Design of Dual-Rate Fuzzy Model-based Digital Controller Using Intelligent Digital Redeisgn

Do Wan Kim*, Jin Bae Park*, and Young Hoon Joo**

*Department of Electrical and Electronic Engineering, Yonsei University, Seodaemun-gu, Seoul, 120-749, Korea,
**School of Electronic and Information Engineering, Kunsan National University, Kunsan, Chonbuk, 573-701, Korea.

Abstract: This paper proposes a novel and efficient intelligent digital redesign technique for a Takagi-Sugeno (TS) fuzzy system. The term of intelligent digital redesign involves converting an existing analog fuzzy-model-based controller into an equivalent digital counterpart in the sense of state matching. In this paper, we suggest the discretization method based on the dual-rate sampling approximation is first proposed, and then attempt to globally match the states of the overall closed-loop TS fuzzy system with the pre-designed analog fuzzy-model-based controller and those with the digitally redesigned fuzzy-model-based controller. To show the feasibility and the effectiveness of the proposed method, a computer simulation is provided.

Keywords: Fuzzy-model-based control, intelligent digital redesign, linear matrix inequality.

1. Introduction

Nowadays, owing to the recent development of the microprocessor and its interfacing hardware, digital controller is popularly utilized for controlling complex dynamical systems such as aircrafts [1], robots [2], hard disc drivers [3], and chaotic systems [9, 10]. Especially, various research results on the Takagi-Sugeno (T-S) fuzzy-model-based digital control method, which is popularly used as an effective technique of the control of nonlinear systems, have been published [4, 5, 6, 7, 8].

One of the digital design approaches, the so-called digital redesign is a controller design procedure for a hybrid control system, where an analog controller is first designed and then converted to an equivalent digital controller [10]. Historically, digital redesign was first studied in detail by Kuo [12]. He proposed a discrete-state matching method and applied it to a simplified one-axis sky-lab satellite system. Recently, Shieh and his colleagues [1, 13] have thoroughly investigated the digital redesign for a class of linear systems. Joo and his colleagues [4, 5, 6, 7, 8] have researched the intelligent digital redesign using the T-S fuzzy-model-based controller for complex nonlinear system.

Unlike the digital redesign for a class of linear systems, an exact approach to the intelligent digital redesign may be impossible due to nonlinear behavior among the T-S fuzzy rules. In specific, under influences of this nonlinear behavior, the discretization as well as state matching gives rise to error .

This paper aims at developing a new global discretization methodology for the T-S fuzzy systems. Our key idea is to utilize the dual-rate sampling schemes. The overall dynamics of the T-S fuzzy system are discretized for the slow-rate sampling period, and then the approximation for the polytopic structure is performed in the fast-rate sampling scheme. To match the state of continuous-data system and digital system, this paper formulates the convex minimization problem and presents derivation of some sufficient conditions in terms of the LMIs for it.

2. Preliminaries

Let us consider the continuous-data T-S fuzzy model

$$R_i : \text{IF } z_1(t) \text{ is about } \Gamma_{i1} \text{ and } \cdots \text{ and } z_p(t) \text{ is about } \Gamma_{ip}$$
$$\text{THEN } \frac{d}{dt}x_c(t) = A_i x_c(t) + B_i u_c(t) \quad (1)$$

where $z_j(t)$, $j \in \mathcal{I}_p = \{1, 2, \dots, p\}$, is the jth premise variable, and Γ_{ij}, $(i, j) \in \mathcal{I}_q (= \{1, 2, \dots, q\}) \times \mathcal{I}_p$, is the fuzzy set of jth premise variable in the ith fuzzy rule. Using the center-average defuzzification, product inference, and singleton fuzzifier, the overall dynamics of (1) is described by

$$\frac{d}{dt}x_c(t) = \sum_{i=1}^{q} \theta_i(z(t))(A_i x_c(t) + B_i u_c(t)) \quad (2)$$

where $w_i(z(t)) = \prod_{j=1}^{p} \Gamma_{ij}(z_j(t))$, $\theta_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{q} w_i(z(t))}$, and $\Gamma_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in Γ_{ij}. We use the following fuzzy-model-based controller.

$$R_i : \text{IF } z_1(t) \text{ is about } \Gamma_{i1} \text{ and } \cdots \text{ and } z_p(t) \text{ is about } \Gamma_{ip}$$
$$\text{THEN } u_c(t) = K_c^i x_c(t) \quad (3)$$

