Attitude Control of Artificial Satellites via Intelligent Digital Redesign

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Abstract: This paper proposes an approach to attitude control artificial satellites with jet-engine. The jet-engine produces on-off thrust, which can be modelled as pulse-width-modulated (PWM) function. Therefore, the problem is converted to design a PWM controller and we develop an efficient technique for this purpose using digital redesign. The digital redesign is a converting technique a well-designed analog controller into the equivalent digital one maintaining the property of the original analog control system in the sense of state-matching. The redesigned digital controller is again converted into PWM controller using the equivalent area principle. We show a computer simulation of the attitude control of artificial satellites.

Keywords: T-S fuzzy systems, digital redesign, PWM control, attitude control

1. Introduction

The use of digital devices in the control of complex dynamical systems such as aircraft [15], robots [19], and chaotic systems [3, 13, 14] becomes popular owing to the increased capability, the reliability, and the reduced cost of modern microprocessors, which results in a hybrid control system, i.e., a continuous-time system with a digital controller. Digital implementation of a controller is indeed quite desirable especially when the designed controller utilizes some sophisticated and advanced control algorithms that require considerable amount of computation efforts. For this reason, digital control of continuous-time systems has been an active research branch in recent years.

The main stream in design approaches to a suitable digital controller is to discretize the continuous-time plant first, and then to determine a digital controller for the discretized plant, which is called the direct digital design approach [11]. Another efficient approach, which is called digital redesign, is a digital controller design procedure, where an analog controller is first designed and then converted to an equivalent digital controller in the sense of state matching [11,12,12–15]. It is noted that these digital redesign schemes basically work only for a class of linear systems. For that reason, it has been highly demanded to develop some intelligent digital redesign methodology for complex nonlinear systems, in which the first attempt was made by Joo *et al.* [3]. They synergistically merged both the Takagi-Sugeno (T-S) fuzzy-model-based control and the digital redesign technique for a class of nonlinear systems. Chang *et al.* extended the intelligent digital redesign to uncertain T-S fuzzy systems [6].

In general, there exist two types of digital controllers: the pulse-amplitude-modulated (PAM) controller and the pulse-width-modulated (PWM) controller. The PWM controller, which produces a series of discontinuous pulses with a fixed amplitude and variable width, has become popular in industry. Interestingly, it is observed that the significance is greatly increased for the attitude control of the artificial satellite with on-off reaction jets because jet-engine produces on-off thrust, which can be modelled as PWM signals.

This paper proposes an PWM control law design technique for T-S fuzzy systems and its application to attitude control of nonlinear artificial satellites with jet-engine using intelligent digital redesign.

Section 2 briefly reviews T-S fuzzy systems. In Section 3, the intelligent digital redesign is discussed. The PWM controller design technique is shown in Section 4. Section 5 includes an computer simulation–attitude control of 3-axis artificial satellites. Conclusions are drawn in Section 6.

2. T-S Fuzzy Systems

Most physical systems are quite complex in practice and have strong nonlinearities so that it is difficult, if not impossible, to build rigorous mathematical models [9]. Fortunately, a certain class of nonlinear dynamical systems can be expressed in some forms of either a linear mathematical model locally, or an aggregation of a set of linear mathematical models.

Consider a nonlinear dynamical system of the following form:

$$\dot{x}_c(t) = f(x_c(t), u_c(t)) \tag{1}$$

where $x_c(t) \in \mathbb{R}^n$ is the state vector, $u_c(t) \in \mathbb{R}^m$ is the control input vector. The subscript 'c' means the analog control, while the subscript 'd' will denotes the digital control in the sequel. The vector field $f : U \subset \mathbb{R}^n \times \mathbb{R}^m \to V \subset \mathbb{R}^n$ on a compact set U is assumed to be affine in $u_c(t)$ and $\mathbf{C}^r(r \geq 1)$. One way to view a T-S fuzzy system is that it performs a nonlinearly interpolated linear mapping $\chi(x_c(t), u_c(t)) : U \subset \mathbb{R}^n \times \mathbb{R}^m \to V \subset \mathbb{R}^n$ so as to satisfy

$$\sup_{x_c(t), u_c(t) \in U} \| f(x_c(t), u_c(t)) - \chi(x_c(t), u_c(t)) \| \leq \epsilon$$

where ϵ is an arbitrary small positive scalar.

