Vibration Suppression Control for a Twin-Drive Geared Mechanical System with Backlash: Effects of Model-Based Control

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Abstract: This paper deals with a control technique of eliminating the transient vibration of a twin-drive geared mechanical system. This technique is based on a model-based control in order to establish the damping effect at the driven machine part. The control model is composed of reduced-order electrical and mechanical parts. This control model estimates a load speed converted to the motor shaft. The difference between the estimated load speed and the motor speed is calculated dynamically and it is added to the velocity command to suppress the transient vibration generated at the load. This control technique is applied to a twin-drive geared system with backlash. In the previous work, the performance of this control method is examined by simulations. In this paper, the effectiveness of this control technique is verified by experiments. The settling time of the residual vibration generated at the loading inertia can be shortened down to about 1/2 of the uncompensated vibration level.

Keywords: Twin-Drive Geared System, Transient Vibration, Backlash, Vibration Control, Model-Based Control, Reduced-Order Model, Delay Element, Damping.

1. INTRODUCTION

In the field of industrial machines, such as cut-off machines and transfer machines, selected motors to meet the specifications often have the larger volume and the heavier weight. In order to cut down the integration space and to meet the required output to drive the driven machine part, plural motors of the same output and the smaller size are often integrated into the power transmission system to compose the output torque of each motor by using gear trains.

However, owing to the existence of the backlash in the gear train, the distributed torque for each motor is not always equal, and several kinds of vibration phenomena are induced in these servo systems. For instance, transient vibrations related to the backlash and the mechanical eigenvalues, the interference of the output torque of each motor, the stabilization problem of the system and so on will be taken up. The occurrence of the transient vibration causes a problem such that the tact time of the system may be lengthened and the positioning accuracy of the end-effector may be deteriorated.

As a typical example of these mechanical systems, a twin-drive geared system is taken up in this paper and an easily realizable control technique is proposed.

For the single-drive geared system with backlash, several kinds of control methods are proposed and their effectiveness is investigated in many papers. For instance, the full-closed loop control using sensors at the end-effectors [1], a torque compensation control using the PD feedback loop with a disturbance observer [2] and a speed control method using the gear torque observer and a feedback gain [3] are well known.

However, for the twin-drive geared mechanical system with backlash, few control methods are proposed, and most of the proposed techniques have not dealt with the driven mechanical part and the interaction between the vibration mode and the backlash.

This paper deals with a control technique to suppress the transient vibration mainly related to the eigenvalues of the mechanical part in the lower-frequency range and the backlash. This technique is based on a model-based control [4], [5]. Referring to the model-based control, the control model as a dynamical compensator is related to the velocity control loop, and it is composed of reduced-order electrical and mechanical parts. The control model for the mechanical part, namely the geared mechanical system with backlash, is composed of the linear reduced-order model without backlash and the delay element which is related to the backlash and acts when changing of the motor's rotational acceleration. This control model calculates the rotational speed of the driven mechanical part, which is converted to the motor shaft. The difference between the estimated rotational speed of the driven machine part and the motor speed is calculated dynamically, and it is added to the velocity command to suppress the transient vibration after being multiplied by a gain. The function of this technique is to establish a damping effect at the driven mechanical part.

This control technique is applied to the twin-drive geared mechanical system composed of two servo motors, spur gears (two pinions and a bull gear) and a loading inertia connected to the bull gear through a torsion-bar.

In [6], the performance of the proposed control technique is examined by simulations on time responses. The time responses show satisfactory control results in reducing the transient vibration of the geared mechanical system with backlash. The settling time of the transient vibration related to the first vibration mode can be shortened down to about 1/2 of the uncompensated vibration level.

In this paper, the effectiveness of the model-based control is verified by experiments. The settling time of the residual vibration generated at the loading inertia can be shortened down to about 1/2 of the uncompensated vibration level.