Using the center-average defuzzification, product inference, and singleton fuzzifier, the overall control law is given by

$$u_c(t) = \sum_{i=1}^{q} \theta_i(z(t)) K_c^i x_c(t) \quad (4)$$

Substituting (4) into (2) yields the state equations of the closed-loop system:

$$\frac{d}{dt}x_c(t) = \sum_{i=1}^{q}\sum_{j=1}^{q} \theta_i(z(t))\theta_j(z(t))(A_i + B_i K_c^j)x_c(t) \quad (5)$$

The digital T-S fuzzy system is denoted by

$$\frac{d}{dt}x_d(t) = \sum_{i=1}^{q} \theta_i(z(t))(A_i x_d(t) + B_i u_d(kT)) \quad (6)$$

for $t \in [kT, kT + T)$. The intelligent digital redesign problem is to find the digital control gains so that the state variables of the digital system (6) are as closely as possible to those of the continuous-data system (5) as the sampling instants for a given input $u(t)$.

3. Intelligent Digital Redesign

3.1. Discretization of Continuous-time T-S Fuzzy System With Dual-Rate Sampling

To match the responses of (6) and (5) at $t = (k+1)T$ for arbitrary initial state $x_c(kT) = x_d(kT)$, the discretized versions of them is required. We first consider the digital T-S fuzzy system (6). If the state transition matrix $\Phi(t, t_0)$ satisfies $\Phi(t,t_0) = \Phi(t,t_1)\Phi(t_1,t_0)$ with initial condition $\Phi(t_0,t_0) = I$ and $\frac{\partial}{\partial t}\Phi(t,t_0) = \sum_{i=1}^{q}\theta_i(z(t))A_i\Phi(t,t_0)$, the general solution of (6) excited by the initial state $x_d(t_0)$ and the input $u(t)$ is given by

$$x_d(t) = \Phi(t,t_0)x_d(t_0) + \int_{t_0}^{t}\Phi(t,\tau)\left(\sum_{i=1}^{q}\theta_i(z(\tau))B_i\right)u_d(\tau)d\tau \tag{7}$$

If an input $u_d(t)$ is generated by a digital computer followed by a digital-to-analog converter, then $u_d(t)$ will be piecewise constant, *i.e.*, $u_d(t) = u_d(kT)$ for $t \in [kT, kT+T)$. Besides, assume that the firing strength $\theta_i(z(t))$ for $t \in [kT, kT+T)$ is $\theta_i(z(kT))$. For this input and $t \in [kT, kT+T)$, (7) is exactly evaluated by

$$\begin{aligned}
x_d(kT+T) &= \Phi(kT+T,kT)x_d(kT) + \int_{kT}^{kT+T}\Phi(kT+T,\tau) \\
&\quad\times\left(\sum_{i=1}^{q}\theta_i(z(\tau))B_i\right)u_d(\tau)d\tau \\
&= \exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\right)x_d(kT) \\
&\quad+\int_{kT}^{kT+T}\exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_i(kT+T-\tau)\right) \\
&\quad\times\left(\sum_{i=1}^{q}\theta_i(z(kT))B_i\right)u_d(kT)d\tau \\
&= \exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\right)x_d(kT) \\
&\quad+\left(\exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\right)-I\right) \\
&\quad\times\left(\sum_{i=1}^{q}\theta_i(z(kT))A_i\right)^{-1}\left(\sum_{i=1}^{q}\theta_i(z(kT))B_i\right) \\
&\quad\times u_d(kT) \\
&\triangleq \mathcal{A}(kT)x_d(kT) + \mathcal{B}(kT)u_d(kT)
\end{aligned} \tag{8}$$

Note that there is no approximation error involved in this derivation and yields the exact solution of (6) at $t = kT$ if the input and the firing strength is piecewise constant. However, it is impossible to utilize (8) for design of the digital controller in terms of LMIs since it is not represented in the polytopic structure unlike the general T-S fuzzy system. Dealing such issue is formulated as follows:

Problem 1: *The pointwise dynamical behavior of the continuous-time T-S fuzzy system are sufficiently satisfied with the following design objectives: The discrete-time T-S fuzzy system must be obtained by discretizing the overall dynamics of the system as accurately as possible, and it must maintain the polytopic structure.*

Our key idea for the Problem 1 is to utilize the dual-rate sampling schemes. The overall dynamics of the T-S fuzzy system is discretized for the slow-rate sampling period T, and then the approximation for the polytopic structure is performed in the fast-rate sampling period $\frac{T}{n}$, where $n > 1$ and n is constant. The following theorem gives the rigorous mathematical approximation to accomplish the Problem 1.

