

A Design Method of Sliding Model Control System Using Parallel Ladder Network of Dynamic Compensators

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Abstract: In this paper, the design method of sliding mode control (SMC) system for SISO linear system is discussed. First, we consider the similarity between the design method of sliding mode hyper plane using the strict positive realness and the characteristics of zeros of feedback system and the design method of simple adaptive control. Based on such a consideration, we propose the new design method of SMC system using parallel dynamic compensator. As a result, SMC system can be constructed only with the derivative of output signal for controlled plant. The performance of SMC system designed by proposed method is confirmed through the numerical example.

Keywords: sliding mode control, strict positive realness, ladder filter, parallel feedforward compensator

1. INTRODUCTION

Sliding Model Control (SMC) method [1]-[10] has been widely noticed because of its superior robust control performance for systems with highly uncertainty. SMC is based on the principle that a nonlinear switching control action holds the state trajectory on the hyper-plane in the state space and makes the convergence of the state to the equilibrium point with sliding mode action. SMC design procedure can be divided into three partitions, that is, (1) a design of hyper plane (2) a design of sliding mode controller (3) a design of countermeasure for chattering phenomenon [9], [10]. Concerning the first part, there exist some schemes. One of such scheme, a design method using the characteristics of zero, calculate the parameters of hyper plane by solving a Riccati algebraic equation which is introduced using the invariable relationship of zeros between the controlled plant and the strictly positive real (SPR) feedback control system. SMC systems usually require all state signals of the controlled plant because the switching function of SMC is constructed with them.

Recently, some SMC methods using the switching function constructed only with output signal of controlled plant has been considered, while they suppose the additional restrictive assumptions. However, it seems that such approach is significant in order to relieve from the measuring of all state signals.

Now, the same characteristics as mentioned above have been used to design the simple adaptive control (SAC) system [11] that is one of model following type direct adaptive control methods based on command generator tracker approach. The SAC system can be constructed with the output feedback controller that can guarantee its stability. In this paper, from the point aim at the identity of design conditions for control system, we consider a design of hyper plane using the parallel ladder network of dynamic compensators proposed so as to expand the applicable system class of SAC method [12]. Here, in order to make clear the point of discussion, we suppose the controlled plant as SISO non-minimum phase system with relative degree larger than 1. In this case, the parallel ladder network can be designed by use of the information of approximate leading coefficient and relative degree of controlled plant transfer function. As a result, the augmented plant directly generates the SMC switch-

ing function as its output signal. Then, the robust SMC can be easily designed using only the first derivative signal of plant output signal.

Furthermore, the control performance of resulting SMC system is examined through its application to Electric-mechanical system example. And we confirm its sufficiently high performance of SMC system designed by proposed method through the comparison with the conventional design method. And the experimental results show the robust control performance for un-modeled dynamics of actuator and that the proposed method makes easy to try again designing SMC system.

2. PROBLEM FORMULATION

To make clear the point at issue, let us consider the single-input n -th order linear time-invariant nonminimum phase system, given by the following transfer function, as controlled plant. And, suppose that the relative degree of controlled plant is larger than 1.

$$G_p(s) = \frac{N(s)}{D(s)} \tag{1}$$

$$N(s) = h_{n-m}s^m + h_{n-m+1}s^{m-1} + \dots + h_{n-1}s + h_n \tag{2}$$

$$D(s) = s^n + p_1s^{n-1} + \dots + p_{n-1}s + p_n \tag{3}$$

where $h_{n-m} > 0, m \geq 2$

Denoting the output signal as $y(t)$, the input signal as $u(t)$ and the state variable as $x(t)$, the state space model of controlled plant(1) can be given as

$$\dot{x}(t) = Ax(t) + bu(t) \tag{4}$$

$$y(t) = c^T x(t). \tag{5}$$

where

$$x_i = \dot{x}_{i-1}(t) - b_{i-1}u(t) \quad (i = 2, \dots, n) \tag{6}$$

$$b_j = \begin{cases} 0 & (j=1, \dots, n-m-1) \\ h_j - \sum_{k=1}^j P_k b_{j-k} & (j=n-m, \dots, n) \end{cases} \tag{7}$$

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 \\ -p_n & -p_{n-1} & \cdots & -p_1 \end{bmatrix} \quad (8)$$

$$b = [0 \ \cdots \ 0 \ b_{n-m} \ \cdots \ b_n]^T \quad (9)$$

$$c^T = [1, 0, \dots, 0] \quad (10)$$

Now, we assume that the state variables can not be detected perfectly and the strict values of plant parameters A, b are un-known. Then, we continue the discussion on the simple design method for SMC system for the above-mentioned controlled plant.

