Workload Allocation Methods in Discrete Manufacturing Systems:

Model and Optimization

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Abstract: Workload programming is allocating suitable workloads of production process according to the needs of products, which would minimize the total cost of both work and stock under some constraint conditions. In this paper, a production process flow chart of discrete manufacturing is presented by a Petri net, and the optimization model of workload-stock is established. An approach of the optimal workloads is provided by means of the integer matrix theory. An example is given to verify this method.

Keywords: manufacturing; discrete system; optimization; integer matrices

1 Introduction

In manufacturing systems, the production management should ensure the procurement of the expected amount of goods in a correct time and the maintenance of the proper stocks in the production process.

As emphasized in GRAL grids (Doumeingts et al, 1993), the function of the Decision System in an enterprise is to manage products, manage resources and plan tasks. In a discrete manufacturing system, the production management should make a workload programming, which could ensure the targeted amount of products and proper stock levels in the production process in order to minimizing the cost of both tasks and stocks.

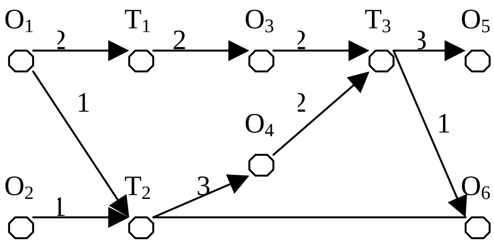
In this paper, the product/ process data of a discrete manufacturing system are given in a Petri net, in which the transitions refer to the manufacturing tasks and the places to the objects with itemized characters (Lecompte et al 2000). The graph is the model of the various manufacturing processes listed. The input objects are the elementary components, and the output objects are the finished products, whereas the intermediate objects are the semi-finished products. The transformations of these stocks are connected by task nodes in the graph. No restriction on the type of process (machining, assembly, disassembly, etc.) is made.

Based on the semantics of the Petri net, a linear model, which characterizes the relationships between tasks and stocks, is presented in section 2. An optimization equation under some system constraint conditions is set up in section 3. An approach, which identifies the workload to be delivered to reach the production targets and ensures proper inventory in every object stages, using the integer programming and integer matrix theory, is developed in section 4. An example is given to verify this method in section 5.

2 Modeling

A discrete manufacturing process can be described by a Petri net [O. T. C] with O (Card O=m) the set of places representing the product items (Components, semi-finished and final products), T (Card T=n) the set of transitions the transformations and C (m*n) the incidence matrix expressing the relationships between objects and transformations. An example of discrete manufacturing process with 6 objects and 3 transformations is shown on Figure 1.

The arrows express the constraints of precedence from a transformation to another, and are weighed by the numbers of objects produced or consumed by the transformations.



$O_u=\{O_1, O_2\}$; $O_x=\{O_3, O_4\}$; $O_y=\{O_5, O_6\}$

$$C=\begin{pmatrix} -2 & -1 & 0 \\ 0 & -1 & 0 \\ 2 & 0 & -2 \\ 0 & 3 & -2 \\ 0 & 0 & 3 \\ 0 & 2 & 1 \end{pmatrix}$$

Fig.1 Model of manufacturing processes

Note that set O can be partitioned in O_u, O_x and O_y, respectively the subset of elementary components, of semi-finished items and of final items.

It is defined the n-dimension workload vector W ($W\in N^n$) whose integer entries stipulate the numbers of transformations and the m-dimension stock level vectors S_0 and S (S_0, $S\in N^m$) whose integer entries refer to the stock levels of each product item before and after work W. In other words, work W pushes the stocks from S_0 into S. The relation between W, S_0 and S is described by the following linear equation (Bourrieres, 1998) so called direct production model:

$$\Delta S=S-S_0=CW$$
$$S_0, S\in N^m \qquad W\in N^n \qquad (1)$$

Where C is a n*m integer matrix.

3 Optimization equation

3.1 Analysis of equation

The vector S can be divided into three parts: the entries (dimension m_y) in O_y denoted by S_y, are targets determined by the product order book and orders from contractors; the entries (dimension m_x) in O_x denoted by S_x are semi-finished stock levels, which aren't predetermined; the entries (dimension m_u) in O_u denoted by S_u are elementary components, which are needed input from other departments.