2. TWIN-DRIVE GEARED MECHANICAL SYSTEM WITH BACKLASH

2.1 Schema of a twin-drive geared system

Fig.1 shows a schematic diagram of a twin-drive geared mechanical system. Two pinion gears which are driven by motors of same output are connected to the bull gear. A loading inertia is connected to a bull gear through a torsion-bar.

2.2 Modeling of backlash

The backlash between the pinion and the bull gears induces the transmission-delay of the motor torque, when the tooth separation occurs. Therefore, the backlash can be modeled into the delay element as shown in Fig.2.

2.3 Equations of motion

In general, as a pinion gear is rigidly connected to a motor shaft, this twin-drive geared mechanical system can be regarded as a 4-mass system. Further, this system is often controlled by the velocity control using the PI action.

With neglecting the damping coefficient at the meshing point of the gear stage, equations of motion of this geared system are written in Eq.(1), when the tooth separation between the pinion and the bull gears does not occur.

$$\begin{aligned}
&J_m\ddot{\theta}_{m1} + K_g R_{g1}(R_{g1}\theta_{m1} - R_{g2}\theta_g) = T_{m1},\\
&J_m\ddot{\theta}_{m2} + K_g R_{g1}(R_{g1}\theta_{m2} - R_{g2}\theta_g) = T_{m2},\\
&J_g\ddot{\theta}_g + K_g R_{g2}(R_{g2}\theta_g - R_{g1}\theta_{m1})\\
&\qquad + K_g R_{g2}(R_{g2}\theta_g - R_{g1}\theta_{m2})\\
&\qquad + C_s(\dot{\theta}_g - \dot{\theta}_l) + K_s(\theta_g - \theta_l) = 0,\\
&J_l\ddot{\theta}_l + C_s(\dot{\theta}_l - \dot{\theta}_g) + K_s(\theta_l - \theta_g) = 0.
\end{aligned} \tag{1}$$

On the other hand, when the tooth separation occurs at the meshing points, equations of motion can be written as

$$\begin{aligned}
&J_m\ddot{\theta}_{m1} = T_{m1},\\
&J_m\ddot{\theta}_{m2} = T_{m2},\\
&J_g\ddot{\theta}_g + C_s(\dot{\theta}_g - \dot{\theta}_l) + K_s(\theta_g - \theta_l) = 0,\\
&J_l\ddot{\theta}_l + C_s(\dot{\theta}_l - \dot{\theta}_g) + K_s(\theta_l - \theta_g) = 0,
\end{aligned} \tag{2}$$

where

k= 1, 2 for motor1 and motor2, respectively,
θ_{mk}= angular rotation of the motor,
θ_g = angular rotation of the bull gear,
θ_l= angular rotation of the driven machine part,
T_{mk}= output torque of the motor,
J_m= moment of inertia of the motor including the pinion,
J_g= moment of inertia of the bull gear,
J_l= moment of inertia of the driven machine part,
R_{g1}, R_{g2} = pitch radiuses of the pinion and the bull gears,
K_g= tooth stiffness of the gear pair,
K_s = torsional stiffness between the bull gear and the loading inertia,
C_s= damping factor between the bull gear and the loading inertia,
$2\delta_1$, $2\delta_2$= the amount of backlash.

Further, when the motor speed is controlled by the PI control, equations related to the motor armature are expressed as

$$L\frac{di_k}{dt} = K_c(K_v e_k + \frac{K_v}{T_i}\int e_k dt - K_{cb} i_k) - R i_k - K_e\omega_{mk}, \tag{3}$$
$$e_k = \omega_{cmd} - \omega_{mk}.$$

The output torque of the motor is expressed as

$$T_{mk} = K_{tk} i_k, \tag{4}$$

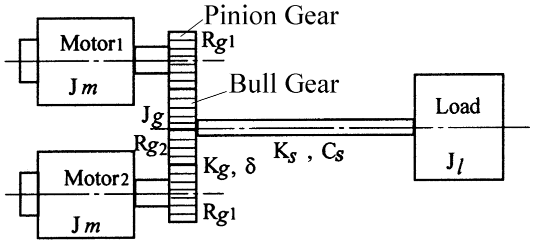


Fig. 1 Schematic diagram of a twin-drive geared system.

