Networked Control System Design Accounting for Time-Delays with Application to Inverted Pendulum

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Abstract: In this paper the networked control systems (NCS) problem is discussed where plants and controllers are distributed and interconnected by a common network. NCS is designed with LQ regulator and applied to an inverted pendulum accounting for the multiple time delays. We are to deals with a networked control system with a single controller, multiple sensors and multiple actuators. Since these parts are distributed, they are interconnected by communication networks. An NCS with LQ regulator is designed and applied to an inverted pendulum as a benchmark plant to check its performance under time delays induced by the network. Network induced delays are composed of two parts. One is the delay from controller to plant, and another is from plant to controller. They are assumed to be constant in this paper, and the plant and controller are discretized. To apply the LQ regulator the NCS model is transformed to a standard model with delayed states as state variable. And real network induced delay is measuring in TCP/IP network assuming that two delays are constant.

Keywords: NCS, Network induced delay, LQR, Inverted Pendulum

1. INTRODUCTION

A control system communicating with sensors and actuators over a communication network is called Networked Control Systems (NCS). In these days, the industrial control systems are requiring that decentralization of control, modularity and low costs. So analysis, modeling and control of NCS have been investigated for several decades. There are many different network types in control systems: Ethernet bus, ControlNet, and DeviceNet, and, etc. These common-bus network architectures for distributed control systems offer advantages in industrial area. These specific control problems have been a major trend in industrial area.[1-5]

By the way, the communication architecture change from point-to-point to common-bus introduces different forms of time delay uncertainty between sensors, actuators, and controller. Such time delays could be constant, bounded, or random depended on the network protocols or network status. Related to the time-delay on NCS research has been studied for several years. Analyses of time-delay systems have been investigated mainly in the frequency domain for single-delay cases. For multi-delay cases, the functional approach or LMI (Linear Matrix Inequality) is used. In functional approach, the controller design uses robust or stochastic control approach.

This article describes the networked control systems problem with network induced delay. Network induced delays are composed of two parts. One is the delay from controller to plant, and another is from plant to controller. They are assumed to be constant in this paper, and the plant and controller are discretized for the computer implementation. In this paper, networked control system with constant delays is designed for an inverted pendulum, which is an unstable systems frequently used as an benchmark plant. Therefore, time-delays are very important factors in the NCS to be designed. To implement NCS for inverted pendulum, the time delays are tested in the local area network environment. The control algorithm is based on the standard LQ regulator[6-8]. To apply the LQ regulator the NCS model is transformed to a standard model with delayed states as state variable.

This paper is organized as follows: In Section 2, the NCS problem it to be formulated. Section 3 describes the system discretization, and Section 4 designs the LQ regulator for the NCS. In Section 5, some simulations applied to an inverted pendulum are performed, and conclusions are given in Section 6.

2. PROBLEM FORMULATION

2.1 The controlled plant

An inverted pendulum is to be taken as the controlled plant, which is known to be nonlinear and unstable and so frequently used as a benchmark plant in control system design problems. The inverted pendulum used in this paper is mounted on guided rail shown in Fig. 1. So the cart position and the rod angle must be controlled at the same time. Non-NCS LQR has been used for inverted pendulum. The information of the cart position and the rod angle is converted by hardware AD/DA device.

The linearized model of the inverted pendulum at the equilibrium point is described by the continuous state-space equation as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{\alpha} \\ \ddot{x} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -4.5 & -16.8 & 0 \\ 0 & 46.9 & 55.3 & 0 \end{bmatrix} \begin{bmatrix} x \\ \alpha \\ \dot{x} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 3.8 \\ -12.4 \end{bmatrix} V_{in}. \qquad (1)$$

2.2 Network formulation

Suppose that networked control systems are composed of two computers and two HUBs, where one computer is used as the controller and another as sensor and actuator. An NCS is to be designed accounting for the network induced delays, and some network induced delays are generated and tested. The test program is a modified ping program based on TCP/IP. The network induced delays are taken as 16~30ms between two computers and two hubs, that is, the sensor computer – hub 1– hub 2 – the control computer.

