Fuzzy Sliding Mode Control for Uncertain Nonlinear Systems Using Fuzzy Models

SamJun Seo*, DongSik Kim**

*Department of Electrical & Electronic Engineering, Anyang University, Korea
(Tel : +82-31-467-0874; E-mail:ssj@aycc.anyang.ac.kr)
**Division of Information and Technology Engineering, Soonchunhyang University, Korea
(Tel : +81-41-530-1370; E-mail:dongsik@sch.ac.kr)

Abstract: Fuzzy sliding mode controller for a class of uncertain nonlinear dynamical systems is proposed and analyzed. The controller's construction and its analysis involve sliding modes. The proposed controller consists of two components. Sliding mode component is employed to eliminate the effects of disturbances, while a fuzzy model component equipped with an adaptation mechanism reduces modeling uncertainties by approximating model uncertainties. To demonstrate its performance, the proposed control algorithm is applied to an inverted pendulum. The results show that both alleviation of chattering and performance are achieved.

Keywords: fuzzy model, nonlinear system, fuzzy sliding mode control, inverted pendulum

1. INTRODUCTION

Variable structure control systems constitute a class of nonlinear feedback control systems whose structure changes depending on the state of the system. Although, neither structure is necessarily stable, their combination results in a sliding mode, that is, the system trajectory slides along a sliding surface. Variable structure control can be applied very well in the presence of model uncertainties, parameter fluctuations and disturbances provided that the upper bounds of their absolute values are known. The sliding mode control is especially appropriate for the tracking control of robot manipulators and also for motors whose mechanical loads change over a wide range.[1]

Despite the benefits of VSC control, this suffers from two major shortcomings. First, the insensitivity property of a VSC system is present only when the system is in the sliding mode. The second shortcoming is control chattering. To overcome the first disadvantage , that is to reduce the reaching time, the use of high gain control signal was suggested[2]. On the other hand, a time-varing switching surface was suggested in order to eliminate the reaching phase, where the initial tracking control errors were assumed to be zero[3]. However, this assumption rules out many practical situations in which the system initial conditions are located arbitrarily.

The problem for chattering has been addressed by many researchers. In [3], the discontinuous control is approximated inside a boundary layer located around the switching surface. However, although chattering can be reduced, robustness and tracking accuracy are compromised.

The fuzzy logic controller, based on Zadeh's fuzzy set theory has firstly been developed by Mamdani and his coworkers about twenty years ago, and has successfully been applied to many commercial products and industrial systems.[4][5] The main advantages of fuzzy logic controller are it can control the complex ill-defined systems by converting the linguistic control strategy of operators' experience or experts' knowledge into an automatic control strategy without knowing the mathematical model of the controlled systems.

However, at present there still have some problems in the design of fuzzy logic controllers. Control rules that are the most important factor in FLC are generally obtained from intuition and experience of the experts, and such rules represented by linguistic rule sets or fuzzy relation. But there is always something difficult to obtain from such control rules and this makes the design of the controller difficult and the response trajectory of the controlled system is unpredictable.

Recently, fuzzy logic control has emerged as a paradigm of intelligent control capable of dealing with complex and ill-defined system[6]. New results have been made recently to identify the connection between fuzzy logic and variable structure control[7,8,9]. It has been shown that fuzzy logic control is a general form of variable structure control. This connection has suggested the integration of the two control approaches in control system design applications[8,9,10].

In this paper, we will introduce a fuzzy sliding mode controller(FMSC) which is designed by the techniques of both fuzzy logic controller(FLC) and variable structure controller(VSC). In the conventional VSC, when the sliding mode occurs, the state trajectories of the control system will kept on a prespecified switching hyperplane. In other words, the controlled system has insensitive properties to uncertainties of the process. However, owing to the sampling action of digital implementation and delay of switch device, a realized VSC may have chattering. But in our design method the chattering phenomenon will be attenuated.

