Application of Derivative State Constrained Optimal $\mathcal{H}_2$ Controller for Disk Drive Read System

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Abstract This paper presents the design technique for controlling the oscillation in the Disk Drive Read System via Derivative State Constrained (DSC)-Optimal H_2 Controller. The Optimal H_2, DSC-Optimal H_2 and Incorporating of Stability Degree Specification DSC Optimal H_2 are discussed. The results among these schemes are compared to verify the merit of DSC that effectively suppresses the oscillation in oscillatory system. The suggestions of how to select the weights of optimal controls are given.

Keywords Disk Drive Read System Controls, DSC-Optimal H_2 Controller, Degree of Stability

1. INTRODUCTION

Consider the basic diagram of a disk drive shown in Fig. 1.

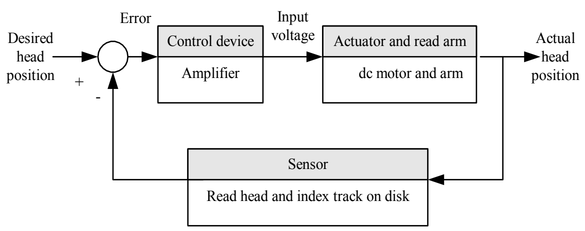

Fig. 1 Closed-loop control system for disk drive

The goal of the reader device is to position the reader head in order to read the data stored on a track on the disk. The variable to accurately control is the position of the reader head mounted on a slider device. The disk rotates at a speed of between 1800 and 7200 rpm, and the head flies above the disk at a distance of less than 100 nm [1].

2. DISK DRIVE READ SYSTEM MODEL

A model for disk drive read system can be considered as a two-mass system with a spring flexure shown in Fig. 2.

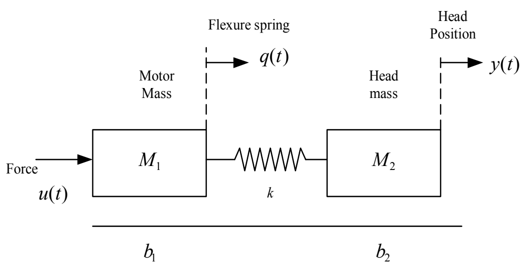

Fig.2. Model of the two-mass system with a spring flexure

Typical parameters for the two mass system are given in Table 1. To develop the state variable model, we choose the state variables as $x_1 = q$ and $x_2 = y$. Then, in matrix form

Table 1 Typical Parameter of the Two-Mass Model

Parameter	Symbol	Value
Motor mass	M_1	$20\text{ g} = 0.02\text{ kg}$
Flexure spring	k	$10 \le k \le \infty$
Head mounting	M_2	$0.5\text{ g} = 0.0005\text{ kg}$
Head position	$x_2(t)$	variable in mm
Friction at mass 1	b_1	410×10^{-3} kg/m/s
Field resistance	R	$1\ \Omega$
Field inductance	L	1 mH
Motor constant	K_m	$125\text{ N}\cdot\text{m/A}$
Friction at mass 2	b_2	4.1×10^{-3} kg/m/s

$$\frac{d}{dt}x(t) = Ax(t) + B_2u(t)$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k/M_1 & k/M_1 & -b/M_1 & 0 \\ -k/M_2 & -k/M_2 & 0 & -b/M_2 \end{bmatrix}, \quad x = \begin{bmatrix} q \\ y \\ \dot{q} \\ \dot{y} \end{bmatrix}, \text{ and } B_2 = \begin{bmatrix} 0 \\ 0 \\ 1/M_1 \\ 0 \end{bmatrix}.$$

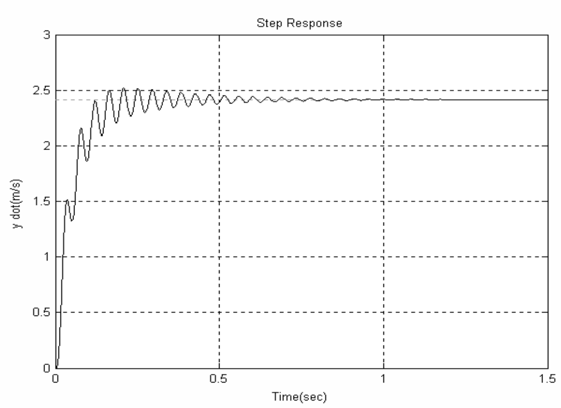

Fig. 3 Response of $\dot{y}$ for a step input for the two-mass model with $k = 10$.