Theorem 1: *The digital TS fuzzy system* (6) *can be converted to the following well-approximated pointwise dynamical behavior:*

$$\begin{aligned}
x_d(kT+T) \approx &\underbrace{\sum_{i_1=1}^{q}\sum_{i_2=1}^{q}\cdots\sum_{i_n=1}^{q}}_{n}\theta_{i_1i_2\cdots i_n}(z(kT))\left(\mathcal{G}_{i_1i_2\cdots i_n}x_d(kT)\right. \\
&\left.+\mathcal{H}_{i_1i_2\cdots i_n}u_d(kT)\right)
\end{aligned} \tag{9}$$

where $G_i^{\frac{1}{n}} = \exp(A_i\frac{T}{n})$, $H_i^{(\frac{1}{n})} = \left(G_i^{\frac{1}{n}} - I\right)A_i^{-1}B_i$, $\mathcal{G}_{i_1i_2\cdots i_n} = \prod_{l=1}^{n}G_{i_l}^{\frac{1}{n}}$, $\mathcal{H}_{i_1i_2\cdots i_n} = H_{i_1}^{(\frac{1}{n})} + \sum_{l=2}^{n}\left(\prod_{m=1}^{l-1}G_{i_m}^{\frac{1}{n}}\right)H_{i_l}^{(\frac{1}{n})}$, $\theta_{i_1i_2\cdots i_n} = \prod_{l=1}^{n}\theta_{i_l}(z(kT))$, *and* $(i_1, i_2, \cdots, i_n) \in \underbrace{\mathcal{I}_q\times\cdots\times\mathcal{I}_q}_{n}$.

Proof: Start from question, namely approximation of (8). $\mathcal{A}(kT)$ and $\mathcal{B}(kT)$ of (8) are finely approximated as follows:

$$\begin{aligned}
\mathcal{A}(kT) &= \exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\right) \\
&= \exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_i\frac{T}{n}\right)^n \\
&\approx \left(\sum_{i=1}^{q}\theta_i(z(kT))G_i^{\frac{1}{n}}\right)^n \\
&= \underbrace{\sum_{i_1=1}^{q}\sum_{i_2=1}^{q}\cdots\sum_{i_n=1}^{q}}_{n}\theta_{i_1i_2\cdots i_n}\mathcal{G}_{i_1i_2\cdots i_n}
\end{aligned} \tag{10}$$

and

$$\begin{aligned}
\mathcal{B}(kT) &= \left(\exp\left(\sum_{i=1}^{q}\theta_i(z(kT))A_iT\right)-I\right) \\
&\quad\times\left(\sum_{i=1}^{q}\theta_i(z(kT))A_i\right)^{-1}\left(\sum_{i=1}^{q}\theta_i(z(kT))B_i\right) \\
&\approx\left(\left(\sum_{i=1}^{q}\theta_i(z(kT))G_i^{\frac{1}{n}}\right)^n-I\right)\left(\sum_{i=1}^{q}\theta_i(z(kT))A_i\right)^{-1} \\
&\quad\times\left(\sum_{i=1}^{q}\theta_i(z(kT))B_i\right) \\
&=\left(\left(\sum_{i=1}^{q}\theta_i(z(kT))G_i^{\frac{1}{n}}\right)^n-I\right) \\
&\quad\times\left(\sum_{i=1}^{q}\theta_i(z(kT))G_i^{\frac{1}{n}}-I\right)^{-1}\left(\sum_{i=1}^{q}\theta_i(z(kT))G_i^{\frac{1}{n}}-I\right) \\
&\quad\times\left(\sum_{i=1}^{q}\theta_i(z(kT))A_i\right)^{-1}\left(\sum_{i=1}^{q}\theta_i(z(kT))B_i\right)
\end{aligned}$$

$$\approx \left(\left(\sum_{i=1}^{q} \theta_i(z(kT)) G_i^{\frac{1}{n}} \right)^n - I \right) \left(\sum_{i=1}^{q} \theta_i(z(kT)) G_i^{\frac{1}{n}} - I \right)^{-1}$$
$$\times \sum_{i=1}^{q} \theta_i(z(kT)) \left(G_i^{\frac{1}{n}} - I \right) A_i^{-1} B_i$$
$$= \left(I + \sum_{i=1}^{q} \theta_i(z(kT)) G_i^{\frac{1}{n}} + \cdots + \left(\sum_{i=1}^{q} \theta_i(z(kT)) G_i^{\frac{1}{n}} \right)^{n-1} \right)$$
$$\times \sum_{i=1}^{q} \theta_i(z(kT)) H_i^{(\frac{1}{n})}$$
$$= \underbrace{\sum_{i_1=1}^{q} \sum_{i_2=1}^{q} \cdots \sum_{i_n=1}^{q}}_{n} \theta_{i_1 i_2 \cdots i_n}(z(kT)) \mathcal{H}_{i_1 i_2 \cdots i_n} \tag{11}$$