Assume there exist q pairs of $v_i = (A_i, B_i)$ which represent the local dynamic behavior of (1), such that the matrix polytope

$$\mathcal{F} = \mathbf{Co}\left\{ [A_1, B_1], \ldots, [A_q, B_q] \right\}$$

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contains the domain U, where **Co** denotes a convex hull of the set $V = \left\{v_1, \ldots, v_q\right\}$, and $A_i \in \mathbb{R}^{n\times n}$, and $B_i \in \mathbb{R}^{n\times m}$. Therefore, one can find an adequate mapping at time instant t with ϵ of the form:

$$\chi(x_c(t), u_c(t)) = A(\theta)x_c(t) + B(\theta)u_c(t) \qquad (2)$$

where $A(\theta)$ ranges over a matrix polytope

$$A(\theta) \in \mathbf{Co}\{A_1, \ldots, A_q\},$$

and $B(\theta) \in \mathbf{Co}\{B_1, \ldots, B_q\}$ with $\sum_{i=1}^{q} \theta_i = 1,\ \theta_i \geq 1$. The key idea of the T-S fuzzy inference system is to determine the coefficients θ_i in the convex combination of the given vertices V by virtue of the available qualitative knowledge from domain experts, which are quantified by 'IF-THEN' rule base. More precisely, the ith rule of the T-S fuzzy system is formulated in the following form:

$$\begin{aligned} R^i : &\text{IF } z_1(t) \text{ is about } \Gamma_1^i \text{ and } \cdots \text{ and } z_n(t) \text{ is about } \Gamma_n^i \\ &\text{THEN } \dot{x}_c(t) = A_i x_c(t) + B_i u_c(t) \end{aligned} \qquad (3)$$

where R^i denotes the ith fuzzy inference rule, $z_h(t)$ is the premise variable, $\Gamma_h^i, i = 1, \ldots, q, h = 1, \ldots, n$, is the fuzzy set of the hth premise variable in the ith fuzzy inference rule. Using the center-average defuzzification, product inference, and singleton fuzzifier, the global dynamics of this T-S fuzzy system (3) is described by

$$\dot{x}_c(t) = \sum_{i=1}^{q} \theta_i(z(t))(A_i x_c(t) + B_i u_c(t)) \qquad (4)$$

in which

$$\omega_i(z(t)) = \prod_{h=1}^{n} \Gamma_h^i(z_h(t)), \qquad \theta_i(z(t)) = \frac{\omega_i(z(t))}{\sum_{i=1}^{q} \omega_i(z(t))}$$

and $\Gamma_h^i(z_h(t))$ is the membership value of the hth premise variable $z_h(t)$ in Γ_h^i. Some basic properties of $\theta_i(t)$ are:

$$\theta_i(z(t)) \geq 0, \quad \sum_{i=1}^{q} \theta_i(z(t)) = 1 \qquad (5)$$

Throughout this paper, a well-constructed continuous-time state-feedback fuzzy-model-based control law is assumed to be pre-designed, which will be used in redesigning the digital control law. The controller rule is of the following form:

$$\begin{aligned} R^i : &\text{IF } z_1(t) \text{ is about } \Gamma_1^i \text{ and } \cdots \text{ and } z_n(t) \text{ is about } \Gamma_n^i, \\ &\text{THEN } u_c(t) = K_c^i x_c(t) \end{aligned} \qquad (6)$$

The defuzzified output of the controller rules is given by

$$u_c(t) = \sum_{i=1}^{q} \theta_i(z(t)) K_c^i x_c(t) \qquad (7)$$

The overall continuous-time closed-loop T-S fuzzy system is

$$\dot{x}_c(t) = \sum_{i=1}^{q}\sum_{j=1}^{q} \theta_i(z(t))\theta_j(z(t)) \left(A_i + B_i K_c^j\right) x_c(t) \qquad (8)$$

3. Intelligent Digital Redesign

This section first investigates the discretization of the continuous-time T-S fuzzy systems, and then the new global intelligent digital redesign method is presented by using a convex optimization technique.