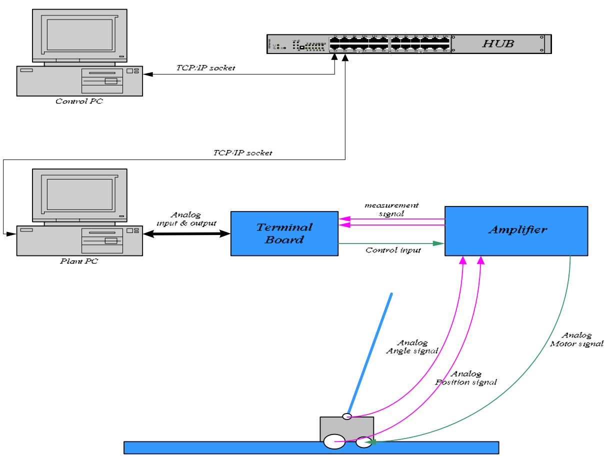

Fig. 1 Simplified NCS for an inverted pendulum.

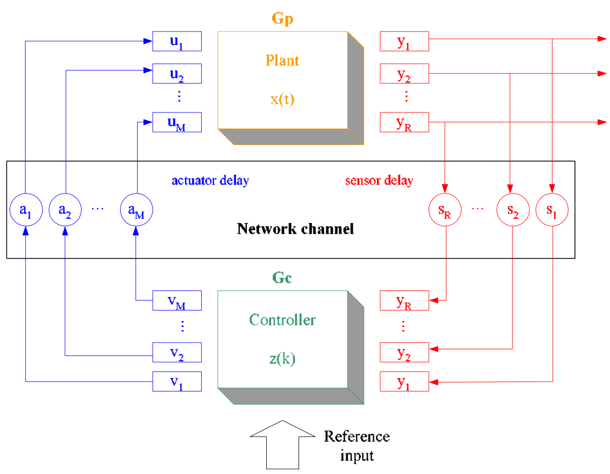

Fig. 2 The block diagram of an NCS

3. DISCRETIZED SYSTEM

Fig.2 is the block diagram of an NCS. This diagram has been proposed by Feng-Li Lian [1]. This is MIMO model with multi sensor input and multi actuator output. Also each input, output node was considered different delay scale. Such signals are discrete signal. In order to construct closed loop system, continuous system is discretized considering the system's time delay. The discretized closed-loop model has been proposed in [1]. The continuous plant model is as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A}_p\mathbf{x}(t) + \mathbf{B}_p\mathbf{u}(t),$$
$$\mathbf{y}(t) = \mathbf{C}_p x(t). \tag{2}$$

where $\mathbf{A}_p, \mathbf{B}_p, \mathbf{C}_p$ are continuous system matrices.

Eq. (2) is converted to

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \sum_{j=0}^{1}\mathbf{B}_k^j\mathbf{v}_{k-j}. \tag{3}$$

where $\mathbf{A} \equiv \exp(\mathbf{A}_p T),$

$$\Gamma(T,q) \equiv \exp\left(\mathbf{A}_p(T-q)\right)\mathbf{B}_p,$$
$$\mathbf{B}_m^1(k) \equiv \int_0^{a_m(k)}\Gamma_m(T,q)dq,$$
$$\mathbf{B}_m^1(k) \equiv \int_0^{a_m(k)}\Gamma_m(T,q)dq.$$

Considering the sampling of the sensor Eq. (3) is converted to

$$\mathbf{x}\left((k+1-\delta)T\right) = \mathbf{A}_\delta\mathbf{x}_k + \sum_{j=0}^{1}\mathbf{B}_{\delta k}^j\mathbf{v}_{k-j} \tag{4}$$

let T (sampling time)=10 ms,
a (actuator delay)=1 ms,
s_1, s_2, s_3, s_4 (sensor delay)=3, 4, 5, 6 ms