■

In the proposed discretization method, two approximations are performed as follows:

$$\exp\left(\sum_{i=1}^{q} \theta_i(z(kT)) A_i \frac{T}{n} \right) \approx \sum_{i=1}^{q} \theta_i(z(kT)) G_i^{\frac{1}{n}} \tag{12}$$

$$\left(\sum_{i=1}^{q} \theta_i(z(kT)) G_i^{\frac{1}{n}} - I \right) \left(\sum_{i=1}^{q} \theta_i(z(kT)) A_i \right)^{-1}$$
$$\times \left(\sum_{i=1}^{q} \theta_i(z(kT)) B_i \right) \approx \sum_{i=1}^{q} \theta_i(z(kT)) H_i^{(\frac{1}{n})} \tag{13}$$

From (12), introduce approximation error defined as

$$e_1 = \left\| \exp\left(\sum_{i=1}^{q} \theta_i(z(kT)) A_i \frac{T}{n} \right) - \sum_{i=1}^{q} \theta_i(z(kT)) G_i^{\frac{1}{n}} \right\|_2 \tag{14}$$

Applying Taylor series expansion from the right-hand side gives

$$e_1 = \frac{T^2}{n^2} \left\| \left(\frac{1}{2!} \left(\sum_{i=1}^{q} \theta_i(z(kT)) A_i \right)^2 \right.\right.$$
$$\left.\left. - \frac{1}{2!} \sum_{i=1}^{q} \theta_i(z(kT)) A_i^2 \right) + \cdots \right\|_2 = \mathcal{O}\left(\frac{T^2}{n^2} \right) \tag{15}$$

In the same manner, approximation error in (13) is

$$e_2 = \left\| \left(\sum_{i=1}^{q} \theta_i(z(kT)) G_i^{\frac{1}{n}} - I \right) \left(\sum_{i=1}^{q} \theta_i(z(kT)) A_i \right)^{-1} \right.$$
$$\left. \times \left(\sum_{i=1}^{q} \theta_i(z(kT)) B_i \right) - \sum_{i=1}^{q} \theta_i(z(kT)) H_i^{(\frac{1}{n})} \right\|_2 = \mathcal{O}\left(\frac{T}{n} \right) \tag{16}$$

From (15) and (16), approximation error clearly goes to zero as n approaches to infinity.

Remark 1: *Note that the proposed pointwise behavior* (9) *yields not only small error but also forms itself into the polytopic structure.*

Remark 2: *In [7,8], the approximations are also performed at above two cases, and the approximation results are exactly equal to* (12) *and* (13) *when* $n = 1$. *Therefore, our discretized version* (9) *under the choice of a sufficiently large* n *yields smaller approximation error than it of [7,8].*

Corollary 1: *The continuous-data closed-loop T-S fuzzy system* (5) *can also be the following well-approximated pointwise dynamical behavior:*

$$x_c(kT+T) \approx \underbrace{\sum_{i_1=1}^{q} \sum_{i_2=1}^{q} \cdots \sum_{i_n=1}^{q}}_{n} \underbrace{\sum_{j_1=1}^{q} \sum_{j_2=1}^{q} \cdots \sum_{j_n=1}^{q}}_{n} \theta_{i_1 i_2 \cdots i_n}(z(kT))$$
$$\times \theta_{j_1 j_2 \cdots j_n}(z(kT)) \mathcal{M}_{i_1 i_2 \cdots i_n j_1 j_2 \cdots j_n} x_c(kT) \tag{17}$$

where $\mathcal{M}_{i_1 i_2 \cdots i_n j_1 j_2 \cdots j_n} = \prod_{l=1}^{n} \exp\left((A_{i_l} + B_{i_l} K_{j_l}) \frac{T}{n} \right)$ *and* $(i_1, i_2, \cdots, i_n, j_1, j_2, \cdots, j_n) \in \underbrace{\mathcal{I}_q \times \cdots \times \mathcal{I}_q}_{2n}$.

Proof: It can be straightforwardly proved by Theorem 1. ■

3.2. Digital Redesign Using State Matching

Let the digital control law for (6) take the following form:

$$u_d(kT) = \underbrace{\sum_{i_1=1}^{q} \sum_{i_2=1}^{q} \cdots \sum_{i_n=1}^{q}}_{n} \theta_{i_1 i_2 \cdots i_n}(z(kT)) K_d^{i_1 i_2 \cdots i_n} x_d(kT) \tag{18}$$

for $t \in [kT, kT+T)$, where $K_d^{i_1 i_2 \cdots i_n}$ is the digital control gain matrix to be redesigned. Substituting (18) into the discretized version (9) of (6) yields the state equations of the closed-loop system:

$$x_d(kT+T) \approx \underbrace{\sum_{i_1=1}^{q} \sum_{i_2=1}^{q} \cdots \sum_{i_n=1}^{q}}_{n} \underbrace{\sum_{j_1=1}^{q} \sum_{j_2=1}^{q} \cdots \sum_{j_n=1}^{q}}_{n} \theta_{i_1 i_2 \cdots i_n}(z(kT))$$
$$\times \theta_{j_1 j_2 \cdots j_n}(z(kT)) \left(\mathcal{G}_{i_1 i_2 \cdots i_n} + \mathcal{H}_{i_1 i_2 \cdots i_n} K_d^{j_1 j_2 \cdots j_n} \right)$$
$$\times x_d(kT) \tag{19}$$

Therefore, we can attempt to match the state $x(kT+T)$ of (19) and (17) under the assumption that $x_c(kT) = x_d(kT)$ More precisely, it is necessary to determine constant matrices $K_d^{i_1 i_2 \cdots i_n}$ such that the following matrix equality constraints be satisfied

$$\mathcal{M}_{i_1 i_2 \cdots i_n j_1 j_2 \cdots j_n} = \mathcal{G}_{i_1 i_2 \cdots i_n} + \mathcal{H}_{i_1 i_2 \cdots i_n} K_d^{j_1 j_2 \cdots j_n} \tag{20}$$

then, the state $x_d(kT+T)$ of (19) closely matches the state $x_c(kT+T)$ of (17), provided that their initial condition are the same. This problem can become a convex optimization problem such as

$$\left\| \mathcal{M}_{i_1 i_2 \cdots i_n j_1 j_2 \cdots j_n} - \left(\mathcal{G}_{i_1 i_2 \cdots i_n} + \mathcal{H}_{i_1 i_2 \cdots i_n} K_{j_1 j_2 \cdots j_n}^d \right) \right\| < \gamma I \tag{21}$$

hence can be numerically solved in terms of LMIs. In this aspect, the following theorem provides a solution to the convex optimization problem.

Theorem 2: *If there exist constant matrixes* $K_{i_1 i_2 \cdots i_n}^d$ *and a possibly small positive scalars* γ *such the following gen-*

eralized eigenvalue problem (GEVP) has solutions

$$\underset{K^d_{i_1 i_2 \cdots i_n}}{Minimize} \quad \gamma \quad \text{subject to}$$

$$\begin{bmatrix} -\gamma I & * \\ \mathcal{M}_{i_1 i_2 \cdots i_n j_1 j_2 \cdots j_n} - \mathcal{G}_{i_1 i_2 \cdots i_n} - \mathcal{H}_{i_1 i_2 \cdots i_n} K^d_{j_1 j_2 \cdots j_n} & -\gamma I \end{bmatrix} < 0, \quad i_1, i_2 \cdots, i_n, j_1, j_2 \cdots, j_n \in \mathcal{I}_q \tag{22}$$

then $x_d(kT+T)$ *of* (19) *closely matches the state* $x_c(kT+T)$ *of* (17)*, where '*$*$*' denotes the transposed element in symmetric positions.*

Proof: Consider the convex optimization problem (21) From the definition of the induced-2 norm, the following inequalities hold

$$\left(\mathcal{M}_{i_1 i_2 \cdots i_n j_1 j_2 \cdots j_n} - \mathcal{G}_{i_1 i_2 \cdots i_n} - \mathcal{H}_{i_1 i_2 \cdots i_n} K^d_{j_1 j_2 \cdots j_n}\right)^T \times \left(\mathcal{M}_{i_1 i_2 \cdots i_n j_1 j_2 \cdots j_n} - \mathcal{G}_{i_1 i_2 \cdots i_n} - \mathcal{H}_{i_1 i_2 \cdots i_n} K^d_{j_1 j_2 \cdots j_n}\right) < \gamma^2 I \tag{23}$$

Using Schur complement, can be represented by

$$\begin{bmatrix} -\gamma I & * \\ \mathcal{M}_{i_1 i_2 \cdots i_n j_1 j_2 \cdots j_n} - \mathcal{G}_{i_1 i_2 \cdots i_n} - \mathcal{H}_{i_1 i_2 \cdots i_n} K^d_{j_1 j_2 \cdots j_n} & -\gamma I \end{bmatrix} < 0 \tag{24}$$