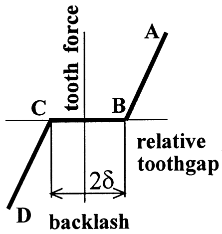


Fig. 2 Modeling of backlash.

where

ω_{cmd}= velocity command,
ω_{mk}= rotating speed of the motor,
ω_g= rotating speed of the bull gear,
ω_l= rotating speed of the driven machine part,
e_k = error,
i_k = current of the armature,
R = motor armature resistance,
L = motor armature inductance,
K_{tk}= torque constant,
K_e= voltage constant,
K_c= current loop gain,
K_{cb}= current feedback gain,
K_v= proportional gain of the PI control,
T_i= integral time constant of the PI control.

According to Eqs.(1)~(4), a block diagram of the twin-drive geared mechanical system can be expressed as Fig.3.

3. REDUCED-ORDER CONTROL MODEL

The control model of dynamical compensator is composed of linear reduced-order mechanical and electrical models related to the velocity control loop and the delay-element related to the backlash.

3.1 Linear reduced-order model

3.1.1 Modeling of mechanical part This paper deals with a case such that the residual vibration is mainly dominated by the first vibration mode and the higher order vibration modes are apart from the first one. The 4-mass system shown in Fig.3 is transformed into a 2-mass system shown in Fig.4 by considering only the first vibration mode. In this linear reduced-order model, the natural angular frequency ω_n and the damping ratio γ_n are expressed as

$$\begin{aligned}
&\omega_n = \sqrt{K_g^m(1/J_m^m + 1/J_l^m)},\\
&\gamma_n = \frac{C_g^m(1/J_m^m + 1/J_l^m)}{2\omega_n},
\end{aligned} \tag{5}$$

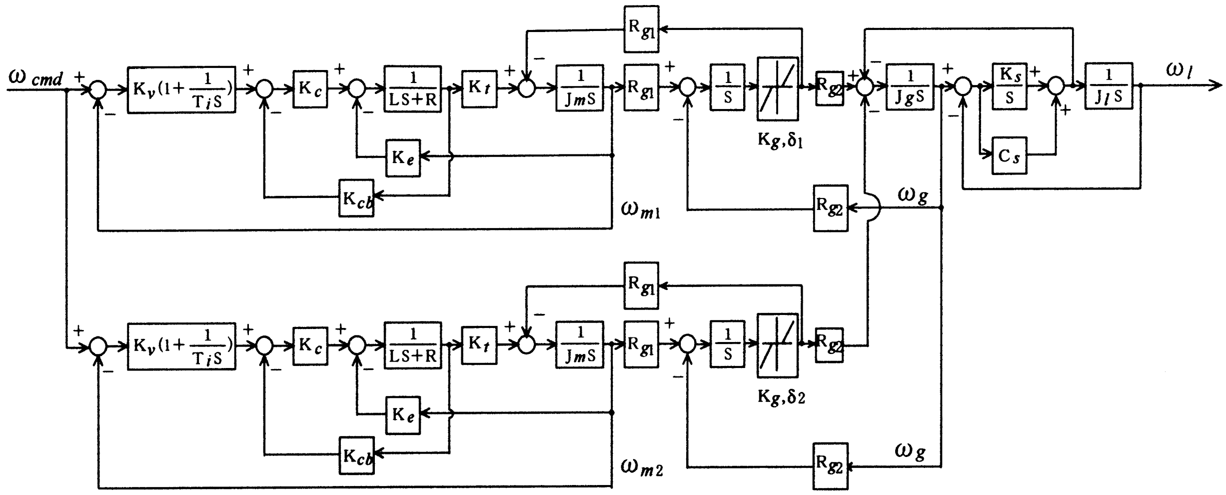


Fig. 3 Block diagram of the twin-drive geared system.