Then, discretized inverted pendulum's state equation is

$$\mathbf{x}(kT-s_r) = \begin{bmatrix} 1 & -0.0001 & 0.0066 & 0 \\ 0 & 1.0008 & 0.0010 & 0.0060 \\ 0 & -0.0216 & 0.9194 & -0.0001 \\ 0 & 0.1857 & 0.2140 & 1.0004 \end{bmatrix}\mathbf{x}_{k-1} + \begin{bmatrix} 0.0001 \\ -0.0002 \\ 0.0111 \\ -0.0244 \end{bmatrix}\mathbf{v}_{k-1}\begin{bmatrix} 0 \\ -0.0001 \\ 0.0071 \\ -0.0236 \end{bmatrix}\mathbf{v}_{k-2} \tag{5}$$

4. LQ CONTROLLER

Since the NCS problem requires a multivariable control law, the LQ regulator is adopted in this paper. In order to apply the above discretized system equation to the LQ optimal control design technique, some new state variables are required. So the new state variable $\mathbf{z}(k)$ is also taken as follows:

$$\mathbf{z}(k) = \left[\mathbf{x}_s(k)^T, \mathbf{v}(k-1)^T, \mathbf{v}(k-2)^T\right]^T \tag{6}$$

$$\mathbf{z}(k+1) = \begin{bmatrix} \mathbf{A}_{x_s} & \mathbf{B}_{x_s}^1 & \mathbf{B}_{x_s}^2 \\ 0 & 0 & 0 \\ 0 & \mathbf{I} & 0 \end{bmatrix}\mathbf{z}(k) + \begin{bmatrix} \mathbf{B}_{x_s}^0 \\ \mathbf{I} \\ 0 \end{bmatrix}\mathbf{v}(k). \tag{7}$$

By Eqs. (6)~(7), the new state augmented equation for the inverted pendulum is described by

$$z(k+1) = \begin{bmatrix} 1 & -0.0002 & 0.0092 & 0 & 0.0001 & 0 \\ 0 & 1.0023 & 0.0026 & 0.0100 & -0.0004 & 0 \\ 0 & -0.0415 & 0.8453 & -0.0002 & 0.0168 & 0 \\ 0 & 0.04576 & 0.5095 & 1.0023 & -0.0517 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} z(k) + \begin{bmatrix} 0.0001 \\ -0.0002 \\ 0.0147 \\ -0.0363 \\ 1 \\ 0 \end{bmatrix} v(k). \tag{8}$$

Non NCS optimal control cost function is

$$J = \int\left(x'Q_s x + rv^2\right)dt. \tag{9}$$

where $r = 0.0003$,

$$Q_s = \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

And the cost function and Q, R for NCS optimal controller are as follows :

$$V=\sum_{k=0}^{N}\left\{\left[\mathbf{x}_s(k)-\mathbf{d}_s(k)\right]^T\mathbf{Q}_s\left[\mathbf{x}_s(k)-\mathbf{d}_s(k)\right]+\sum_{t=0}^{2}\mathbf{v}(k-i)^T\mathbf{R}_i\mathbf{v}(k-i)\right\}. \quad (10)$$

where $\mathbf{R}=r$,

$$\mathbf{Q}=\begin{bmatrix}\mathbf{Q}_s & 0 & 0\\ 0 & \mathbf{R}_1 & 0\\ 0 & 0 & \mathbf{R}_2\end{bmatrix}$$

$\mathbf{d}(k)$: desired trajectory

Then, the optimal control law is

$$\mathbf{v}^*(k)=-\mathbf{K}_s(k)\left[\mathbf{x}_s(k)-\mathbf{d}_s(k)\right]-\sum_{i=1}^{2}\mathbf{K}_v^i(k)\mathbf{v}^*(k-i). \quad (11)$$

This optimal controller utilize the measured state as well as past inputs as the controller inputs.