■

In the Theorem 2, the error of GEVP(22) can be defined as

$$e_3 = \left\|\mathcal{M}_{i_1 i_2 \cdots i_n j_1 j_2 \cdots j_n} - \mathcal{G}_{i_1 i_2 \cdots i_n} - \mathcal{H}_{i_1 i_2 \cdots i_n} K^d_{j_1 j_2 \cdots j_n}\right\| = \mathcal{O}(\frac{T}{n}) \tag{25}$$

in which applying Taylor series expansion from $\mathcal{M}_{i_1 i_2 \cdots i_n j_1 j_2 \cdots j_n}$, $\mathcal{G}_{i_1 i_2 \cdots i_n}$, and $\mathcal{H}_{i_1 i_2 \cdots i_n}$ of the right-hand side gives $\mathcal{M}_{i_1 i_2 \cdots i_n j_1 j_2 \cdots j_n} = \left(I + (A_{i_1} + B_{i_1} K^d_{j_1})\frac{T}{n} + \mathcal{O}\left(\frac{T^2}{n^2}\right)\right) \times \left(I + (A_{i_2} + B_{i_2} K^d_{j_2})\frac{T}{n} + \mathcal{O}\left(\frac{T^2}{n^2}\right)\right) \cdots \left(I + (A_{i_n} + B_{i_n} K^d_{j_n})\frac{T}{n} + \mathcal{O}\left(\frac{T^2}{n^2}\right)\right)$, $\mathcal{G}_{i_1 i_2 \cdots i_n} = \left(I + A_{i_1}\frac{T}{n} + \mathcal{O}\left(\frac{T^2}{n^2}\right)\right)\left(I + A_{i_2}\frac{T}{n} + \mathcal{O}\left(\frac{T^2}{n^2}\right)\right) \cdots \left(I + A_{i_n}\frac{T}{n} + \mathcal{O}\left(\frac{T^2}{n^2}\right)\right)$, and $\mathcal{H}_{i_1 i_2 \cdots i_n} = H_{i_1}^{(\frac{1}{n})} + \sum_{l=2}^{n}\left(\prod_{m=1}^{l-1} G_{i_m}^{\frac{1}{n}}\right) H_{i_l}^{(\frac{1}{n})}$. Therefore, because of $e_3 = \mathcal{O}(\frac{T}{n})$, e_3 is expected to small value for sufficiently large n.

4. Intelligent Digital Redesign Example: The Chen's Chaotic Attractor

In this section, we use the results in Section 3. to the digital control problem of the Chen's chaotic attractor. The dynamics of the Chen's chaotic attractor are as follows:

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} a(x_2(t) - x_1(t)) \\ (c-a)x_1(t) - x_1(t)x_3(t) + cx_2(t) \\ x_1(t)x_2(t) - bx_3(t) \end{bmatrix} \tag{26}$$

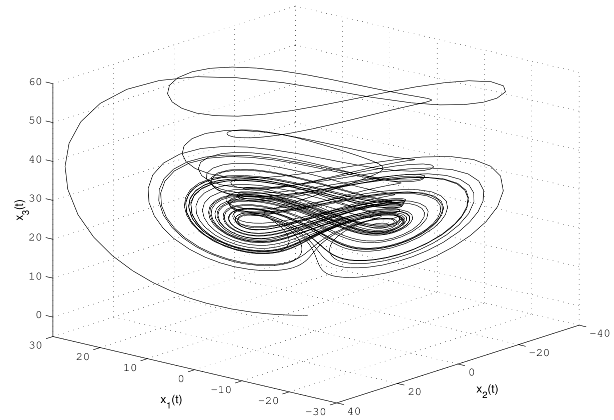

Fig. 1. Trajectory of the Chen's chaotic attractor.

where $a = 35$, $b = 3$, and $c = 28$. The corresponding T-S fuzzy model of the system in (26) is expressed as follows:

R^1 : IF $x_1(t)$ is about Γ_1^1,

$$\text{THEN} \frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = A_1 \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + B_1 u_d(t)$$

R^2 : IF $x_1(t)$ is about Γ_1^2,

$$\text{THEN} \frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = A_2 \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + B_2 u_d(t)$$

where

$$A_1 = \begin{bmatrix} -a & a & 0 \\ c-a & c & -x_{1min} \\ 0 & x_{1min} & -b \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} -a & a & 0 \\ c-a & c & -x_{1max} \\ 0 & x_{1max} & -b \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{27}$$

and the membership functions are

$$\Gamma_1^1(x_1(t)) = \frac{-x_1(t) + x_{1max}}{x_{1max} - x_{1min}}, \quad \Gamma_1^2(x_2(t)) = \frac{x_1(t) - x_{1min}}{x_{1max} - x_{1min}}. \tag{28}$$

where Γ_j^i are positive semi-definite for all $x \in [x_{1min}, x_{1max}](= [-30, 30])$. With $x_1(0) = 1$, $x_2(0) = 1$, and $x_3(0) = -1$, the trajectory of this system is shown in Fig 4. From Theorem 5 in [14], the gain matrices for the analog fuzzy model-based controller is obtained as follows:

$$K_c^1 = \begin{bmatrix} -5.1763 & 204.8118 & 131.8175 \end{bmatrix}$$
$$K_c^2 = \begin{bmatrix} -5.1763 & 204.8118 & -131.8175 \end{bmatrix}$$