The control objective of basic SMC is a stabilization of the system (4), that is, the realization of the following relationship:

$$\lim_{t \rightarrow \infty} x(t) = 0 \quad (11)$$

by use of the sliding mode phenomena caused by switching action of control input. The first step is a design of switching function which is characterise a hyper plane on which the sliding mode generates. By a conventional design method [1]-[3], the switching function $\sigma(t)$ is given as

$$\sigma(t) = s^T x(t) \quad (12)$$

,if the state variable $x(t)$ is detectable. And, s^T is determined from (A, b, c) . However, these conditions do not hold as mentioned above. So, we can not design the switching function $\sigma(t)$ in the form of (12).

Our goal is to design a SMC without use of the state signal which drives the system trajectory onto a prespecified switching surface maintains the trajectory on this surface and forces the state variable to go asymptotically to zero in spite of the presence of uncertainties.

3. DESIGN METHOD

3.1. Design of the switching function using parallel compensators

The controlled plant, given by the transfer function (1) / the state space model (4)(5), is assumed to satisfy the following conditions.

[Assumption 1]

1. The upper bound of the relative degree of $G_p(s)$ $\gamma^* \geq n - m \geq 2$ is known.
2. All zeros of $G_p(s)$ are stable.
3. Positive constants δ_1 and δ_2 such as $h_{n-m} \geq \delta_1, h_n/p_n \geq \delta_2$ are known.
4. Output signal $y(t)$ and its time derivative signal $\dot{y}(t)$ are detectable

Then, we now introduce a parallel ladder network of dynamic compensators for the control plant shown in Fig.1. And let the transfer function of parallel ladder network $F(s)$ be defined by:

$$F(s) = \sum_{i=1}^{\gamma^*-1} F_i(s) \quad (13)$$

$$F_i(s) = \frac{f_i}{D_i(s)}, \quad i = 1, 2, \dots, \gamma^* - 1 \quad (14)$$

$$\text{deg}D_i(s) = \gamma^* - i \quad (15)$$

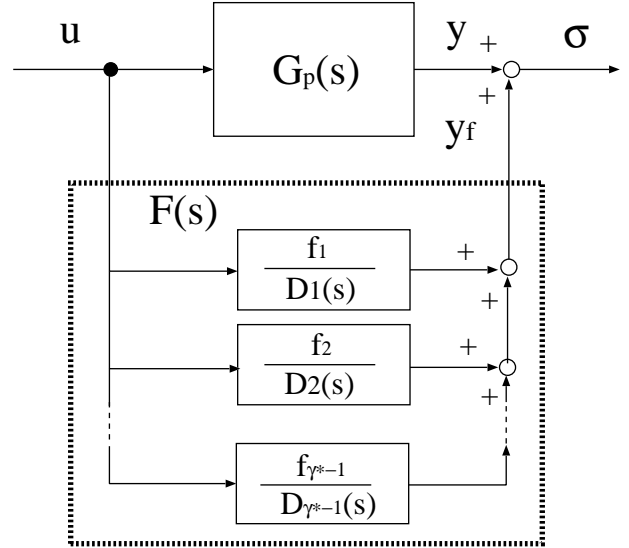


Fig.1 Augmented plant with PFC

$$\delta_1 \gg f_1 \gg f_2 \gg \cdots \gg f_{\gamma^*-1} > 0 \quad (16)$$

$$\delta_2 \gg |F(j0)| \quad (17)$$

$D_i(s)$: monic stable polynomial

$F(s)$ has been proposed as PFC(PFC:Parallel Feedforward Compensator) in order to apply a simple adaptive control method to the controlled plant satisfying the assumption 1 [12].