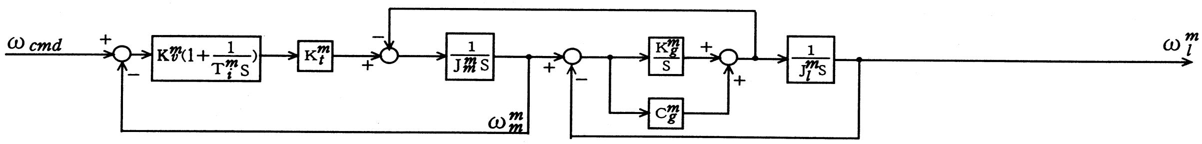


Fig. 4 Block diagram of the reduced-order system.

where

$$J_m^m = 2J_m + \left(\frac{R_{g1}}{R_{g2}}\right)^2 J_g, J_l^m = \left(\frac{R_{g1}}{R_{g2}}\right)^2 J_l,$$
$$K_s^m = \left(\frac{R_{g1}}{R_{g2}}\right)^2 K_s, C_s^m = \left(\frac{R_{g1}}{R_{g2}}\right)^2 C_s. \tag{6}$$

Here, the superscript "*m*" shows that parameters belong to the model. Defining the inertia ratio R_n as $R_n = J_l^m / J_m^m$ and transforming Eq.(5), J_l^m, K_s^m and C_s^m can be expressed as

$$J_l^m = R_n J_m^m,$$
$$K_s^m = \frac{R_n J_m^m}{1+R_n}\omega_n^2, \tag{7}$$
$$C_s^m = \frac{2R_n J_m^m}{1+R_n}\gamma_n \omega_n.$$

Using this expression, the reduced order model can be easily obtained from not only design data but also measured data.

3.1.2 Modeling of electrical part Next, a reduced order electrical model can be obtained. Here, the effect of the counter electromotive force is ignored. Further, considering that the angular cut-off frequency ω_c of the current control loop is much higher than the first natural angular frequency of the mechanical system and the current loop gain within ω_c is about 1.0, the current control system composed of the current control loop and the torque constant is expressed as a proportional gain K_t^m. As a result, a simplified PI control system is expressed as shown in Fig.4. Expressing the natural angular frequency ω_e and the damping ratio ζ_e of the electrical part as Eq.(8), the parameters of reduced-order model can be easily adjusted.

$$\omega_e = \sqrt{\frac{K_v^m K_t^m}{T_i^m J_m^m}},$$
$$\varsigma_e = \frac{1}{2}\sqrt{\frac{T_i^m K_v^m K_t^m}{J_m^m}}, \tag{8}$$

where $K_v^m = K_v$ and $T_i^m = T_i$.

3.2 Estimation of rotating speed of the loading inertia

With operating the delay element on the estimated linear rotational speed $\omega_l^m(t)$, the estimated rotating speed of the driven machine part $\omega_{nlk}^m(t)$ can be expressed as

$$\omega_{nlk}^m(t) = \omega_l^m(t - L_{dk}), \quad k = 1, 2, \tag{9}$$

where L_{dk} represents the dead time.

4. MODEL-BASED CONTROL SYSTEM

Fig.5 shows a block diagram of the model-based control system related to the velocity control loop. Using the relationship of Eqs.(5)~(8), the block diagram of the reduced-order model shown in Fig.4 is transformed into the dotted-line part in Fig.5.