3.1. Discretization of the Continuous-time T-S Fuzzy Systems

This subsection discusses the discretization of the continuous-time T-S fuzzy systems. Consider a class of T-S fuzzy systems governed by

$$\dot{x}_d(t) = \sum_{i=1}^{q} \theta_i(z(t))(A_i x_d(t) + B_i u_d(t)) \qquad (9)$$

where $u_d(t) = u_d(kT)$ is the piecewise-constant control input vector to be determined in the time interval $[kT, kT + T)$, where $T > 0$ is a sampling period. For the digital control of the continuous-time T-S fuzzy system, the digital fuzzy-model-based controller is employed. Let the fuzzy rule of the digital control law for the system (9) take the following form:

$$\begin{aligned} R^i : &\text{IF } z_1(kT) \text{ is about } \Gamma_1^i \text{ and } \cdots \text{ and } z_n(kT) \text{ is about } \Gamma_n^i, \\ &\text{THEN } u_d(t) = K_d^i x_d(kT) \end{aligned} \qquad (10)$$

for $t \in [kT, kT + T)$, where K_d^i is the digital control gain matrix to be redesigned for the ith rule, and the overall control law is given by

$$u_d(t) = \sum_{i=1}^{q} \theta_i(z(kT)) K_d^i x_d(kT) \qquad (11)$$

for $t \in [kT, kT + T)$.

The digital redesign problem is to find digital controller gains in (11) from the analog gains in (7), so that the closed-loop state $x_d(t)$ in (9) with (11) can closely match the closed-loop state $x_c(t)$ in (8) at all sampling time instants $t = kT, k = 1, 2, \ldots,$. Thus it is more convenient to convert the T-S fuzzy system into discrete-time version for derivation of the state matching condition.

There are several methods for discretizing a linear time-invariant (LTI) continuous-time system. Unfortunately, these discretization methods cannot be directly applied to the discretization of the continuous-time T-S fuzzy system since the defuzzifed output of the T-S fuzzy system is not LTI but implicitly time-varying [7]. Moreover, it is further desired to maintain the polytopic structure of the discretized T-S fuzzy system for the construction of the digital fuzzy-model-based controller. Thus we need a mathematical foundation for the discretization of the continuous-time T-S fuzzy system.

Assumption 1: Assume that the firing strength of the ith rule, $\theta_i(z(t))$ is approximated by their values at time kT, that is,

$$\theta_i(z(t)) \approx \theta_i(z(kT))$$

for $t \in [kT, kT + T)$. Consequently, the nonlinear matrices $\sum_{i=1}^{q} \theta(z(t))A_i$ and $\sum_{i=1}^{q} \theta(z(t))B_i$ can be approximated as constant matrices $\sum_{i=1}^{q}\theta(z(kT))A_i$ and $\sum_{i=1}^{q}\theta(z(kT))B_i$, respectively, over any interval $[kT, kT+T)$.

If a sufficiently small sampling period T is chosen, Assumption 1 is reasonable.

Theorem 1: The pointwise dynamical behavior of the T-S fuzzy system (9) can be efficiently approximated by

$$x_d(kT+T) \approx \sum_{i=1}^{q}\theta(z(kT))(G_i x_d(kT) + H_i u_d(kT)) \quad (12)$$

where $G_i = \exp(A_i T)$ and $H_i = (G_i - I)A_i^{-1}B_i$.