We now wish to find an equivalent digital system whose states will match those of the continuous-time controlled T-S fuzzy Chen's chaotic attractor at the sampling instants for the same input and initial states. Applying the digital redesign method, where n is selected as 3, discussed in the preceding section yields the following gain matrices for the

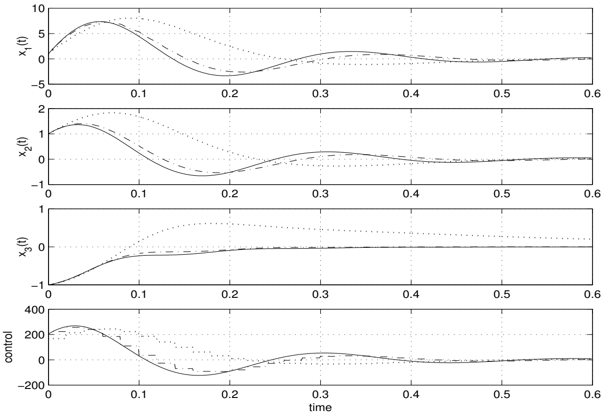

Fig. 2. The time responses of the controlled Chen's chaotic attractor (Sampling time T is 0.002 sec., solid line continuous-time control, dotted line: digital control by [4], and dash-dotted line: digital control by the proposed method.)

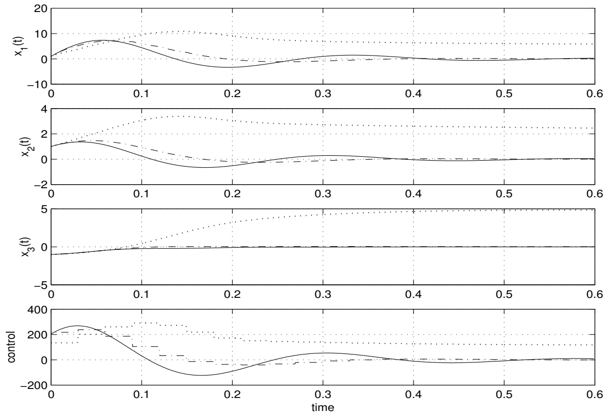

Fig. 3. The time responses of the controlled Chen's chaotic attractor (Sampling time T is 0.003 sec., solid line continuous-time control, dotted line: digital control by [4], and dash-dotted line: digital control by the proposed method.)

sampling period $T = 0.02s$.

$$K_d^{111} = \begin{bmatrix} -14.9745 & 179.1430 & 173.8455 \end{bmatrix}$$
$$K_d^{211} = \begin{bmatrix} -17.7989 & 251.3367 & 63.7664 \end{bmatrix}$$
$$K_d^{121} = \begin{bmatrix} -18.0453 & 251.9124 & 64.9188 \end{bmatrix}$$
$$K_d^{221} = \begin{bmatrix} -17.8002 & 249.9941 & -64.1442 \end{bmatrix}$$
$$K_d^{112} = \begin{bmatrix} -17.6624 & 254.4623 & 64.6591 \end{bmatrix}$$
$$K_d^{212} = \begin{bmatrix} -18.0192 & 250.9625 & -64.2822 \end{bmatrix}$$
$$K_d^{122} = \begin{bmatrix} -17.6873 & 256.7548 & -64.9179 \end{bmatrix}$$
$$K_d^{222} = \begin{bmatrix} -14.9745 & 179.1430 & -173.8455 \end{bmatrix}$$

When another relatively loner sampling period $T = 0.03$ sec. is selected, the gain matrices are obtained as

$$K_d^{111} = \begin{bmatrix} -17.1200 & 141.6405 & 184.0835 \end{bmatrix}$$
$$K_d^{211} = \begin{bmatrix} -32.4597 & 261.1590 & 77.8177 \end{bmatrix}$$
$$K_d^{121} = \begin{bmatrix} -20.5240 & 263.5645 & 76.5096 \end{bmatrix}$$
$$K_d^{221} = \begin{bmatrix} -22.0743 & 259.6616 & -75.2516 \end{bmatrix}$$
$$K_d^{112} = \begin{bmatrix} -18.3166 & 265.2251 & 73.4829 \end{bmatrix}$$
$$K_d^{212} = \begin{bmatrix} -21.7103 & 261.3568 & -75.0983 \end{bmatrix}$$
$$K_d^{122} = \begin{bmatrix} -27.5157 & 268.9302 & -77.4603 \end{bmatrix}$$
$$K_d^{222} = \begin{bmatrix} -17.1200 & 141.6405 & -184.0835 \end{bmatrix}$$