In the compensating control system, the difference between the load speed ω_{nlk}^m which is estimated at the motor shaft and the motor speed ω_{mk} is dynamically calculated, and it is multiplied by the gain K_{bk}. Finally, $K_{bk}(\omega_{nlk}^m - \omega_{mk})$ where k=1, 2 is added to the velocity command ω_{cmd} as

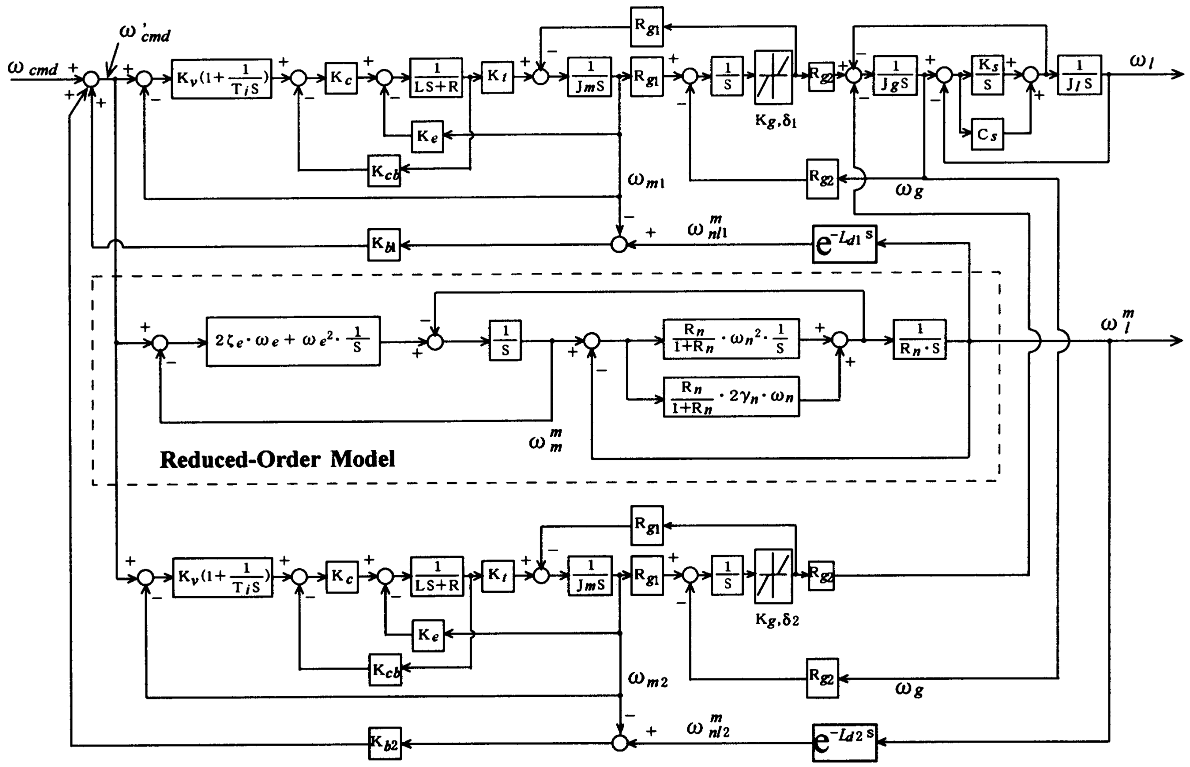


Fig. 5 Block diagram of model-based control for the twin-drive geared system.

$$
\begin{aligned}
\omega'_{cmd}(t) &= \omega_{cmd}(t) + K_{b1}\left(\omega^{m}_{nl1}(t) - \omega_{m1}(t)\right) \\
&\quad + K_{b2}\left(\omega^{m}_{nl2}(t) - \omega_{m2}(t)\right) \\
&= \omega_{cmd}(t) + K_{b1}\left(\omega^{m}_{l}(t - L_{d1}) - \omega_{m1}(t)\right) \\
&\quad + K_{b2}\left(\omega^{m}_{l}(t - L_{d2}) - \omega_{m2}(t)\right).
\end{aligned} \tag{10}
$$

5. SIMULATION ON SUPPRESSION OF TRANSIENT VIBRATION [6]

The step response is calculated in order to verify the suppression effects on the transient vibrations. Simulation conditions are shown in Table 1. The amount of backlash $2\delta_1$ and $2\delta_2$ are respectively set to 0.085mm and 0.23mm according to JIS B 1703. In simulations, the velocity command is changed from 0 to 1000min^{-1}, then from1000 to 0min^{-1} as a step input. The values of K_{b1}, K_{b2} are set to 0.3 and the values of L_{d1}, L_{d2} are set to 4.8ms according to the previous work [5] on the stability and the frequency respose.