Proof: See [7]. ■

The discretized T-S fuzzy system (12) contains the discretization error with the order of $\mathcal{O}\left(T^2\right)$, which is tolerable under the choice of a sufficiently small sampling period, and vanishes as T approaches zero. Notice that the error induced in this discretization procedure is smaller than the first-order truncated Taylor series expansion of (9). Hence, the discretized version of the closed-loop system with (12) and (11) is constructed to yield

$$x_d(kT+T) \approx \sum_{i=1}^{q}\sum_{j=1}^{q}\theta_i(z(kT))\theta_j(z(kT))\left(G_i + H_i K_d^j\right)x_d(kT) \quad (13)$$

Corollary 1: The pointwise dynamical behavior of the continuous-time closed-loop T-S fuzzy system (8) can also be approximately discretized as

$$x_c(kT+T) \approx \sum_{i=1}^{q}\sum_{j=1}^{q}\theta_i(z(kT))\theta_j(z(kT))\Phi_{ij}x_c(kT) \quad (14)$$

where $\Phi_{ij} = \exp\left(\left(A_i + B_i K_c^j\right)T\right)$.

Proof: It can be straightforwardly proved by Theorem 1. ■

3.2. New Intelligent Digital Redesign Based on Global State Matching Concept

Our goal is to develop an intelligent digital redesign technique for T-S fuzzy systems so that the global dynamical behavior of (9) with the digitally redesigned fuzzy-model-based controller may retain that of the closed-loop T-S fuzzy system with the existing analog fuzzy-model-based controller, and the stability of the digitally controlled T-S fuzzy system is secured.

Comparing (13) with (14), to obtain $x_c(kT+T) = x_d(kT+T)$ under the assumption of $x_c(kT) = x_d(kT)$, it is necessary to determine the digital control gain matrices K_d^i such that the following matrix equality constraints should be satisfied

$$\Phi_{ij} = G_i + H_i K_d^j, \qquad i,j = 1,2,\ldots,q. \quad (15)$$

then, the state $x_d(t)$ closely matches the state $x_c(t)$ globally, provided that their initial conditions are the same, that is, $x_c(0) = x_d(0) = x_0$.

Remark 1: Equation (15) may be solved for matrices K_d^i if the dimension of the state vector is not larger than that of the control input vector and $H_i, i = 1,2,\cdots,q$, is nonsingular, which is unusual even in LTI systems. To avoid such awkwardness, various techniques for finding the approximate solution to (15) have been developed for LTI cases [11–15]. Notice that Joo *et al.* [3] and Chang *et al.* [6] applied one of them to each subsystem in T-S fuzzy systems. Unfortunately, such methods only allow the local state matching. In addition, it should be emphasized that the equalities (15) are hardly fulfilled in many cases of fuzzy-model-based control, since each variable K_d^i should satisfy q different matrix equality constraints (15). Besides, it is commonly believed that the stability analysis of the sampled-data T-S fuzzy system is difficult to directly examine because of the hybridism of the system state.

Problem 1: Given a well-constructed gain matrices K_c^i for the stabilizing analog fuzzy-model-based controller (7), find gain matrices $K_d^i, i = 1,\cdots,q$, for the digital fuzzy-model-based control law (11) such that the following constraints are satisfied:

i) Minimize γ subject to $\left\|\Phi_{ij} - G_i - H_i K_d^j\right\| < \gamma$, $i,j = 1,2,\ldots,q$, in the sense of the induced 2-norm measure.
ii) The discretized closed-loop system (13) is globally asymptotically stable in the sense of Lyapunov criterion.

Theorem 2: If there exist symmetric positive definite matrix Q, symmetric positive semi-definite matrix O, constant matrices F_i and a possibly small positive scalar γ such that the following generalized eigenvalue problem (GEVP) has solutions

$$\underset{Q,O,F_i}{Minimize} \quad \gamma \quad \text{subject to}$$

$$\begin{bmatrix} -\gamma Q & \star \\ \Phi_{ij}Q - G_iQ - H_iF_j & -\gamma I \end{bmatrix} < 0, \quad (16)$$

$$\begin{bmatrix} -Q + (q-1)O & \star \\ G_iQ + H_iF_i & -Q \end{bmatrix} < 0, \qquad i,j = 1,2,\ldots,q. \quad (17)$$

$$\begin{bmatrix} -Q - O & \star \\ \frac{G_iQ + H_iF_j + G_jQ + H_jF_i}{2} & -Q \end{bmatrix} < 0, \quad (18)$$

$$i = 1,\ldots,q-1,\ j = i+1,\ldots,q.$$

then, the state $x_d(kT)$ of the discretized version (13) of the T-S fuzzy system (9) controlled via the redesigned digital fuzzy-model-based controller (11) closely matches the state $x_c(kT)$ of the discretized version of the analogously controlled T-S fuzzy system (14). Furthermore, the discretized T-S fuzzy system (13) is globally asymptotically stabilizable in the sense of Lyapunov stability criterion, where $\star$ denotes the transposed element in symmetric positions.