This paper is organized as follows. Section 2 gives some background on fuzzy sliding mode control. Section 3 presents the new fuzzy sliding mode control algorithm, In section 4, an application of the above results to an inverted pendulum illustrated. Finally, section 5 presents some conclusions

2. BACKGROUNS

2.1 Basic concepts of VSC

Let a nonlinear dynamic system be represented by the state equation:

$$\dot{x}(t) = f(x,t) + g(x,t)u \tag{1}$$

where $x \in R^n$ is the state vector of the system, $f, g : R^+ \times R^n$ are vector fields, $u \in R$ is the control input to the system. The VSC design consists in achieving the following steps:

1) Design a switching manifold S in the state space to represent a desired system dynamics, which is of lower order than the dimension of the given plant; S is defined by

$$S = \{x \in R^n | s(x) = 0\} \quad (2)$$

2) Design a variable structure control

$$u(t) = \begin{cases} u^+(x) & when \ \ s(x) > 0 \\ u^-(x) & when \ \ s(x) < 0 \end{cases} \quad (3)$$

such that any state x out side the switching surface is driven to reach this surface in finite time, that is the condition $s(x) = 0$ is satisfied in finite time.

To specify the reaching condition a Lyapunov function is used. Let the Lyapunov function candidate be defined as:

$$V(x,t) = \frac{1}{2} s^2(x) \quad (4)$$

where $s(x)$ is the switching surface and $x \in R^n$ is the state vector of the system. Then the reaching condition for existence of the sliding mode motion of the system under consideration is given as follows:

$$\dot{V}(x,t) = s(x)\dot{s}(x) < 0 \quad for \ \ x \in R^n - S \quad (5)$$

The control of the dynamics of a VSC system in the reaching mode may be made possible by specifying the dynamics of the switching function $s(x)$. More specifically, the dynamics of the switching function $s(x)$ are described by a differential equation of the form:

$$\dot{s}(x) = -K \,\text{sgn}(s) \quad K > 0 \quad (6)$$

Note that we no longer to verify the reaching condition because it is inherent in the differential equation of the function $s(x)$. By specifying the dynamics of the function $s(x)$, we can predetermine the speed with which the system state approaches the switching manifold.

After choosing the dynamics of the reaching mode, we now determine the associated control law. Differentiating $s(x)$ with respect to time along the trajectory of (1) gives

$$\dot{s}(x) = \frac{\partial s}{\partial x} f(x) + \frac{\partial s}{\partial x} g(x) u = -K \,\text{sgn}(x) \quad (7)$$

Solving (7) for the control law gives

$$u = -\left[\frac{\partial s}{\partial x} g(x)\right]^{-1} \left[\frac{\partial s}{\partial x} f(x) + K \,\text{sgn}(x)\right] = u_{eq} + u_s \quad (8)$$

where the existence of the inverse of the matrix $\left[\frac{\partial s}{\partial x} g(x)\right]^{-1}$ is a necessary condition.

2.2 Fuzzy Sets and Fuzzy Logic

A fuzzy set is a generation of the classical notion of a set. Whilst the characteristic function of a classical set can take values of either 0 or 1, which means that an object either belongs to or does not belong to a given set, the characteristic function(called membership function in fuzzy set theory) of a fuzzy set can take on values in the interval [0, 1].

Approximate reasoning is one of the most important concepts of fuzzy logic. It represents inference rules whose premises contain fuzzy propositions, Unlike inference in classical logic, in its computation inference, approximation reasoning uses fuzzy set which represents the meaning of a collection of fuzzy propositions. For instance, let membership function μ_A and μ_R represent the meaning of a fuzzy proposition " x is A " and meaning of an if-than fuzzy rule " x is A then y is B ", respectively, where A and B are fuzzy sets. Then, one can computer the membership function representing the meaning of the conclusion " y is B ". Now consider a collection of L fuzzy if-then rules:

$$Rule^{(l)}: \text{if } x \text{ is } A^{(l)} \text{ then } y \text{ is } B^{(l)}, \ \ l = 1, \cdots, L \quad (9)$$

where $A^{(l)}$ and $B^{(l)}$, $l = 1, \cdots, L$, are fuzzy sets of the variable *x*, then the fuzzy set *B* representing y resulting from the firing of the fuzzy rules is given by []:

$$\mu_B(y) = \max_l \min(\mu_{A^{(l)}}(x^*), \mu_{B^{(l)}}(y)) \ \ \forall y \quad (10)$$