The equation (1) can be divided into three integer matrix equations:

$$S_y=C_yW \qquad (2)$$
$$S_x=C_xW+C_{x0} \qquad (3)$$
$$S_u=C_uW \qquad (4)$$

The contents of workload allocating are deciding a proper product plan S_y, calculating the optimal W, and obtaining S_u.

3.2 Constraint conditions

Let now the constraints on workload and stock levels be considered. Given the limited capacity of the resources that will be involved in the achievement of the workload, there is a constraint condition for vector W:

$$0 \leq W \leq W_l \quad (5)$$

where W_l is the capacity vector of the manufacture system.

The stock levels (mainly S_x) are constrained too, so that S is to be kept between upper bound S_l and lower bound (0 in general). And consequently adjustable variation S_x verifies:

$$0 \leq S_x \leq S_l$$

or

$$0 \leq S_{x0}+C_xW \leq S_l$$

$$-S_{x0} \leq C_xW \leq S_l-S_{x0} \quad (6)$$

3.3 cost function

Suppose K_1 is a n-dimension vector whose ith entry is the work expense of ith workload W_i, so that the expense of manufacturing process is K_1W. Suppose K_2 is an m_x-dimension vector, whose ith entry is the stock expense of ith stock place S_x, so the total expense of both works and stocks is

$$J=K_1W+K_2S_x$$

$$=K_1W+K_2(C_xW+S_{x0})$$

$$=(K_1+K_2C_x)W+K_2S_{x0}$$

Because S_{x0} is a fixed vector, minimizing the total expense is equal to minimizing the cost function:

$$J=KW \quad (7)$$

where $K=K_1+K_2C_x$.

The purpose of the workload programming is finding proper tasks W*, which satisfies equation (2) and minimizes equation (7) under constrain conditions (5) and (6).

4 Integer matrix equations

The necessary and sufficient condition of integer matrix equation $S_y=C_yW$ solvable is that integer matrix $[C_y\ 0]$ equivalences $[C_y\ S_y]$. i.e. one matrix can become other by the elementary transformation. According to integer matrix theory, if C_y is a m_y*n integer matrix ($n>m_y$), there exists a unimodular integer matrix M that

$$C_yM=[C_0\ 0] \quad (8)$$

where M is an integer matrix, whose determinant is +1 or –1; the inverse of M is an integer matrix too. C_0 is a m_y*m_y integer matrix, that is the maximal left factor of integer matrix C_y; i.e. C_0 is a left factor of C_y, and any left factor of C_y is left factor of C_0. C_0 can be gotten from C_y through some integer elementary transformations. One of standard forms of C_0 is a lower triangular matrix. No lose generality, the rank of C_y can be regarded as m_y, so the rank of C_0 is also m_y. C_0 is a nonsingular square matrix.

Let matrix M_1 be the front m_y columns of matrix M, M_2 be the other columns of M, i.e. $M=(M_1\ M_2)$, and equation (8) can be written:

$$C_yM_1=C_0$$

$$C_yM_2=0 \quad (9)$$

The integer matrix equation $C_yW=S_y$ is solvable if C_0 is a left factor of S_y, or $C_0^{-1}S_y$ is an integer vector. C_0^{-1} is an m_y*m_y fraction matrix. If the least common denominator of the entries in ith column of C_0^{-1} is d_i. A sufficient condition for $C_0^{-1}S_y$ integer vector, evidently, is that the ith entry of S_y is a multiple of d_i. So that the expected outcome products can be adjusted to ensure that the equation (2) exist integer solutions. The adjusted S_y is called a feasible product plan.

If S_y is feasible, the integer solution set of the equation (9) is

$$W=M_1C_0^{-1}S_y+M_2Z \qquad (10)$$

where Z is an arbitrary $(n-m_y)$-dimension integer vector. So the cost function is

$$J=KW=K\,M_1C_0^{-1}S_y+KM_2Z \qquad (11)$$

Because $K\,M_1C_0^{-1}S_y$ is a fixed integer, the optimization problem becomes: finding an integer vector Z that minimizes KM_2Z under the constrain conditions that W satisfied the inequality (5) and (6).

Summed up, the optimization process approach includes 3 steps:

Step 1: In accordance with the product order book, giving a feasible expected final product amount, which could ensure the integer equation (2) solvable.

Step 2: Calculating the diophantine equation (8), whose solution set W is gotten.