5. DELAY SIMULATION AND RESULT

5.1 Simple delay simulation

Now, we simulate the NCS for the inverted pendulum. As simulation parameter, we have taken T (sampling time) = 10 ms, a (actuator delay) = 1 ms, s_1, s_2, s_3, s_4 (sensor delay) = 3, 4, 5, 6 ms. And $\mathbf{R}=0.0003$,

$$\mathbf{Q}=\begin{bmatrix}0.025 & 0 & 0 & 0 & 0 & 0\\ 0 & 2 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & \mathbf{R}_1 & 0\\ 0 & 0 & 0 & 0 & 0 & \mathbf{R}_2\end{bmatrix}.$$

where $\mathbf{R}_1=0.0002, \mathbf{R}_2=0.00001\cdot$

Then, NCS state equation is Eq. (8). Non NCS optimal LQR controller gain is

$$\mathbf{K}=\begin{bmatrix}-31 & -141 & -34 & -14\end{bmatrix},$$

and NCS optimal LQR controller gain is

$$\mathbf{K}=\begin{bmatrix}-19.4134 & -125.180 & -47.7710 & -19.1043 & 0.2652 & 0\end{bmatrix}.$$

5.2 Tested delay simulation

Network induced time delays are tested by modified ping program in the building with local area network. Tested network route is from the sensor computer to the control computer through two hubs. Tested network induced time delays are almost 10 ~ 18 ms, and sometimes, they are 40ms over. So more than 40ms time delays are not considered in this paper. Real inverted pendulum's sampling time is 40ms. Let a (actuator delay) = 10 ms, s_1, s_2, s_3, s_4 (sensor delay) = 16, 17, 18, 19 ms. Then discretized state equatioin and closed equations are changed, and also NCS LQR control gain is changed. By Eq. 7, the state equation accounting for the time delays is given as follows:

$$z(k+1)=\begin{bmatrix}1 & -0.0017 & 0.0236 & 0 & 0.0013 & 0\\ 0 & 1.0202 & 0.0213 & 0.0302 & -0.0043 & 0\\ 0 & -0.1069 & 0.6032 & -0.0017 & 0.0580 & 0\\ 0 & 1.3209 & 1.3131 & 1.0202 & -0.1902 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\end{bmatrix}z(k)$$

$$+\begin{bmatrix}0.0002\\ -0.0005\\ 0.0285\\ -0.0819\\ 1\\ 0\end{bmatrix}v(k)$$

where $\mathbf{R}=0.0003$, $\mathbf{R}_1=0.0002, \mathbf{R}_2=0.00001$,

$$\mathbf{Q}=\begin{bmatrix}0.025 & 0 & 0 & 0 & 0 & 0\\ 0 & 2 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0.0002 & 0\\ 0 & 0 & 0 & 0 & 0 & 0.00001\end{bmatrix}.$$

Then, NCS optimal LQR controller gain is

$$\mathbf{K}=\begin{bmatrix}-14.3213 & -73.7166 & -25.1758 & -10.4192 & 0.6429 & 0\end{bmatrix}.$$