3. STATE-SPACE SOLUTIONS TO H_2 FAMILY CONTROL PROBLEMS

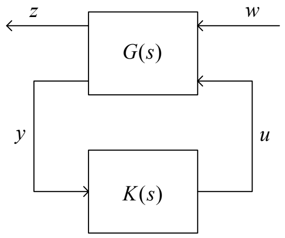

Fig. 4 The block diagram of Generalized Plant and Controller

$$\left.\begin{array}{rcl} \frac{d}{dt}x(t) &=& Ax(t) + B_1 w(t) + B_2 u(t) \\ z(t) &=& C_1 x(t) + D_{11} w(t) + D_{12} u(t) \\ y(t) &=& C_2 x(t) + D_{21} w(t) + D_{22} u(t) \end{array}\right\} \Leftrightarrow G(s) = P(s) \quad (1)$$

In Fig. 4, $G(s)$ is the generalized plant (the plant in a control problem plus all weighting functions). The signal w contains all external inputs, including disturbances, sensor noise, and commands. The output z is an error signal. y is the measured variables. u is the control input.

The generalized plant $G(s)$ is assumed to satisfy the following standard assumptions:

$$\text{A-1}_{\text{standard}}\Big) \begin{cases} (A, B_2) \text{ is stabilizable and} \\ (C_2, A) \text{ is detectable} \end{cases}$$

$$\text{A-2}_{\text{standard}}\Big) \begin{cases} D_{11} = D_{22} = 0, \\ D_{12} \text{ has full column rank with } D_{12}^T D_{12} = I, \text{ and} \\ D_{21} \text{ has full row rank with } D_{21}^T D_{21} = I \end{cases}$$

$$\text{A-3}_{\text{standard}}\Big) \begin{cases} \begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} \text{ has full column rank, and} \\ \begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} \text{ has full row rank, } \forall \omega \in \mathbb{R} \end{cases}$$

3.1 Standard H_2 Solution for Regulator Problem

The solution to standard Optimal H_2 Controller can be expressed as

$$K_2(s) = \left[\begin{array}{c|c} A + B_2 F_2 + L_2 C_2 & -L_2 \\ \hline F_2 & 0 \end{array}\right], \quad (2)$$

where

$$F_2 = -(B_2^T X_2 + D_{12}^T C_1),$$

$$X_2 = Ric(\mathbf{H}_2) \geq 0,$$

$$\mathbf{H}_2 = \begin{bmatrix} A - B_2 D_{12}^T C_1 & -B_2 B_2^T \\ -C_1^T C_1 + C_1^T D_{12} D_{12}^T C_1 & -(A - B_2 D_{12}^T C_1)^T \end{bmatrix},$$

$$L_2 = -(Y_2 C_2^T + B_1 D_{21}^T),$$

$$Y_2 = Ric(\mathbf{J}_2) \geq 0,$$

$$\mathbf{J}_2 = \begin{bmatrix} (A - B_1 D_{21}^T C_2)^T & -C_2^T C_2 \\ -B_1 B_1^T + B_1 D_{21}^T D_{21} B_1^T & -(A - B_1 D_{21}^T C_2) \end{bmatrix}.$$

3.2 H_2 Family Solutions for Integral Servo Problem

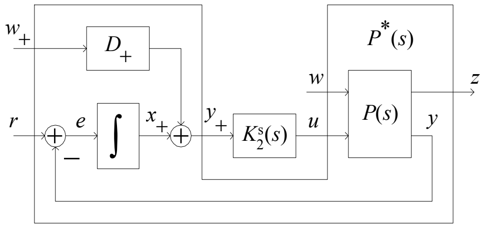

Fig. 5 Augmented System

The structure for H_2 family integral servo control problems is shown in Fig. 5. By the same manner as the obtaining controller $K_2(s)$ in Eqn. (2), the solutions for H_2 family integral servo control problems are obtained as follows:

Standard Optimal H_2 Integral Servo

$$K_2^s(s) = \left[\begin{array}{c|c} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} F_2^s + L_2^s \begin{bmatrix} 0 & I \end{bmatrix} & -L_2^s \\ \hline F_2^s & 0 \end{array}\right], \quad (3)$$

$$F_2^s = -\begin{bmatrix} B_2^T & 0 \end{bmatrix} X_2^s, \quad L_2^s = -Y_2^s \begin{bmatrix} 0 \\ I \end{bmatrix},$$

Derivative State Constrained Optimal H_2 Integral Servo

$$K_{2\dot{x}}^s(s) = \left[\begin{array}{c|c} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} F_{2\dot{x}}^s + L_{2\dot{x}}^s \begin{bmatrix} 0 & I \end{bmatrix} & -L_{2\dot{x}}^s \\ \hline F_{2\dot{x}}^s & 0 \end{array}\right], \quad (4)$$

Derivative State Constrained Optimal H_2 Integral Servo Incorporated with Stability Degree Specification

$$K_{2\dot{x}\alpha}^s(s) = \left[\begin{array}{c|c} \begin{bmatrix} A & 0 \\ -C_2 & 0 \end{bmatrix} + \begin{bmatrix} B_2 \\ 0 \end{bmatrix} F_{2\dot{x}\alpha}^s + L_{2\dot{x}\alpha}^s \begin{bmatrix} 0 & I \end{bmatrix} & -L_{2\dot{x}\alpha}^s \\ \hline F_{2\dot{x}\alpha}^s & 0 \end{array}\right]. \quad (5)$$

4. APPLICATION OF DERIVATIVE STATE CONSTRAINED OPTIMAL $\mathcal{H}_2$ CONTROLLER FOR DISK DRIVE READ SYSTEM

The augmented disk drive read system in section 2 used for finding the Derivative State Constrained Optimal H_2 Integral Servo Incorporated with Stability Degree Specification Controller can be written as follows:

$$\frac{d}{dt}\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \\ \hline \dot{x}_+(t) \end{bmatrix} = \left[\left[\begin{array}{cccc|c} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -k/M_1 & k/M_1 & -b_1/M_1 & 0 & 0 \\ k/M_2 & -k/M_2 & 0 & -b_2/M_2 & 0 \\ \hline 0 & 0 & 0 & -1 & 0 \end{array}\right] + \alpha I\right] \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \\ \hline \dot{x}_+(t) \end{bmatrix}$$

$$
+\left[\begin{array}{ccc:c:c:c}
\begin{bmatrix} \bar{B}_{1,11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & 0 & 0 & \left[\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k/M_1 & k/M_1 & -b/M_1 & 0 \\ k/M_2 & -k/M_2 & 0 & -b_2/M_2 \end{bmatrix}+\alpha I\right]\begin{bmatrix} \bar{D}_{11,11} & & & \\ & \bar{D}_{11,22} & & \\ & & \bar{D}_{11,33} & \\ & & & \bar{D}_{11,44} \end{bmatrix} & 0 & 0 \\ \hdashline
0 & 0 & 1 & -\left[\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}+\alpha I\right]\begin{bmatrix} \bar{D}_{11,11} & & & \\ & \bar{D}_{11,22} & & \\ & & \bar{D}_{11,33} & \\ & & & \bar{D}_{11,44} \end{bmatrix} & 0 & 0
\end{array}\right]\dot{w}(t)+\begin{bmatrix} 0 \\ 0 \\ 1/M_1 \\ 0 \\ 0 \end{bmatrix}\dot{u}(t)
$$