To determine the corresponding crisp value of *y*, a defuzzification procedure is applied to the inferred fuzzy set *B*. One of the most used defuzzification scheme is the center-of –area method. Applying center-of-area method to (10) yields:

$$y^* = \frac{\sum_{l=1}^{L} y_l \min(\mu_{A^{(l)}}(x^*), \mu_{B^{(l)}}(y))}{\sum_{l=1}^{L} \min(\mu_{A^{(l)}}(x^*), \mu_{B^{(l)}}(y))} \quad (11)$$

where y_l represents a crisp value for which the membership function $\mu_{B^{(l)}}$ reaches its maximum, $l = 1, \cdots, L$.

2.3 Takagi-Sugeno-Kang fuzzy model

One can construct a Takagi-Sugeno-Kang fuzzy model if local description of the dynamical system to be controlled is available in the terms of linear models.

$$\dot{x}(t) = A_i x(t) + B_i u(t), \qquad i = 1, 2, \cdots, r \quad (12)$$

where the state vector is $x(t) \in \mathrm{R}^{\mathrm{n}}$, the control input is $u(t) \in \mathrm{R}^{\mathrm{m}}$, and the matrices A_i and B_i are of appropriate dimensions. The above information is then fused with available IF-THEN rules where the *i*th rule can have the form

$$\text{Rule}\, i: \ \text{IF } x_1(t) \text{ is } \mathrm{F}_1^i \text{ AND } \cdots \text{ AND } x_n(t) \text{ is } \mathrm{F}_n^i$$
$$\text{THEN } \dot{x}(t) = A_i x(t) + B_i u(t)$$

where F_j^i, $j = 1, 2, \cdots, \mathrm{n}$, is the *j*th fuzzy set of the *i*th rule. Let $\mu_j^i(x_j)$ be the membership function of the fuzzy set F_j^i and let

$$w^i = w^i(x) = \prod_{j=1}^{\mathrm{n}} \mu_j^i(x_j) \quad (13)$$

Then, given a pair ($x(t), u(t)$), the resulting fuzzy system model is inferred as the weighted average of the local models and has the form

$$\begin{aligned} \dot{x} &= \frac{\sum_{i=1}^{r} w^i (A_i x + B_i u)}{\sum_{i=1}^{r} w^i} \\ &= \sum_{i=1}^{r} \alpha_i (A_i x + B_i u) \\ &= \left(\sum_{i=1}^{r} \alpha_i A_i\right) x + \left(\sum_{i=1}^{r} \alpha_i B_i\right) u \\ &= \mathrm{A}(\alpha) x + B(\alpha) u \end{aligned} \quad (14)$$

where for $i = 1, 2, \cdots, r$

$$\alpha_i = \frac{w^i}{\sum_{i=1}^{r} w^i}.$$

Note that for $i = 1, 2, \cdots, r$

$$\alpha_i \geq 0, \qquad \sum_{i=1}^{r} \alpha_i = 1$$

and

$$\alpha = [\alpha_1 \;\; \alpha_2 \;\; \cdots \;\; \alpha_r]^T \in [0,1]^r$$

Tanaka showed that the fuzzy model (14) can also be used to model a class of feedforward neural networks, as well as to construct fuzzy neural controller

3. FUZZY SLIDING MODE CONTROL

3.1 Fuzzy sliding mode Control using fuzzy models

In [12], Tanaka proposed a Lypunov-based method for constructing globally stabilizing state feedback controller for the fuzzy system model given (15)

$$\dot{x}(t) = A_i x(t) + B_i u(t) + f + B_i h \tag{15}$$

we assume that

$$\begin{aligned} \|f\| &= \|f(\alpha^i, x)\| \le \alpha_f \|x\| \\ \|h\| &= \|h(\alpha^i, x, u)\| \le \beta_x \|x\| + \beta_u \|u\| \end{aligned} \tag{16}$$

where α_f β_x and β_u are known nonnegative constants. We further assume that the matrix A_i is asymptotically stable. If this is not the case, we break u_{eq} into two parts $u_{eq} = u_1 + u_2$ and begin our design with $u_1 = -K_1 x$ such that the matrix $A_i - B_i K_1$ is asymptotically stable. Then our goal is to construct a linear state feedback controller u_{eq} that would make the closed loop system globally asymptotically stable for arbitrary *f* and *h* that satisfy the norm bounds given by (16).