Fig.6 shows simulation results. Fig.6 indicates that the proposed model-based control suppresses the residual vibration of the load. The settling time, namely the time interval between ∇ and ▼, is reduced down to about 1/2.

Further, Fig.6 indicates that the higher order vibration generated at the gear stage can be suppressed with reducing the transient vibration mainly related to the first vibration mode by using the proposed model-based control.

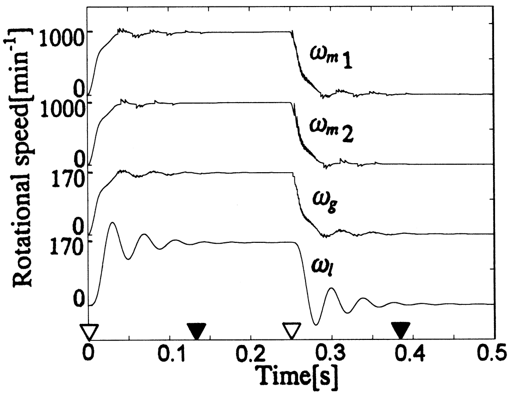
Table 1 Simulation and experimental conditions.

Parameter		Value	Unit
Moment of inertia	J_m	1.572×10^{-5}	kg·m^2
	J_g	1.054×10^{-3}	
	J_l	5.160×10^{-4}	
Tooth stiffness	K_g	$135.6\times 10^{+6}$	N/m
Torsional stiffness	K_s	12.173	N·m/rad
Damping coefficient	C_s	0.008	N·m·sec/rad
Gear reducer			
Reduction ratio	R_g	5.88	R_{g2}/R_{g1}=50/8.5
Module	m	1.0	
Velocity loop gain	K_v	0.015	A/(rad/sec)
Integral time constant	T_i	0.06	sec
Torque constant	K_t	0.316	N·m/A
Voltage constant	K_e	0.316	V/(rad/sec)
Phase resistance	R	4.5	Ω
Phase inductance	L	0.019	H
Current loop gain	K_c	118.84	V/A
Current feedback gain	K_{cb}	1.0	–
Feedback gain	$K_{b1,2}$	0 or 0.3	–
Reduced-order model			
Electrical part			
Natural frequency	ω_e	37.5	rad/sec
Damping ratio	ζ_e	2.555	–
Mechanical part			
Natural frequency	ω_n	168.2	rad/sec
Damping ratio	γ_n	0.058	–
Inertia ratio	R_n	0.241	–

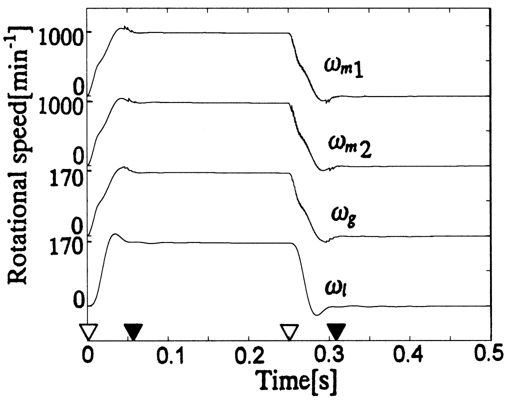
6. EXPERIMENTAL RESULTS AND CONSIDERATIONS

6.1 Experimental set-up

Fig.7 shows a schematic diagram of the experimental set-up. Two pinion gears which are driven by motors of same output are connected to the bull gear. A loading inertia is connected

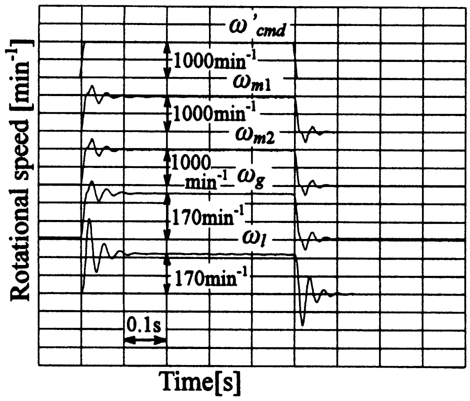


(a) Without model-based control.