Step 3: According to the constrain conditions (5) and (6), the optimal solution W* is obtained which minimizes the cost function J

5 An example

Suppose a manufacturing system is shown as Fig.1, where the expected products $S_y^{\wedge}=(S_5\ S_6)^T=(70\ 55)^T$; the prime semi-finished stocks $S_{x0}=(S_{30}\ S_{40})=(10\ 0)^T$; the unit work costs of $W=(W_1\ W_2\ W_3)$ are $K_1=(2, 3, 3)$; the unit stock costs of stores S_3 and S_4 are $K_2=(0.5\ 0.5)$. The constraint conditions are $0\leq W\leq(40\ 35\ 30)^T$ for works and $0\leq S_x\leq(50\ 50)$ for semi-finished stocks.

From Fig.1 there is

$$C_y=\begin{pmatrix}0&0&3\\0&2&1\end{pmatrix}$$

Transform Cy into the standard form by right-multiplying an unimodular integer matrix:

$$\begin{pmatrix}0&0&3\\0&2&1\end{pmatrix}\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}=\begin{pmatrix}3&0&0\\1&2&0\end{pmatrix}$$

where

$$M_1=\begin{pmatrix}0&0\\0&1\\1&0\end{pmatrix}\quad M_2=\begin{pmatrix}1\\0\\0\end{pmatrix}\quad C_0=\begin{pmatrix}3&0\\1&2\end{pmatrix}$$

So that

$$C_0^{-1}=\frac{\begin{pmatrix}2&0\\-1&3\end{pmatrix}}{6}=\begin{pmatrix}1/3&0\\-1/6&1/2\end{pmatrix}$$

The least common denominator of first column is 6, that of second column is 2. So that the expected products $S_y^{\wedge}$ can be adjusted to a feasible value $S_y=(72,\ 56)^T$, which ensures the integer equation solvable.

The allocating workloads are

$$W=\begin{pmatrix}0&0\\0&1\\1&0\end{pmatrix}\begin{pmatrix}1/3&0\\-1/6&1/2\end{pmatrix}\begin{pmatrix}72\\56\end{pmatrix}+\begin{pmatrix}1\\0\\0\end{pmatrix}Z$$

$$=\begin{pmatrix}Z\\16\\24\end{pmatrix}$$

The stocks of semi-finished products are

$$S_x=C_xW+S_{x0}$$

$$=\begin{pmatrix}2&0&-2\\0&3&-2\end{pmatrix}\begin{pmatrix}Z\\16\\24\end{pmatrix}+\begin{pmatrix}10\\0\end{pmatrix}$$

$$=\begin{pmatrix}2Z-38\\0\end{pmatrix}$$

The coefficient of cost function is

$K=K_2C_x+K_1$

$$= \begin{pmatrix} 0.5 & 0.5 \end{pmatrix} \begin{pmatrix} 2 & 0 & -2 \\ 0 & 3 & -2 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 4.5 & 3 \end{pmatrix}$$

The cost function is

$$J=KW= \begin{pmatrix} 2 & 4.5 & 3 \end{pmatrix} \begin{pmatrix} Z \\ 16 \\ 24 \end{pmatrix} =2Z+144$$

From constraint condition $2Z-38\geq0$, it is easy to get that Z=19 makes J achieve minimum. So the optimal workloads are

$W^*=(19 \quad 16 \quad 24)^T$

The lowest cost if

$$J^*=K_1W^*+K_2S_x= \begin{pmatrix} 2 & 3 & 3 \end{pmatrix} \begin{pmatrix} 19 \\ 16 \\ 24 \end{pmatrix} =152$$

and

$$S_u= \begin{pmatrix} -2 & -1 & 0 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 19 \\ 16 \\ 24 \end{pmatrix} = \begin{pmatrix} -54 \\ -16 \end{pmatrix}$$

This means that the demands for elementary components are $S_1=54$, $S_2=16$.

6 Conclusion

This paper presents a discrete manufacturing process model and provides a method to solve the associated equations for quantitative planning. This workload allocation method would minimize the total cost of tasks and stocks under conditions of satisfying the needs of final product orders, and the bounds of production and inventory capacities.

In a multi-stage planning process, the final stock amounts of a former stage are the initial one of the next stage. It is necessary to set up a series of dynamic equations for optimizing whole process. This problem will be discussed in another paper.

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