Suppose that the state space model of PFC is represented by

$$\dot{x}_f(t) = A_f x_f(t) + b_f u(t) \quad (18)$$

$$y_f(t) = c_f^T x_f(t), \quad (19)$$

the state space model of an augmented plant $G_a(s) = G_p(s) + F(s)$ is described as follows.

$$\dot{x}_a(t) = A_a x_a(t) + b_a u(t) \quad (20)$$

$$\sigma(t) = y(t) + y_f(t) = s^T x_a(t) \quad (21)$$

$$A_a = \begin{bmatrix} A & 0 \\ 0 & A_f \end{bmatrix}, b_a = \begin{bmatrix} b \\ b_f \end{bmatrix}, s = \begin{bmatrix} c \\ c_f \end{bmatrix} \quad (22)$$

Under the assumption 1, the following lemma holds.

[lemma 3.1] s satisfies the following design condition to insure that the sliding mode exists and is reachable. Namely, for $\sigma(t)$ given in (21), $\sigma(t) = 0$ gives the sliding switching hyper plane.

$$(C1) \ s^T b_a > 0$$

$$(C2) \ \text{All zeros of } G_a(s) \text{ are stable.}$$

(Proof) According to a reference [12], the PFC parameter selection based on (16),(17) makes the augmented plant satisfying a sufficient condition for ASPR-ness derived by Zeheb [13] such that

$$(1) \ \text{The relative degree is 1.}$$

$$(2) \ \text{The leading coefficient of numerator of transfer function is positive. Namely,}$$

$$s^T b_a = f_{\gamma^*-1} > 0. \quad (23)$$

$$(3) \ \text{All zeros of transfer function are stable.}$$

Comparing the above Zeheb's conditions and the switching function parameter desing condition, the conditions (1) and (2) are equivalent to the condition (C1). And the condition (3) is also same as (C2).

3.2. Design of SMC

Based on the lemma 3.1, the following theorem can be used to construct the control law.

[Theorem 3.1]

If the control is chosen as:

$$u(t) = -\frac{1}{f_{\gamma^*-1}} \left(\frac{dy(t)}{dt} + c_f^T A_f x_f(t) \right) - k(x_a, t) \frac{\sigma(t)}{|\sigma(t)|} \quad (24)$$

where $k(x_a, t) > 0$, then the reaching condition, $\sigma(t)\dot{\sigma}(t) < 0$, is satisfied and the global stability of the switching surface $\sigma(t) = 0$.

(Proof) From equations, (4),(5),(18),(19), (20) and (21), the following relationship holds:

$$\frac{dy(t)}{dt} + c_f^T A_f x_f(t) = s^T A_a x_a(t) \quad (25)$$

Using this relationship and (23), (24) can be rewritten in

$$u(t) = -(s^T b_a)^{-1} s^T A_a x_a(t) - k(x_a, t) \frac{\sigma(t)}{|\sigma(t)|}. \quad (26)$$

Let $V(\sigma)$ given as

$$V(\sigma) = \frac{1}{2} \sigma^2(t) \quad (27)$$

be a candidate of Lyapunov function, the following expression is derived for dV/dt .

$$\begin{aligned} \frac{dV(\sigma)}{dt} &= \sigma(t)\dot{\sigma}(t) \\ &= \sigma(t) \left\{ s^T A_a x_a(t) - s^T b_a (s^T b_a)^{-1} s^T A_a x_a(t) \right. \\ &\quad \left. - s^T b_a k(x_a, t) \sigma(t) / |\sigma(t)| \right\} \\ &= -s^T b_a k(x_a, t) \frac{\sigma(t)^2}{|\sigma(t)|} \\ &= -f_{\gamma^*-1} k(x_a, t) \frac{\sigma(t)^2}{|\sigma(t)|} \end{aligned} \quad (28)$$