The digital control system with the obtained gain matrices is simulated on the digital computer with $T = 0.02$ sec. and $T = 0.03$ sec.. Other available intelligent digital redesign method [4] is also simulated for the comparison. Figure 2 and 3 show the comparisons for $T = 0.01$s and $T = 0.02$s, respectively. These figures show that the proposed digital redesign technique leads to a digital controlled Chen's chaotic attractor with responses that match the continuous-data controlled T-S fuzzy Chen's chaotic attractor very closely as the comparison with [4].

5. Conclusions

In this paper, a new intelligent digital redesign method has been proposed for the digital control of the continuous-time fuzzy system, and its validity and applicability are verified through a computer simulation. Unlike other methods, the proposed method utilizes the discrete-time T-S fuzzy model with the dual-rate sampling. The dual-rate scheme is major factor that improves the accuracy of the state matching as well as the discretization. Therefore, the digital controller designed by the proposed method is expected to maintain the good performance for the relatively long sampling period.

References

[1] L.S. Shieh, W.M. Wang, J. Bain, and J. W. Sunkel, "Design of lifted dual-rate digital controllers for x-38 vehicle," *Jounal of Guidance, Contr. and Dynaics,* Vol. 23, pp. 629-339, Jul., 2000.

[2] C.J. Lee and C. Mavroidis, "PC-based control of robotic and mechatronic systems under MS-Windows NT workstation," *IEEE/ASME Trans. on Mechatronics,* Vol. 6, No. 3, pp. 311-321, 2001.

[3] K. Takaishi and S. Saito, "Seek control and settling control taming actuator resonance of hard disk drives," *IEEE trans. on Magnetics,* Vol. 39, No. 2, pp. 838-843, 2003.

[4] Y. H. Joo, L. S. Shieh, and G. Chen, "Hybrid state-space fuzzy model-based controller with dual-rate sampling for digital control of chaotic systems," *IEEE Trans. on Fuzzy Systems,* Vol. 7, No. 4, pp. 394-408, 1999.

[5] W. Chang, J. B. Park, Y. H. Joo, and G. Chen, "Design of sampled-data fuzzy-model-based control systems by using intelligent digital redesign," *IEEE Trans. on Circ. an Syst. I,* vol. 49, No. 4, pp. 509-517, 2002.

[6] W. Chang, J. B. Park, and Y. H. Joo, "GA-based intelligent digital redesign of fuzzy-model-based controllers," *IEEE Trans. on Fuzzy Systems,* Vol. 11, No. 1, pp. 35-44, 2003.

[7] Z. Li, J. B. Park, and Y. H. Joo, "Chaotifying continuous-time T-S fuzzy systems via discretization," *IEEE Trans. on Circ. an Syst. I,* Vol. 48, No. 10, pp. 1237-1243, 2001.

[8] H. J. Lee, H. B. Kim, Y. H. Joo, and J. B. Park, "A new intelligent digital redesign for T-S fuzzy systems: global approach," *IEEE Trans. on Fuzzy Systems,* to be published, 2003.

[9] J. Xu, G. Chen, and L. S. Shieh, "Digital redesign for controlling chaotic Chua's circuit," *IEEE trans. on Aero. & Electro.,* Vol. 32, No. 8, pp. 1488-1499, 1996.

[10] S. M. Guo, L. S. Shieh, G. Chen, and C. F. Lin, "Effective chaotic orbit tracker: A prediction-based digital redesigned approach," *IEEE trans. on Circ. and Systs. I.,* Vol. 47, No. 11, pp. 1577-1570, 2000.

[11] G. Chen and T. Ueta, "Yet another chaotic attractor," *Int. J. Bifur. Chaos,* Vol. 9, No. 7, pp. 1465-1466, 1999.

[12] B. C. Kuo, *Digital control systems*, Holt, Rinehart and Winston, 1980.

[13] L. S. Shieh, W. M. Wang, M. K. Appu Paniker, "Design of PAM abd PWM digital controllers for cascaded analog systems," ISA Trans., Vol. 37, pp. 201-213, 1998.

[14] K. Tanaka, T. Ikeda, and H. O. Wang,"Fuzzy regulators and fuzzy observers: relaxed stability conditions and LMI-based designs," *IEEE Trans. on Fuzzy Systems,* Vol. 6, No. 2, pp. 250 -265, 1998.