Static Output Feedback Model Predictive Control for Wiener Models with Polytopic Uncertainty Descriptions

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Abstract: In this paper, we proposed static output feedback model predictive control for Wiener models. We adopted polytopic uncertainty description of Wiener Model Predictive Control (WMPC) algorithms for considering output nonlinearities. Robust stability conditions have been presented under which the closed loop stability of static output feedback MPC is guaranteed. The proposed control law is determined from the static output feedback WMPC based on the current estimated state with explicit satisfaction of input constraints.

Keywords: static output feedback, model predictive control, wiener model

1. INTRODUCTION

There are very few design techniques that can be proven to stabilize processes in the presence of nonlinearities and constraints. Model predictive control(MPC)-an optimal control based method-is one of these techniques. MPC refers to a class of computer control algorithms that control the future behavior of a plant through the use of an explicit process model. At each control interval the MPC algorithm computes an open-loop sequence of manipulated variable adjustments in order to optimize future plant behavior. The first input in the optimal sequence is injected into the plant, and the entire optimization is repeated at subsequent control intervals [1].

A special class of nonlinear models is block oriented one in which a linear time invariant dynamic block is preceded and followed by a static non-linearity. When the linear dynamic block is followed by a static output non-linearity the model is referred to as a Wiener model. Although Wiener models only represent a small subclass of all nonlinear models, they have appeared useful in modeling several nonlinear processes encountered in the process industry, for example distillation columns, a heat exchanger and pH neutralization processes. This capability of approximating nonlinear processes has been investigated by Boyd and Chua, who have proven that any time invariant system can be approximated with arbitrary accuracy by a finite dimensional Wiener model.

There are several methods to relax the computational demand of the nonlinear optimization problem for wiener models. Norquay et al. [7] use the specific structure of Wiener models to relax the computational demand. This is done by inverting the static nonlinearity, thus essentially removing it from the control problem, which enables the use of linear MPC techniques for the remaining linear block. Van den Boom and Bloemen [11] presented another MPC algorithm for Wiener models that take the effect of the input and output nonlinearities into account, while still retaining a convex optimization problem. This is done by transforming the static nonlinearities into polytopic uncertainty descriptions. Thus the Wiener model is represented as a linear model with uncertainty, which enables the use of robust linear MPC techniques to control this kind of system. The optimization problem is given by minimization of a linear cost function subject to linear matrix inequalities (LMIs)[13]. By the way, every suggested Wiener model predictive control (WMPC) algorithms are considered on regulation and tracking problems based on state feedback or state observer based form.

This paper is oganized as follows. In section 2, we present the static output feedback Wiener model predictive control algorithm and closed-loop stability. In section 3, we present example to illustrate this algorithm. Finally, in section 4, we conclude this paper.

2. PROBLEM FORMULATION

2.1. System description

Consider following wiener models:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \\ z(k) &= h_i(y_i(k)) \;\; for \;\; i = 1...p \end{aligned} \tag{1}$$

In which A, B, C, D are the system matrices of the linear dynamic block, $u(k) \in \mathbf{R^{nb}}$is the control input,$y(k) \in \mathbf{R^{nc}}$ is the output of the linear block respectively, h_i is the nonlinear mapping from $y(k)$ to $z(k)$ and is the output of the nonlinear block. The numbers of linear block output $y(k)$and nonlinear output $z(k)$ equal to p.

The nonlinearity of the Wiener model is transformed into a polytopic uncertainty description. Assume, without loss of generality that $h_1 \dots h_p$ are polynomials. The nonlinearity can be written as:

$$\begin{aligned} z(k) = h_i(y_i(k)) = H(y)y(k) \\ where \quad H(y) \in \Omega = Co\{H_1, \dots, H_{2^p}\} \end{aligned} \tag{2}$$

in which $H(y)$ is a diagonal matrix because of the special structure of the nonlinearity. When the operating region for $y(k)$ is limited the entries of $H(y)$are bounded by minimum and maximum values. All the possible combinations of the maximum and minimum values of the element of $H(y)$ are used to generate 2^p vertices $\{H_1, \dots, H_{2^p}\}$of the polytopic description Ω which contains the nonlinear matrix $H(y)$. (Co refers to the convex hull).

2.2. Controller Design

Consider the following problem, which minimizes the infinite horizon quadratic object function:

$$\min_{u(k+i|k)=Fz(k+i|k)} \max_{H(y)\in\Omega} J_\infty(k) \quad (3)$$

subject to

$$|u_r(k+i|k)| \leq u_{r,max}, \quad i \geq 0, \quad ,r = 1,2,\ldots,nb \quad (4)$$

where

$$J_\infty(k) = \sum_{i=0}^{\infty}[z(k+i|k)^T Q_1 z(k+i|k) + u(k+i|k)^T Ru(k+i|k)] \quad (5)$$

with $Q_1 > 0$, $R > 0$ we assume that at each sampling time k,a static output feedback law

$$u(k+i|k) = Fz(k+i|k) \quad (6)$$

is used to minimize the maximum value of $J_\infty(k)$, it is easy to derive an upper bound on $J_\infty(k)$. At sampling time k , define a quadratic function

$$V(x) = x^T P(k)x, \quad P(k) > 0. \quad (7)$$

For any $H(y) \in \Omega$, $i \geq 0$ suppose $V(x)$ satisfies the following robust stability constraint:

$$V(x(k+i+1|k)) - V(x(k+i|k)) \geq -[z(k+i|k)^T Q_1 z(k+i|k) + u(k+i|k)^T Ru(k+i|k)] \quad (8)$$

summing (8) from $i = 0$ to $i = \infty$ and requiring $x(\infty|k) = 0$ or $V(\infty,|k) = 0$, it follows that

$$\max_{H(y)\in\Omega} J_\infty(k) \geq V(x(k|k)) \geq \gamma \quad (9)$$

(8) and (9) give an upper bound on $J_\infty(k)$. The condition $V(x(k|k)) \geq \gamma$ in (9) can be expressed equivalently as the LMIs

$$\begin{bmatrix} 1 & x(k|k)^T \\ x(k|k) & Q \end{bmatrix} \geq 0, \quad Q > 0 \quad (10)$$

where $Q = \gamma P(k)^{-1}$ .The robust stability constraint (8) for the system (1) is satisfied if for each vertex of Ω.
inequality (8) become

$$x(k+i|k)^T \Big((A + BFH(y)C)^T P(A + BFH(y)C) - P + C^T H(y)^T F^T RFH(y)C + H(y)^T Q_1 H(y)\Big)x(k+i|k) \leq 0 \quad (11)$$

We see that this equivalent to

$$\begin{bmatrix} Q & QA^T + Y^T B^T & QC^T\hat{Q}^{1/2} & Y^T R^{1/2} \\ AQ + BY & Q & 0 & 0 \\ \hat{Q}^{1/2}CQ & 0 & \gamma I & 0 \\ R^{1/2}Y & 0 & 0 & \gamma I \end{bmatrix} \geq 0 \quad (12)$$

where $Q = \gamma P(k)^{-1}$,$Y = FHCQ$ and $\hat{Q} = H^T Q_1 H$
The input constraints (4) are satisfied if there exists a symmetric matrix such that [2]

$$\begin{bmatrix} X & Y \\ Y^T & Q \end{bmatrix} \geq 0, \quad withX_{jj} \leq u^2_{j,max}, j = 1,2,\ldots,nb \quad (13)$$

2.3. Closed loop stability

The closed-loop system for the system(1) is:

$$x(k+1) = \bar{A}x(k) \quad (14)$$

where $\bar{A} = A + BF$.
The system(14) is stable if there exists a matrix $Q > 0$ such that for all vertices of Ω

$$\begin{bmatrix} Q & \bar{A}^T \\ \bar{A} & Q \end{bmatrix} > 0 \quad (15)$$

by using Schur complement and letting $P = Q^{-1}$,(15) is equivalent to

$$P - \bar{A}^T P\bar{A} > 0 \quad (16)$$

which ensures that the quadratic function $x^T Px$ is monotonically decreasing.

3. SIMULATIONS

In this section, we present an example that illustrate the implementation of this static output feedback WMPC. The system matrices and nonlinearity of the model and the tuning parameters of this algorithms are:

$$A = \begin{bmatrix} 0.9973 & 0 & 0 \\ 0.0713 & 0.0948 & 0 \\ -0.0070 & 0 & 1.000 \end{bmatrix}$$
$$B = \begin{bmatrix} 0.8988 & -0.4480 & -0.1498 \\ 0.0326 & 17.2985 & -0.0054 \\ -0.0031 & -1.7455 & 1.6593 \end{bmatrix}$$
$$C = \begin{bmatrix} 1.000 & 0 & 0 \\ 0 & 1.000 & 0 \\ 0.0051 & 0 & 0.0046 \end{bmatrix}$$

$Q1 = 20, \quad R = 0.1$
$u_{max} = [6 \quad 2 \quad 5]^T$
$H(y) = diag(1+\alpha(k), 1+\alpha(k), 1+\alpha(k))$
where $|\alpha(k)| \leq 0.5$
the initial condition used $x = [13.5, 8.9, -2]^T$, we illustrate simulation results in figure 1.

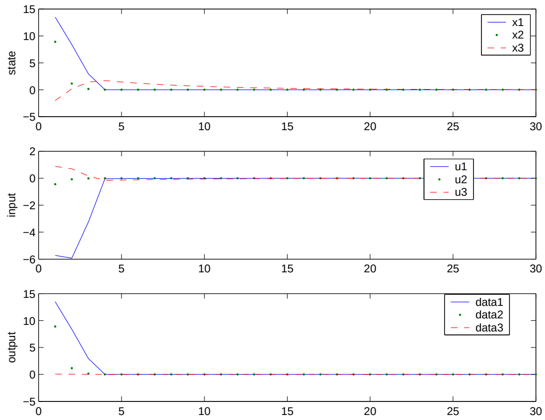

Fig. 1. The closed-loop responses of system for example

4. DISCCUSION

In this paper, we proposed static output feedback model predictive control for Wiener models. We adopted polytopic uncertain description method of Wiener Model predictive control (WMPC) algorithms for considering output nonlinearities. Robust stability conditions have been presented under which the closed loop stability of static output feedback MPC is guaranteed. The proposed control law is determined from the static output feedback WMPC based on the current estimated state with explicit satisfaction of input constraints.

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