With the fact that $k(x_a, t) > 0$, it is obvious that $dV(\sigma)/dt = \sigma(t)\dot{\sigma}(t) < 0$ for $\sigma(t) \neq 0$. Hence, we have

$$\lim_{t \rightarrow \infty} \sigma(t) = 0. \quad (29)$$

The time to reach sliding mode is determined by $-f_{\gamma^*-1} k(x_a, t)$. If the trajectory of augmented plant state reach to the sliding surface ($\sigma(t) = 0$ holds), the equivalent linear system can be described by the following differential equation by the substitution of (26) into (20).

$$\begin{aligned} \dot{x}_a(t) &= A_a x_a(t) - b_a (s^T b_a)^{-1} s^T A_a x_a(t) \\ &= \left\{ A_a - (s^T b_a)^{-1} b_a s^T A_a \right\} x_a(t) \\ &= \hat{A} x_a(t) \end{aligned} \quad (30)$$

According to Young et.al. [8], eigenvalues of \hat{A} are identical with the zeros of augmented plant $G_a(s)$ and origin. From lemma 3.1, since all zeros of augmented plant are stable, the state trajectory of augmented plant converge to origin from arbitrary initial point.

(Remark 1) The perfect information of state variable of controlled plant is unnecessary for the SMC input (24) construction. In a conventional SMC method[], under the assumption 1(4), a state observer is usually introduced in order to estimate the $n-2$ number

of state signal $x_3(t) \sim x_n(t)$. On the other hand, since the proposed method, which utilize $(\gamma^* - 1)$ -th order parallel compensator, accept to avoid in constructing such an observer, we can construct simple SMC system for high order plant with comparatively small relative degree. And, the proposed method require a few plant parameter information such as nominal values of low frequency gain and leading coefficient of numerator of controlled plant transfer function. Hence, we can see that the proposed design method is an easy method for uncertainties of plant parameters (A, b, c).

(Remark 2) As stated in the proof of theorem 3.1, the dynamics of the system in sliding mode along $\sigma(t) = s^T x_a(t) = 0$ is determined by the system zeros of the augmented system represented by the triple (A_a, b_a, s^T) . In the case of the use of PFC represented in (13) ~ (17), the augmented system zeros are located in neighborhood of the $(n-m)$ number of controlled system zeros and the roots of $D_i(s) = 0$. Hence, we know that choosing the roots of $D_i(s) = 0$ directly effects on the dynamics of the system in sliding mode.

(Remark 3) There exists some methods of concrete construction of PFC (13)~(17)[11]. For example, Iwai et.al. [12] proposed to design each dynamic compensator in the following formation.

$$D_{\gamma^*-j}(s) = (s+a)D_{\gamma^*-j+1}(s) \quad (31)$$

$$D_{\gamma^*}(s) = 1 \quad (32)$$

$$a > 0, j = 1, \dots, \gamma^* - 1$$

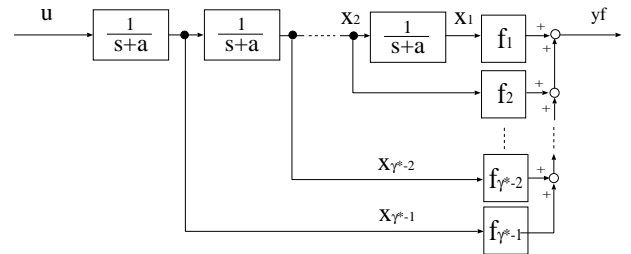


Fig.2 Realization of PFC

In this case, the structure of PFC is ladder network of first order lag compensators. The state variable and parameters of PFC shown in Fig.2 can be given as follows.

$$x^T(t) = \left[\dot{x}_{f1}(t) \quad \dot{x}_{f2}(t) \quad \dots \quad \dot{x}_{f\gamma^*-1}(t) \right] \quad (33)$$

$$A_f = \begin{bmatrix} -a & 1 & 0 & \dots & 0 \\ 0 & -a & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & & & \ddots & -a & 1 \\ 0 & \dots & \dots & 0 & -a \end{bmatrix} \quad (34)$$

$$b_f^T = \left[0 \quad \dots \quad 0 \quad 1 \right] \quad (35)$$

$$c_f^T = \left[f_1 \quad f_2 \quad \dots \quad f_{\gamma^*-1} \right] \quad (36)$$

Then, $c_f^T A_f x_f(t)$ used in (24) is given as

$$c_f^T A_f x_f(t) = \begin{bmatrix} -a \cdot f_1 \\ f_1 - a \cdot f_2 \\ \vdots \\ f_{\gamma^*-2} - a \cdot f_{\gamma^*-1} \end{bmatrix} x_f(t). \quad (37)$$

Fig.3 shows the SMC system which is constructed based on proposed method.