Theorem 1: Suppose that A_i is asymptotically stable and $P = P^T > 0$ is the solution to the Lyapunov matrix equation $A_i^T P + PA_i = -2Q$ for some $Q = Q^T > 0$. Suppose also that $\alpha_f < \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$, $\beta_u < 1$.

Then the equivalent control input $u_2 = -\gamma B_i^T P x$, where

$$\gamma > \frac{\beta_x^2}{4(\lambda_{\min}(Q) - \alpha_f \lambda_{\max}(P))(1-\beta_u)} \tag{17}$$

globally asymptotically stabilizes the uncertain system for arbitrary f and h that satisfy the norm bounds (16).

3.2 Control gain *K*

In this section we present a solution to the sliding mode control design using concepts from fuzzy set theory. In the design of switching control u_s, the control gain *K* has to be selected according to the following rules. At each instant time, *t*, $s(x,t)$ is the algebraic value of the switching function *s*. Then these rules are:

Rule 1: IF $|s(x(t))|$ is *SL* THEN $K(t) = K_L$

Rule 2: IF $|s(x(t))|$ is *SM* THEN $K(t) = K_M$

Rule 3: IF $|s(x(t))|$ is *SS* THEN $K(t) = K_S$

Rule 4: IF $|s(x(t))|$ is *SZ* THEN $K(t) = K_Z$

where *SL*, *SM*, *SS* and *SZ* are fuzzy sets of the variable $|s(x,t)|$. K_L, K_M, K_S and K_Z are different values of the control gain corresponding to a large, medium, small and zero gain value, respectively.

Given the value $|s(x,t)|$ at the instant time, *t*, the value of the control gain *K* at the time *t* is inferred using the above four fuzzy if then rules following procedure.

$$K(t) = \frac{\mu_{SL} K_L + \mu_{SM} K_M + \mu_{SS} K_S + \mu_{SZ} K_Z}{\mu_{SL} + \mu_{SM} + \mu_{SS} + \mu_{SZ}} \tag{18}$$

This strategy of selecting the variable control gain *K(t)* has following advantages over choosing a fixed control gain *K*:

1) A large control gain is applied only when the system state is far away from the sliding mode.
2) When the system state is close to the sliding manifold, a small control gain used.

3.3 Modification of the rules

Control rules that are the most important factor in FLC are generally obtained from intuition and experience of the experts, and such rules represented by linguistic rule sets or fuzzy relation. But there is always something difficult to obtain from such control rules and this makes the design of the controller difficult. Here, a self-organizing fuzzy sliding mode control algorithm using gradient descent method is proposed.

The learning algorithm is to modify the consequent parameter such that the system trajectory stays on the sliding manifold. If the sliding condition, $s\dot{s} < 0$ is, is satisfied, the switching function will converge to zero, leading to the desired dynamics. According to the sliding condition, the consequent parameters should be adjusted in the direction that minimizes the value $s\dot{s}$. These consequent parameters are adjusted to reduced $s\dot{s}$ means that the controller is tuned to satisfy the sliding condition and consequently have a sliding behavior.

According to the gradient descent method, the parameters are updated from

$$\dot{K}_j = -\Gamma \frac{\partial s(t)\dot{s}(t)}{\partial K_j(t)} \tag{19}$$

in which Γ is the learning gain. By the chain rule, (19) becomes

$$\dot{K}_j = -\Gamma \frac{\partial s(t)\dot{s}(t)}{\partial u(t)} \frac{\partial u(t)}{\partial K_j(t)} \tag{20}$$

As u_{eq} depends on current state variable only, $\frac{\partial u_{eq}}{\partial K_j} = 0$.