Proof: See [5]. ■

4. PWM Digital Controller Design

The PWM digital controller can be developed using the aforementioned digitally redesigned controller as follows.

Rewrite the digital control system in an alternative form as

$$\dot{x}_d(t) = \sum_{i=1}^{q}\theta_i(z(t))\left(A_i x_d(t) + \sum_{h=1}^{m}B_i^{(h)}u_d^{(h)}(kT)\right) \quad (19)$$

where $B_i^{(h)}$ is the hth column of B_i and $u_d^{(h)}(kT)$ is the hth component of the digitally redesigned control input vector $u_d(kT)$. The corresponding discrete-time model is

$$\dot{x}_d(kT+T) = \sum_{i=1}^{q} \theta_i(z(kT)) \Big(G_i x_d(t) + \sum_{h=1}^{m} H_i^{(h)} u_d^{(h)}(kT) \Big) \tag{20}$$

where $H_i^{(h)} = (G_i - I)A_i^{-1}B_i^{(h)}$.
Now we consider a PWM controlled T-S fuzzy system be represented as

$$\dot{x}_d(t) = \sum_{i=1}^{q} \theta_i(z(t)) \Big(A_i x_d(t) + \sum_{h=1}^{m} B_i^{(h)} u_{PWM}^{(h)}(kT) \Big) \tag{21}$$

Generally, the PWM control law is mathematically represented by

$$u_{PWM}^{(h)}(t) = \begin{cases} 0, & \text{for } t \in [kT, kT + \tau_k^{(h)}) \\ \mathrm{sgn}(u_d) u_M^{(h)}, & \text{for } t \in [kT + \tau_k^{(h)}, kT + \tau_k^{(h)} + \delta_k^{(h)}) \\ 0, & \text{for } t \in [kT + \tau_k^{(h)} + \delta_k^{(h)}, kT + T) \end{cases} \tag{22}$$

where $\delta_k^{(h)}$ is the firing duration of the predetermined constant control input $u_M^{(h)}$ in the time interval $[kT, kT+T)$ and $\tau_k^{(h)}$ is the firing delay.
One easy way to design the PWM controller is to determine the firing duration $\tau_k^{(h)}$ so that the integration of control input respect to time is the same. This conversion has been widely used in industries for many years. In 1960, R. E. Andeen proved in his paper "The principle of equivalent areas" [20] the validity of this conversion under the assumption that the sampling period is suitably small.

Theorem 3: From the digitally redesigned control input $u_d(t)$, the multi input PWM control input u_{PWM} can be redesigned with the following firing duration $\delta_k^{(h)}$ and the delay $\tau_k^{(h)}$ as follows:

$$\delta_k^{(h)} = T \frac{u_d^{(h)}(kT)}{u_M^{(h)}}, \tau_k^{(h)} = \frac{1}{2}(T - \delta^{(h)})$$

Proof: Consider the following discrete system:

$$\dot{x}_d(kT+T) = \sum_{i=1}^{q} \theta_i(z(kT)) \Big(G_i x_d(t) + \sum_{h=1}^{m} H_i^{(h)} u_d^{(h)}(kT) \Big) \tag{23}$$

