step response when K=10 and P=1.5 ($P < P_c = 1.74$).

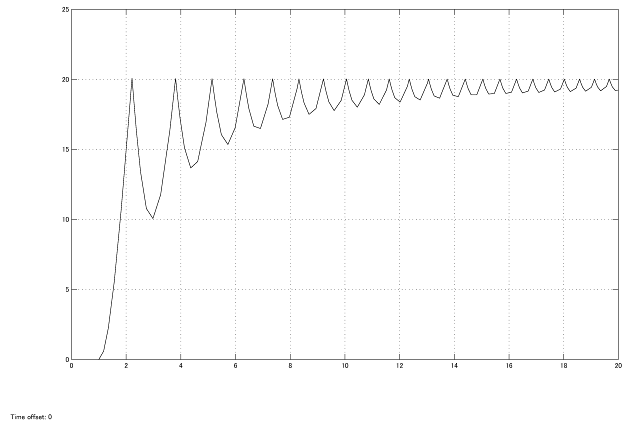


Fig. 11. Step response when τ_2=0.1923 and P=80000 ($P > P_c = 6.55$ but $P < P_m = 81750$).

5. PRELIMINARY APPLICATION TO THE LATEST ACTUAL PROSTHETIC HAND

In this section, results of a preliminary application to an actual myoelectric prosthetic hand is described.

Owing to results of the latest development of the myoelectric prosthetic hand, the dynamics of the servo system has been much improved, and its characteristics has been estimated as $\omega_n = 31.88$, and $\zeta = 0.727$, and other parameters have been the same as described in the previous section.

The resultant control system has been proved to be stable, that is, when $k = 2, \tau_1 = 0.25$, and $\tau_2 = 0.12$, as can be seen from Fig.12.

The stability is, however, easily influenced by change in system parameters. For example, Fig.13(a) shows the step response of the control system when τ_2 is changed from 0.12 to 0.01 and P=20. In this case, P_c was obtained to be 9.29 by the Popov method, whereas P_m to be 14.0 by the proposed method. It is confirmed from Fig.13(a) that the system is unstable.

Figure 13(b) shows the step response of the control system when τ_2=0.01 and P=11.6. This system can not be assured to be stable based on the Popov method, but the system is decided to be stable by the proposed method because $P < P_m$. The step response shown in Fig.13(b) seems to be the one from stable systems, and the final value of the output $\bar{\theta}_f$ becomes 21.2, that is equal to the value calculated by Eq.(21) for estimating the final value of the system.

For comparison, Fig.14 shows the step responses of the corresponding linear control system which was constituted by removing the nonlinear element (dead zone element) from the present control system and the others were all untouched. We can see that the responses of the two systems are quite different except the stability of the system.

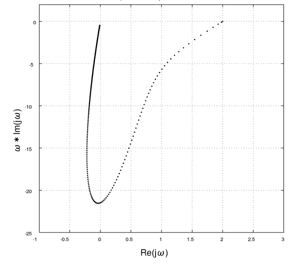
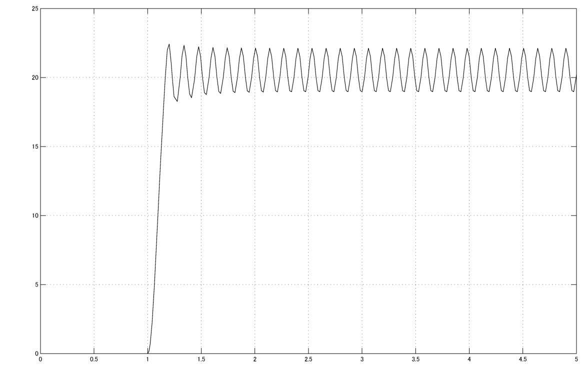
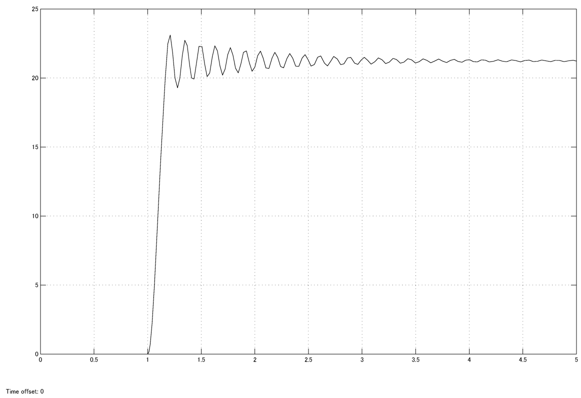


Fig. 12. Popov locus of the latest system when $\tau_2 = 0.12$ ($P_c \simeq \infty$).

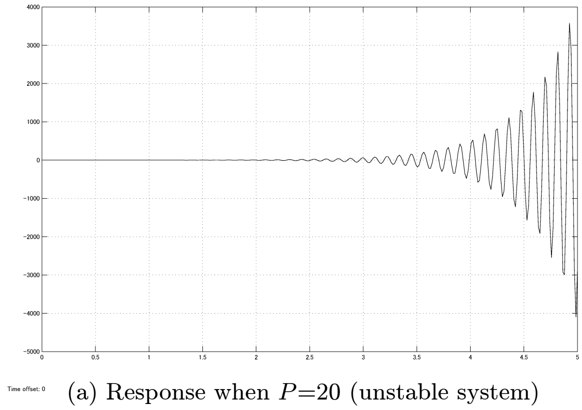


(a) Response when P=20 ($P > P_c = 9.29$ and $P > P_m = 14.0$)

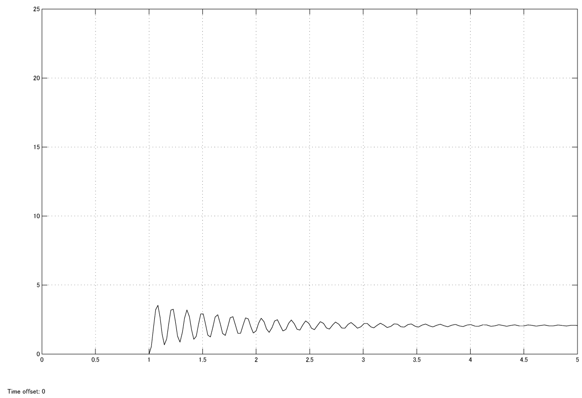


(b) Response when P=11.6 ($P > P_c = 9.29$ but $P < P_m = 14.0$)

Fig. 13. Step response when the value of P was changed.



(a) Response when P=20 (unstable system)



(b) Response when P=11.6 (stable system)

Fig. 14. Step response of the corresponding linear system.

6. CONCLUSION

A method to analyze the stability and dynamics of a nonlinear control system has been proposed. Investigated nonlinear system is a neuromuscular control system of a myoelectric prosthetic hand (PH).

It has been shown in this paper that the nonlinear system with a dead zone can be approximated to a linear feedback system and that well-known methods of analysis and design on linear control systems can be applicable. It is also shown through various simulation results that errors induced by approximation are practically negligible and thus the design methods are quite accurate.

The obtained final results by applying fully the proposed method to the actual myoelectric prosthetic hand will be published in the near future.

Since grasping an object is one of most fundamental actions, the proposed method could be expected to be applicable in many applications.

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