Therefore we have

$$\dot{K}_j = -\Gamma |s| \left[\frac{\partial s}{\partial x} g(x) \right]^{-1} \frac{\partial u_s(t)}{\partial K_j(t)} \tag{21}$$

Substituting (18) into (21), we obtain the following learning law for consequent parameters :

$$\dot{K}_j(t) = -\gamma_k |s| \frac{w_j(t)}{\sum_{i=1}^{m} w_i(t)} \tag{22}$$

In summary, the proposed sliding mode control algorithm using self learning fuzzy logic consists of following steps:

1) Design a switching manifold *S* in the state space represents desired system dynamics as in classical VSC.

2) Compute the control law u_{eq}

a) Stabilize A_i if necessary. Calculate $u_1 = -K_1 x$ so that the matrix $A_i - B_i K_1$ has its eigenvalues in the desired location.

b) Solve the Lyapunov equation $A_i^T P + PA_i = -2Q$ for some $Q = Q^T > 0$. Choosing $Q = I_n$ maximizes the ratio $\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$. Check if $\alpha_f < \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$.

c) Construct the resulting equivalent control

$u_{eq} = u_1 + u_2$.

3) Update the control gain K_j using learning algorithm (22)

4) Compute the control law u_s

4. COMPUTER SIMULATIONS

We apply the stabilization algorithm given in the previous section to balance an inverted pendulum mounted on a cart. The equations of motion for the pendulum are

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{g\sin(x_1) - amlx_2^2 \sin(2x_1)/2 - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)} \quad (23)$$

where x_1 is the angle of the pendulum from the vertical line, x_2 is the angular velocity of the pendulum, u is the control force applied to the cart, and $g = 9.8m/s^2$ is the magnitude of the acceleration due to gravity.

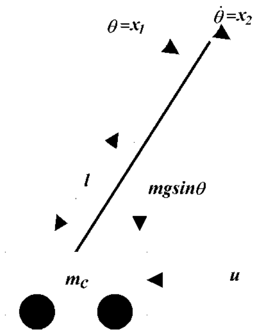

Fig.1 The inverted pendulum system

Numerical values of the parameters are given in Table 1.

Table 1 The parameters of the inverted pendulum

	symbol	Value
Acceleration of gravity	g	$9.8m/s^2$
Mass of cart	m_c	1kg
Mass of pole	m	0.5kg
Length of pole	l	0.5m

The control objective is to maintain $x_1 = 0$ and $x_2 = 0$. In the design example we choose the switching surface given by

$$s(x) = 10x_1 + x_2 = 0 \quad (24)$$

In order to find the equivalent control input u_{eq} we consider modeling equation (14) as our truth model; that is, we are confident that these equations adequately represent the behavior of the system to be controlled. We use the truth model in our computer simulations to evaluate the performance of the design. We design controllers using a design model, which can sometimes be obtained by simplifying the truth model. In our examples, we assume that, by using insight and experience we obtained the following two rules describing the plant dynamics:

Rule 1: IF x_1 is about 0

THEN $\dot{x}(t) = A_1 x(t) + B_1 u(t)$

Rule 2: IF x_1 is about $\pm\pi/4$

THEN $\dot{x}(t) = A_2 x(t) + B_2 u(t)$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ \frac{g}{4l/3 - aml} & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ -\frac{a}{4l/3 - aml} \end{bmatrix}$$

and

$$B_1 = \begin{bmatrix} 0 \\ -\frac{a\cos(x_1)}{4l/3 - mal\cos^2(x_1)} \end{bmatrix}_{x_1 = \pm\pi/4} .$$

We use membership function of the form

$$w_1(x_1) = \frac{1 - 1/(1 + \exp(-7(x_1 - \pi/8)))}{(1 + \exp(-7(x_1 + \pi/8)))}$$

$$w_2(x_1) = 1 - w_1(x_1)$$

The corresponding fuzzy system model is

$$\dot{x} = (\alpha_1 A_1 + \alpha_2 A_2)x + (\alpha_1 B_1 + \alpha_2 B_2)u \quad (25)$$

where $\alpha_i = w_i, i = 1, 2$, because $w_1 + w_2 = 1$ for all t. We then proceed to construct a stabilizing linear controller according to the algorithm given in the previous section. We first represent model (25) as

$$\dot{x} = (A_1 + \alpha_2(A_2 - A_1))x + (B_1 + \alpha_2(B_2 - B_1))u \quad (26)$$