The discretized system of PWM control system is

$$\begin{aligned} \dot{x}_d(kT+T) = & \sum_{i=1}^{q} \theta_i(z(kT)) \Big(G_i x_d(kT) \\ & + \sum_{h=1}^{m} \int_{kT+\tau_k^{(h)}}^{kT+\tau_k^{(h)}+\delta_k^{(h)}} e^{A_i(kT+T-\lambda)} B_i^{(h)} u_{PWM}^{(h)}(\lambda) d\lambda \Big) \\ = & \sum_{i=1}^{q} \theta_i(z(kT)) \Big(G_i x_d(kT) + \sum_{h=1}^{m} H_{PWM_i}^{(h)} u_M^{(h)} \Big) \end{aligned} \tag{24}$$

where $H_{PWM_i}^{(h)} = e^{A_i(T-\tau_k^{(h)}-\delta_k^{(h)})}(e^{A_i\delta_k^{(h)}} - I)A_i^{-1}B_i^{(h)}$. To match the state $x_d(t)$ controlled by the digitally redesigned controller and the state $x_d(t)$ controlled by the PWM controller, we let

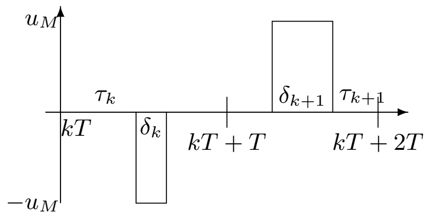

Fig. 1. typical PWM control input

$$H_i^{(h)} u_d^{(h)}(kT) = H_{PWM_i}^{(h)} u_{PWM}^{(h)}(t)$$

Hence $(G_i - I)A_i^{-1}B_i^{(h)}u_d^{(h)}(kT) = e^{A_i(T-\tau_k^{(h)}-\delta_k^{(h)})}(e^{A_i\delta_k^{(h)}} - I)A_i^{-1}B_i^{(h)}u_M^{(h)}$. Taking the second-order Taylor series expansion of the matrix exponential function, we have

$$T\Big(I + \frac{1}{2}A_i T\Big)u_d^{(h)}(kT) \approx \delta_k^{(h)}\Big(I + \frac{1}{2}A_i(2T - 2\tau_k^{(h)-\delta_k^{(h)}}\Big)u_M^{(h)}$$

Solving this yields

$$\delta_k^{(h)} = T\frac{u_d^{(h)}(kT)}{u_M^{(h)}}$$

and

$$\tau_k^{(h)} = \frac{1}{2}(T - \delta^{(h)})$$

The proof is completed. ■

Figure 4shows a typical PWM control input signal.

5. Attitude Control of 3-Axis Artificial Satellites

The nonlinear equations of motion in terms of components along the body-fixed control axes can be written as follows:

$$I\dot{\omega} + \omega \times I\omega = M$$

In the rigid body axis frame is coincident with the principal-axis reference frame, the above dynamic equation can be written as

$$\dot{\omega}_x = \frac{(I_y - I_z)}{I_x}\omega_y\omega_z + \frac{M_x}{I_x} \tag{25}$$

$$\dot{\omega}_y = \frac{(I_z - I_x)}{I_y}\omega_z\omega_x + \frac{M_y}{I_y} \tag{26}$$

$$\dot{\omega}_z = \frac{(I_x - I_y)}{I_z}\omega_x\omega_y + \frac{M_z}{I_z} \tag{27}$$

First, in order to construct the T-S fuzzy system, the nonlinear term $\omega_z\omega_x$ should be expressed as a convex sum of the state as follows:

$$\begin{aligned} \omega_z\omega_x &= \Gamma_1^1(\omega_x)\omega_z + \Gamma_1^2(\omega_x)\cdot\alpha_1\omega_z \\ 1 &= \Gamma_1^1(\omega_x) + \Gamma_1^2(\omega_x) \end{aligned}$$

Solving these we have

$$\begin{cases} \Gamma_1^1(\omega_x) = \frac{\omega_x - \alpha_1}{1-\alpha_1} \\ \Gamma_1^2(\omega_x) = \frac{1-\omega_x}{1-\alpha_1} \end{cases} \tag{28}$$