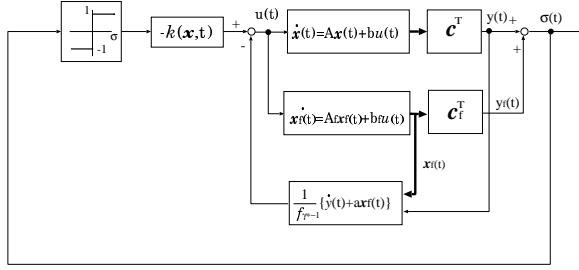


Fig.3 Proposed SMC System

3.3. Measure for chattering phenomenon

It is well known that the control law which satisfy the sliding condition are discontinuous across the surface $\sigma(t)$, thus leading to control chattering. Chattering is, in general, highly undesirable in practice, since it involves extremely high control activity, and may excite high-frequency dynamics neglected in the course of modelling. Slotine and Sastry [9] suggested a solution to this problem by smoothing out the control discontinuity in a thin boundary layer neighbouring the switching surface. Here, the control law (24) can be modified to have the form

$$u(t) = -\frac{1}{f_{\gamma-1}} \left(\frac{dy(t)}{dt} + c_f^T A_f x_f(t) \right) - k \frac{\sigma(t)}{|\sigma(t)| + \varepsilon} \quad (38)$$

with smoothing function[10], where ε denotes sufficiently small positive constant which is a smoothing parameter. And $k(x_a, t)$ is determined by positive constant $k > 0$.

4. EXAMPLE

4.1. Plant model

A helicopter model (Fig.4) will be used to be demonstrate the proposed design method. There exists an equilibrium point $p = 0$ for a steady thrust $F_{pc} = \frac{M_e g R_c}{R_p}$ [N] produced by rotor. Suppose that Δp denote a argument of pitch angle for equilibrium point and

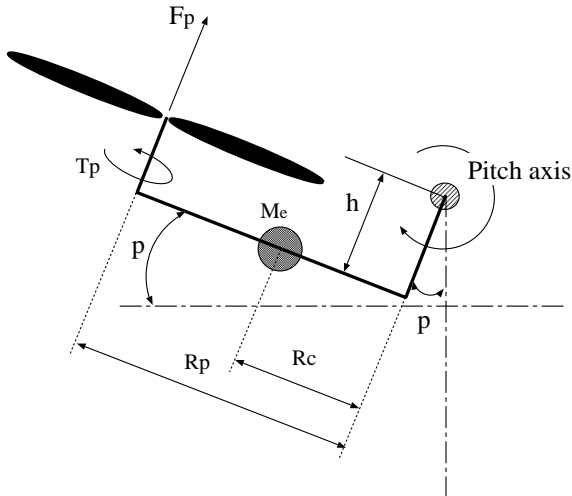


Fig.4 Schematic diagram of controlled plant

$\Delta F_p(t)$ denote variation of thrust, an equation of motion can be derived as

$$J_p \ddot{\Delta p}(t) + B_p \dot{\Delta p}(t) + M_e g h \Delta p(t) = R_p \Delta F_p(t). \quad (39)$$

Consider the different type models for the rotor dynamics described by follows.

[Rotor Model I]

Suppose that the thrust is proportional to a command voltage of DC motor, namely,

$$\Delta F_p(t) = K_{Mv} \Delta v(t). \quad (40)$$

[Rotor Model II]

Suppose that the thrust is proportional to an angular velocity of DC motor and the DC motor dynamics is represented by first order lag model where an input is the command voltage and an output is the angular velocity of DC motor, namely,

$$\Delta F_p(t) = K_{Mr} \Delta r(t) \quad (41)$$

$$\tau_M \dot{\Delta r}(t) = -\Delta r(t) + K_{rv} \Delta v(t) \quad (42)$$

where $K_{Mv} = K_{Mr} K_{rv}$.