The uncertain elements of system model (26) satisfy the matching condition. Hence, we can take $\beta_u = 0.9703$ because $\alpha_2 \in [0,1]$ for all t whenever $|x_1| \le \pi/2$. Suppose that the desired eigenvalues of $A_1 - b_1 k_1$ are $\{-2,-2\}$. For this choice of desired eigenvalues we find $k_1 = [-120.67 \ \text{-}22.67]$. Solving the Lyapunov equation $(A_1 - b_1 k_1)^T P + P(A_1 - b_1 k_1) = -2I_2$ yields

$$P = \begin{bmatrix} 2.25 & 0.25 \\ 0.25 & 0.3125 \end{bmatrix}$$

We find $\tilde{\gamma} = \beta_x^2 / (4\lambda_{\min}(Q)(1 - \beta_u)) = 47910$. Let $\gamma = 50000$. Then the equivalent control input is

$$u_{eq} = -(k_1 + \gamma b_1^T P)x = [2325.7 \ \ 2777.7]x \quad (27)$$

The learning laws are given by

$$\dot{K}_j(t) = -\gamma_k |s| \frac{w_j(t)}{\sum_{i=1}^{m} w_i(t)} \quad j = L, M, S, Z \quad (26)$$

For traditional sliding mode control the control gain K was kept constant and equal to 5. For the proposed control algorithm, the membership functions of the fuzzy subsets *SZ*, *SS*, *SM* and *SL* are shown in Figure 2.

The simulation results of the application of the proposed control design approach are depicted in Figure 3. The learning gain γ_k was 20. It is clear from the simulation results that the proposed control approach reduces the chattering while maintaining a very small tracking error.

5. CONCLUSIONS

In this paper, fuzzy sliding mode controller for a class of uncertain nonlinear dynamical systems is proposed and analyzed. The controller's construction and its analysis involve sliding modes. The proposed controller consists of two components. Sliding mode component is employed to eliminate the effects of disturbances, while a fuzzy model component equipped with an adaptation mechanism reduces modeling uncertainties by approximating model uncertainties. To demonstrate its performance, the proposed control algorithm is applied to an inverted pendulum. The results show that both alleviation of chattering and performance are achieved.

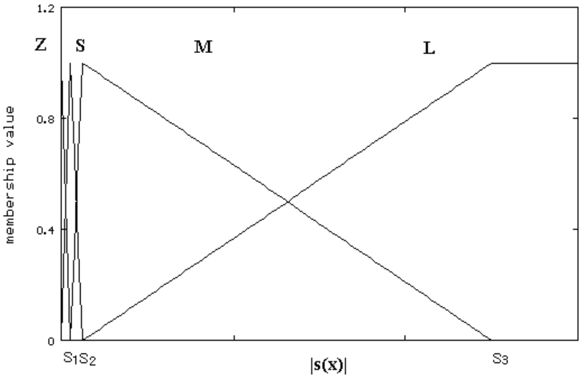

Fig.2 The membership functions of $|s(x)|$

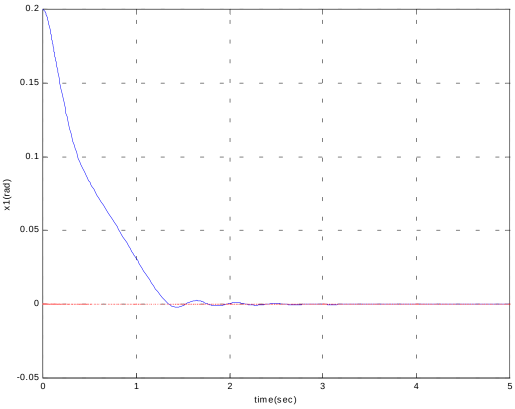

Fig.3 Angular displacement of pole

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