Table 1. Simulation parameters

parameters	value	unit
I_x	3668.0	kgm^2
I_y	970.0	kgm^2
I_z	3156.0	kgm^2

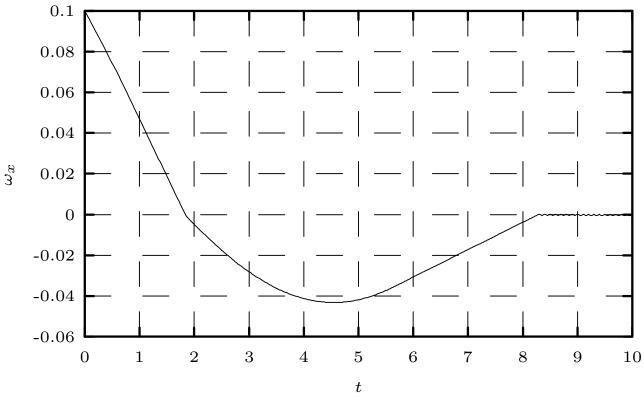

Fig. 2. x-axis angle

Using similar procedure with the other nonlinear terms, the T-S fuzzy system of the above nonlinear equations can be obtained as follows:

$$
\begin{aligned}
R^1 &: \text{IF } \omega_x \text{ is about } \Gamma_1^1, \text{ and } \omega_y \text{ is about } \Gamma_2^1, \\
&\quad \text{THEN } \dot{\omega} = A_1\omega + B_1 M \\
R^2 &: \text{IF } \omega_x \text{ is about } \Gamma_1^2, \text{ and } \omega_y \text{ is about } \Gamma_2^1, \\
&\quad \text{THEN } \dot{\omega} = A_2\omega + B_2 M \\
R^3 &: \text{IF } \omega_x \text{ is about } \Gamma_1^1, \text{ and } \omega_y \text{ is about } \Gamma_2^2, \\
&\quad \text{THEN } \dot{\omega} = A_3\omega + B_3 M \\
R^4 &: \text{IF } \omega_x \text{ is about } \Gamma_1^2, \text{ and } \omega_y \text{ is about } \Gamma_2^2, \\
&\quad \text{THEN } \dot{\omega} = A_4\omega + B_4 M
\end{aligned}
$$

where

$$
A_1 = \begin{bmatrix} 0 & 0 & \frac{(I_y - I_z)}{I_x} \\ 0 & 0 & \frac{(I_z - I_x)}{I_y} \\ \frac{(I_x - I_y)}{I_z} & 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 & \alpha_1 \frac{(I_y - I_z)}{I_x} \\ 0 & 0 & \frac{(I_z - I_x)}{I_y} \\ \alpha_1 \frac{(I_x - I_y)}{I_z} & 0 & 0 \end{bmatrix},
$$

$$
A_3 = \begin{bmatrix} 0 & 0 & \frac{(I_y - I_z)}{I_x} \\ 0 & 0 & \alpha_2 \frac{(I_z - I_x)}{I_y} \\ \frac{(I_x - I_y)}{I_z} & 0 & 0 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 0 & \alpha_1 \frac{(I_y - I_z)}{I_x} \\ 0 & 0 & \alpha_2 \frac{(I_z - I_x)}{I_y} \\ \alpha_1 \frac{(I_x - I_y)}{I_z} & 0 & 0 \end{bmatrix}
$$

and $B_1 = B_2 = B_3 = B_4 = \text{diag}\left\{\frac{1}{I_x}, \frac{1}{I_y}, \frac{1}{I_z}\right\}$. The simulation parameters are given in Table 5.[**?**]. Simulation results are shown in Figs. 2–7. As are shown in the figures, all state variables are well controlled.

6. Conclusions

This paper has discussed the PWM controller design method using intelligent digital redesign. The PWM controller is converted from the digitally redesigned controller using the equivalent area principle. An example-attitude control of artificial satellite shows the effectiveness of the proposed method.