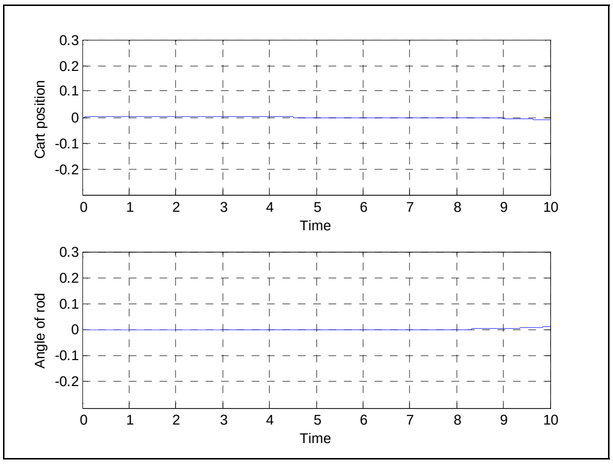

Fig. 3 Non NCS LQ control result

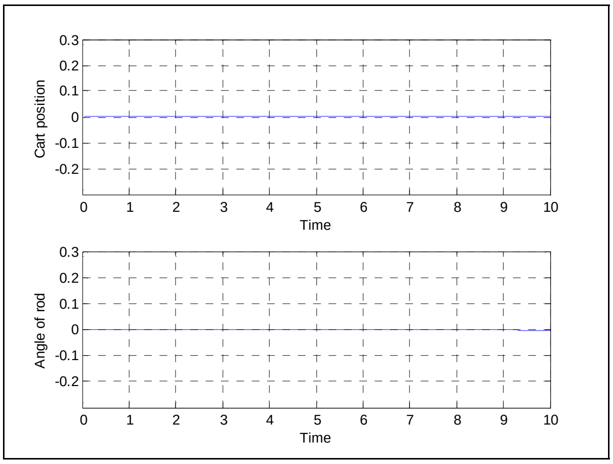

Fig. 4 NCS LQ control result

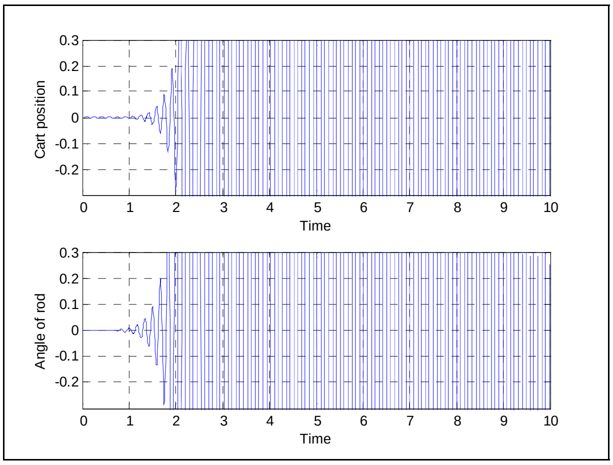

Fig. 5 Non NCS LQ control result with tested delay

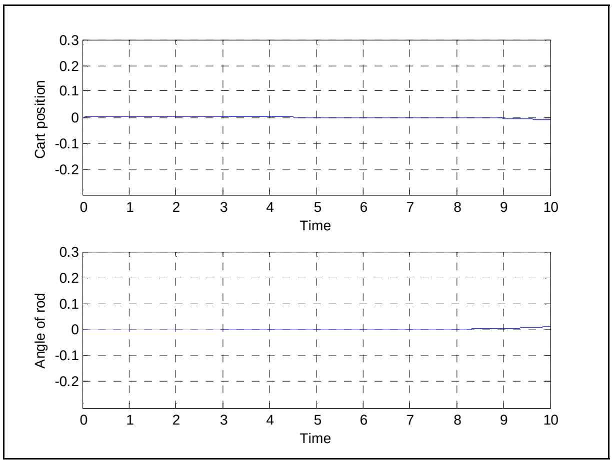

Fig. 6 NCS LQ control result with tested delay

5.3 Simulation results

Fig. 3 ~ 6 show the network induced delay's effect. It is shown that the non-NCS LQR works well with short time delays in Fig. 3, but that the tested delays of 16~19 ms in the actuator and the sensor make the controller useless as shown in Fig. 5. On the other hand, the NCS LQR accounting for the time delays works well in the case of tested delays as well as in the case of short time delays, which can be shown in Fig. 4 and 6.

6. CONCLUSION

In this paper, an NCS is designed and implemented using two computers and two hubs, and applied to an inverted pendulum as the benchmark plant. Because of the characteristics of guided inverted pendulum, LQR is selected as the control law. Since the network based control problems are to generate network induced delays, NCS LQR technique is applied to the linearized inverted pendulum with assumption of the constant delay. Since the delay characteristics are very important and various, simple time delay and various tested time delays are generated in simulations for the NCS designed. And via some simulations, it is shown that the NCS works well even in the case of various time delays.

It is noted that the tested time delays should be less than the sampling time in this paper. More complex or wide range network will make random and complicated time delays even larger that the sampling time. The implementation of NCS for the larger time delays requires further works. The NCS designed is to be connected to DeviceNet with CAN Bus.

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