Rotor model I can be approximated to Rotor Model II, if the time constant τ_M is negligible small. Then, we have the following two types models for controlled plant.

[Helicopter Model I]

State Space (SS) Model-I:

$$\dot{x}_p(t) = A_p x_p(t) + b_p u_p(t) \quad (43)$$

$$y_p(t) = c^T x_p(t) \quad (44)$$

$$A_p = \begin{bmatrix} 0 & 1 \\ -\frac{M_e g h}{J_p} & -\frac{B_p}{J_p} \end{bmatrix}, b_p = \begin{bmatrix} 0 \\ \frac{R_p}{J_p} \end{bmatrix}$$

$$c_p^T = \begin{bmatrix} 1 & 0 \end{bmatrix}, x_p^T(t) = \begin{bmatrix} \Delta p(t) & \dot{\Delta p}(t) \end{bmatrix}$$

$$u_p(t) = K_{Mv} \Delta v(t)$$

Transfer Function (TF) Model-I:

$$G_{p1}(s) = \frac{R_p K_{Mv}}{J_p s^2 + B_p s + M_e g h} \quad (45)$$

[Helicopter Model II]

SS Model-II:

$$\begin{bmatrix} \dot{x}_p(t) \\ \dot{u}_p(t) \end{bmatrix} = \begin{bmatrix} A_p & b_p \\ 0 & -\frac{1}{\tau_M} \end{bmatrix} \begin{bmatrix} x_p(t) \\ u_p(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_{Mr} K_{rv}}{\tau_M} \end{bmatrix} \Delta v(t) \quad (46)$$

$$y_p(t) = \begin{bmatrix} c^T & 0 \end{bmatrix} \begin{bmatrix} x_p(t) \\ u_p(t) \end{bmatrix} \quad (47)$$

TF Model-II:

$$G_{p1}(s) = \frac{R_p K_{Mv}}{(\tau_M s + 1)(J_p s^2 + B_p s + M_e g h)} \quad (48)$$

The plant parameters are shown as follows.

$$A_p = \begin{bmatrix} 0 & 1 \\ -4.155 & -0.488 \end{bmatrix}, b_p = \begin{bmatrix} 0 \\ 4.115 \end{bmatrix}$$

$$\frac{1}{\tau_M} = 3.70, \frac{K_{Mv}}{\tau_M} = 5.97$$

4.2. Design of SMC

Based on the Helicopter Model I, since the relative degree of controlled plant is 2, PFC can be determined by a first order lag system. Taking into account of the fact that $\delta_1 \approx 6$ and $\delta_2 \approx 1.5$, the following compensator is chosen.

$$F_1(s) = \frac{1}{s+1} \quad (49)$$

Based on the Helicopter Model II, the parameters of PFC (14) is given as

$$D_1(s) = D_2(s) = \frac{1}{s+1}, \quad f_1 = 1, \quad f_2 = 0.8 \quad (50)$$

in according to Remark 3. Let the resulting PFC to be $F_2(s)$. The switching gain function are chosen as $k(x_a, t) = 0.2$. And the following simulation results are obtained for the initial conditions such as $x_p(0) = [0.1, 0]^T$ and $\Delta v(0) = 0$.

4.3. Results

Fig.5 shows the simulation result of SMC designed by use of the control law (24) and $F_1(s)$ as the PFC based on the Helicopter Model I (Case-1).