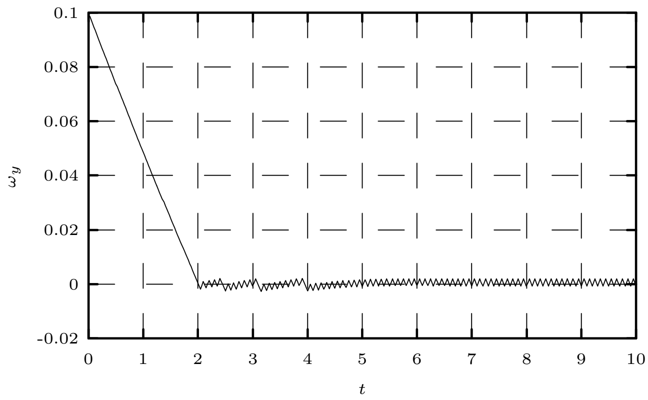

Fig. 3. y-axis angle

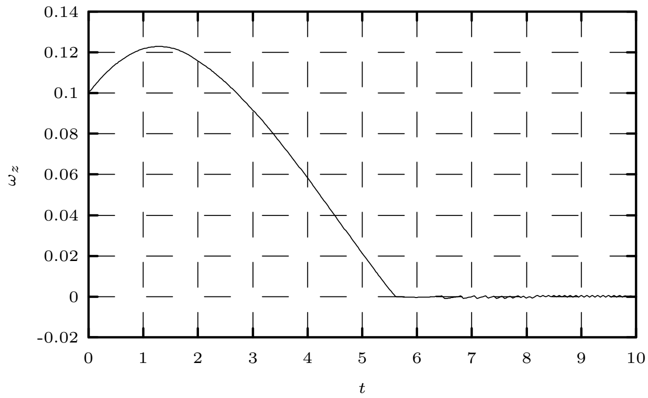

Fig. 4. z-axis angle

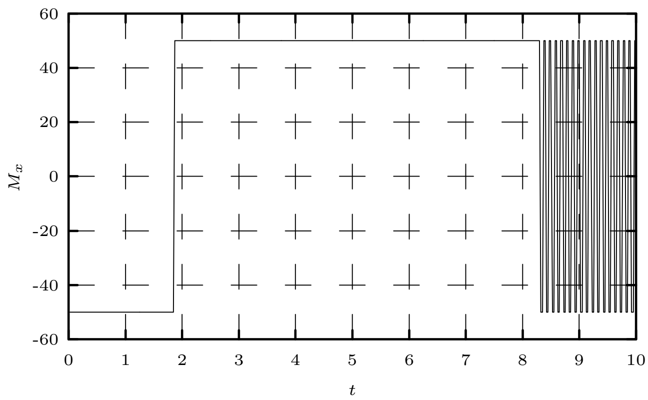

Fig. 5. x-axis torque

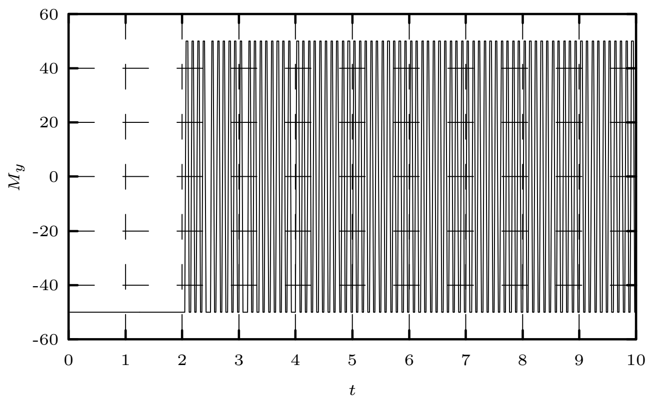

Fig. 6. y-axis torque

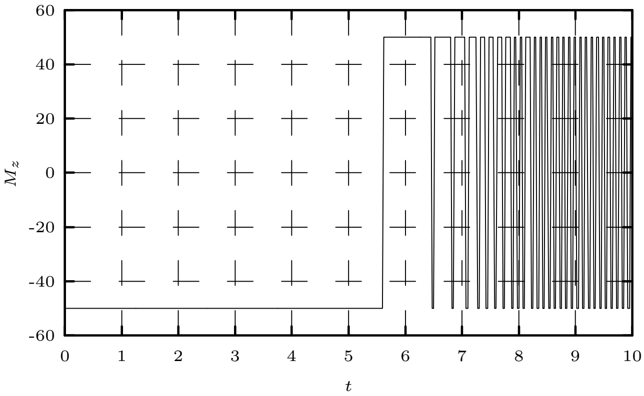

Fig. 7. z-axis torque

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