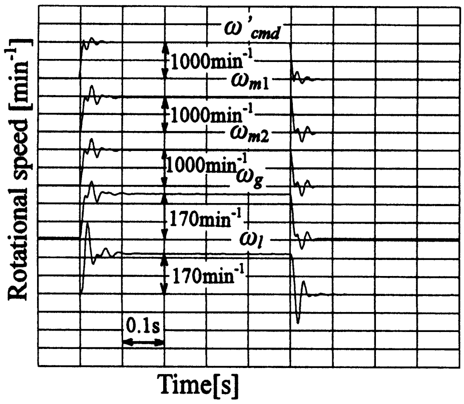


(b) With model-based control.

Fig. 6 Simulation results ($2\delta_1$=0.085, $2\delta_2$=0.23mm).



(a) Without model-based control.



(b) With model-based control.

Fig. 8 Experimental results ($2\delta_1$=$2\delta_2$=0.085mm).

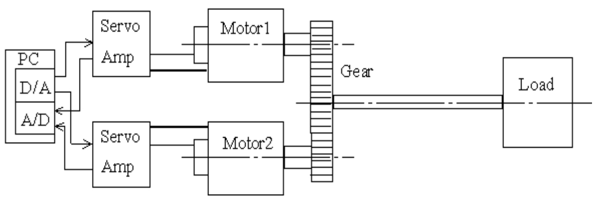


Fig. 7 Schematic diagram of the experimental set-up.

to a bull gear through a torsion-bar. The amount of backlash $2\delta_1$, $2\delta_2$ is set to 0.085mm according to JIS B 1703. Physical parameters of the experimental set-up are shown in Table 1. The linear natural frequency of the mechanical system is about 26Hz.

6.2 Construction of the control system

The control system is composed of servo amplifiers and a personal computer (PC) as a controller as shown in Fig.7. The velocity command is generated from the PC and it is outputted into the servo amplifiers by D/A converter. The velocities of the motor1 and the motor2 are inputted into the PC by A/D converter.

The algorithm of the model-based control is installed into the PC. The sampling period of the compensating action is set to 1.0ms. The value of K_{b1}, K_{b2} are set to 0.3 and the values of L_{d1} and L_{d2} are set to 5.0ms according to the simulation results of the step response.

6.3 Effects on residual vibration

In experiments, a trapezoidal velocity profile is assigned. The constant acceleration in the start phase is 1000min^{-1}/10ms. The cruise velocity is 1000min^{-1}. The constant deceleration in the arrival phase is -1000min^{-1}/10ms.

Fig.8 shows experimental results. Fig.8 indicates that the proposed model-based control suppresses the residual vibration of the loading inertia in the starting and the arrival phases. As a result, the settling time in the arrival phase is reduced down to about 1/2 (from 110ms to 50ms) of the uncompensated vibration level.

7. CONCLUSIONS

A model-based control technique is proposed to eliminate the transient vibration generated in the twin-drive geared mechanical system with backlash. The control model of dynamical compensator is composed of linear reduced-order mechanical and electrical models related to the velocity control loop and the delay-element related to the backlash. In referring to the construction, this model-based control loop can be easily integrated into the position control loop as an inner loop.

The effectiveness of this control method was verified by the experiments. The experiment on the time responses showed satisfactory control results in reducing the transient vibration of the loading inertia. As a result, the settling time in the arrival phase is reduced down to about 1/2 of the uncompensated vibration level.

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