In the case-1, PFC is designed without mistaking the relative degree of plant. So, it is obtained sufficiently well control performance such that the switching function $\sigma(t)$ converge to zero and hold on the sliding mode after $t = 0.5(s)$ and the settling time of plant response is about 3.5(s) while there exists about 15% overshoot as shown in Fig.5 (a)~(b). We can note that the sliding mode is realized by the high frequency switching of input (Fig.5 (c)). So, in the case that the control law (26) is used instead of (24), the above chattering of input is removed where ε is chosen as 0.0001. (Fig.6)

Next, Fig.7 shows the simulation result of SMC designed by use of the control law (24) and $F_1(s)$ as the PFC based on the Helicopter Model II (Case-2). While the PFC is designed with mistaking the relative degree of plant, the switching function $\sigma(t)$ asymptotically converges to zero as shown in Fig.7 (b). This result suggest that the proposed SMC method with parallel ladder network compensator has robustness for uncertainty of plant model structure. On the other hand, it should be paid attention that control performance get worse such that the oscillation occurs in the transient state response (Fig.7(a)). In the case that the PFC is designed as $F_2(s)$ with correct recognition with the relative degree of the Helicopter Model II (Case-3), we can note that the control performance gets better than Case-II as shown in Fig.8. While there exists small oscillation in the transient state response, its magnitude is reduced rather than Case-2. And the settling time of plant output is recovered as same as Case-1, namely, it is about 3.5 (s).

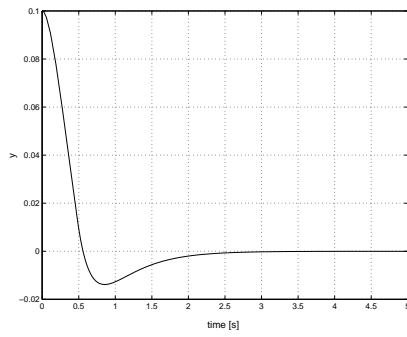
5. CONCLUSION

The new sliding mode control system design method is proposed, which is combining with the parallel ladder network of dynamic compensator guaranteeing the almost strict positive realness (ASPR-ness). This method is based on the equivalence of the sliding mode surface existing condition to the ASPR condition. Since the resulting SMC law consists of feedback of the plant output, the first time derivative of it and the state signal of parallel ladder network type compensator so called PFC (Parallel Feed-forward Compensator), we obtain some advantages such as the simple structure of SMC system, the disuse of state observer for high order plant

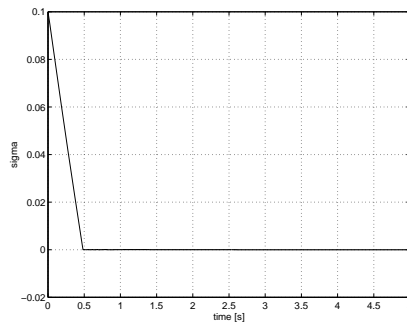
with relative degree which is larger than 2, the flexibility of PFC parameter tuning. And, in this study, the robust control performance and the above advantages are confirmed through some numerical simulations. The extension of the proposed method to multi-input multi-output system theory and to the model following and/or servo control theory will be shown in near future.

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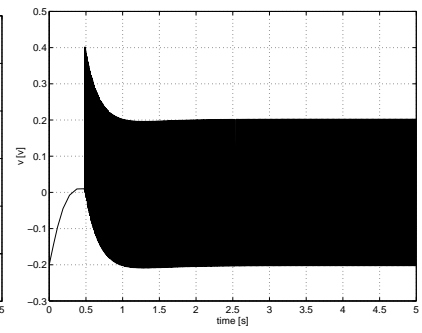
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(a) Response of plant $y_p(t)$

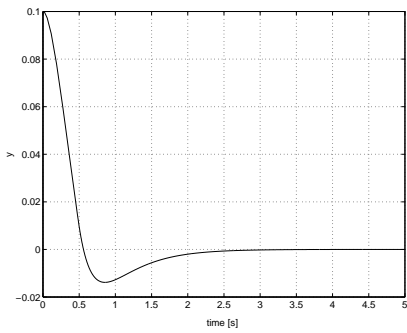


(b) Response of $\sigma(t)$

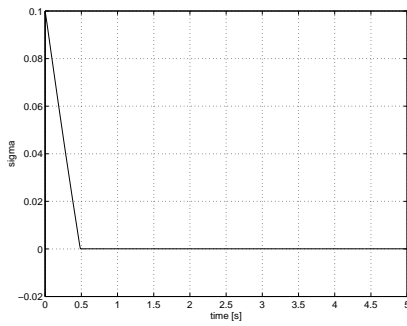


(c) Control input $\Delta v(t)$ [v]