$$
\dot{z}^*(t)=\left[\begin{array}{c:c}
\begin{bmatrix} \bar{C}_{1,11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & 0 \\
0 & 1 \\ \hdashline
\begin{bmatrix} \bar{D}_{11,11} & & & \\ & \bar{D}_{11,22} & & \\ & & \bar{D}_{11,33} & \\ & & & \bar{D}_{11,44} \end{bmatrix}\left[\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k/M_1 & k/M_1 & -b_1/M_1 & 0 \\ k/M_2 & -k/M_2 & 0 & -b_2/M_2 \end{bmatrix}+\alpha I\right] & 0 \\
-\bar{D}_{11+}\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^2 & 0 \\ \hdashline
0 & 0
\end{array}\right]\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \\ \hdashline \dot{x}_+(t) \end{bmatrix}+\left[\begin{array}{c}
0 \\ 0 \\ \hdashline
\begin{bmatrix} \bar{D}_{11,11} & & & \\ & \bar{D}_{11,22} & & \\ & & \bar{D}_{11,33} & \\ & & & \bar{D}_{11,44} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1/M_1 \\ 0 \end{bmatrix} \\
0 \\ \hdashline
1
\end{array}\right]\dot{u}(t),
$$

$$
\dot{y}_+(t)=\left[\begin{array}{cccc:c} 0 & 0 & 0 & 0 & 1 \end{array}\right]\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \\ \dot{x}_+(t) \end{bmatrix}+\left[\begin{array}{cccc:cc:c} 0 & 0 & 0 & 0 & 0 & \bar{D}_{11+} & 1 \end{array}\right]\dot{w}(t)
$$

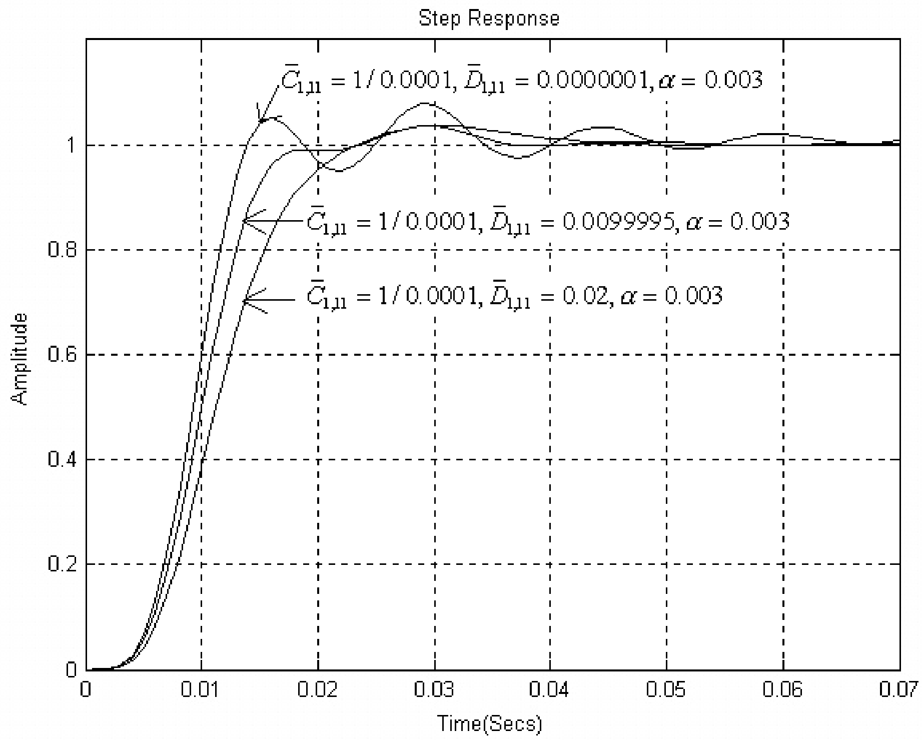

Fig. 6 Actions of Derivative State Constraints

The step responses of $y(t)$; the head position of disk drive read system in Fig. 6 show off the oscillation can effectively be suppressed by using the derivative state constraining (DSC) weights. However, the heavy DSC weights by novice design engineers may caused of long rises time. In order to hold and/or recovers the rise time, the Stability Degree Specification α is then incorporated.

5. CONCLUSIONS

The derivative state constrained optimal H_2 integral servo incorporated with stability degree specification for disk drive read system has been proposed. Of cause, totally of three schemes are included. It is obviously seen that the oscillations in oscillatory systems can effectively be suppressed by using the DSC weights. The determination of design parameters and robustness properties are remained for the future study.

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