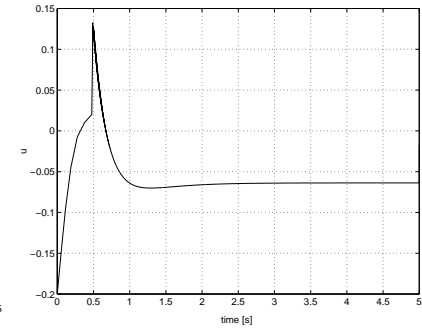
Fig.5 Simulation result (Case-1)



(a) Response of plant $y_p(t)$

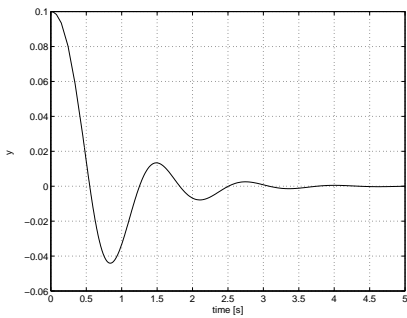


(b) Response of $\sigma(t)$

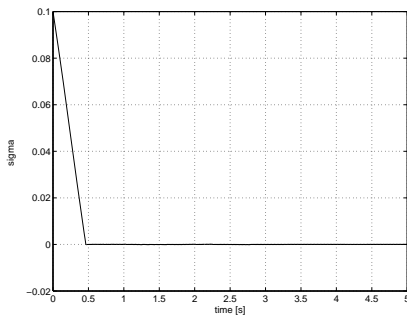


(c) Control input $\Delta v(t)$ [v]

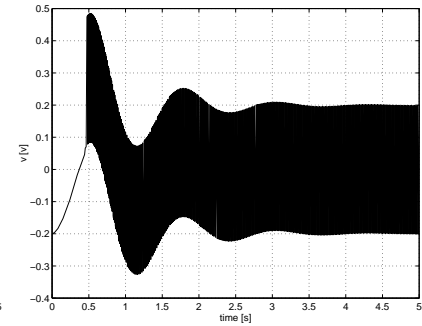
Fig.6 Simulation result (Case-1 using controller (38))



(a) Response of plant $y_p(t)$

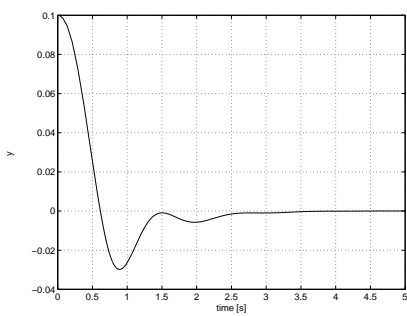


(b) Response of $\sigma(t)$

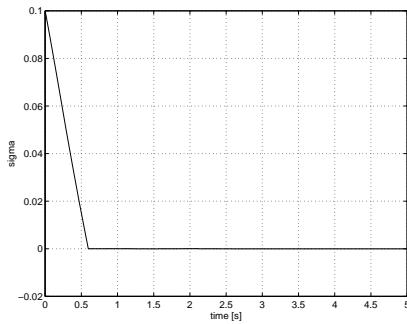


(c) Control input $\Delta v(t)$ [v]

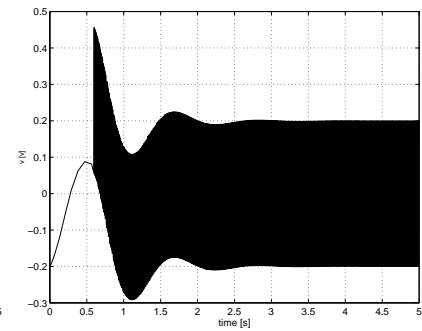
Fig.7 Simulation result (Case-2)



(a) Response of plant $y_p(t)$



(b) Response of $\sigma(t)$



(c) Control input $\Delta v(t)$ [v]

Fig.8 Simulation result (Case-3)