

Local Obstacle Avoidance of Nonholonomic Wheeled Mobile Robots in Trajectory Tracking

Young Ho Lee* and Jong Hyeon Park**

* Department of Precision Mechanical Engineering, Hanyang University, Seoul, Korea
(Tel : +82-2-2297-3786; E-mail: windkebi@ihanyang.ac.kr)

**School of Mechanical Engineering, Hanyang University, Seoul, Korea
(Tel : +82-2-2290-0435; E-mail: jongpark@hanyang.ac.kr)

Abstract: In this paper, we propose an obstacle avoidance technique in trajectory tracking of nonholonomic wheeled mobile robots. Input-output linearized backstepping controller is used in trajectory tracking, and repulsive type control input for obstacle avoidance is added to it. The added input is generated by fuzzy logic. And we do not add the two inputs directly but combine them via fuzzy logic, which determines the ratings of each input. Some simulations are performed to show that with the proposed algorithm, the mobile robot can track its reference trajectory even if there are multiple obstacles on the trajectory of robot.

Keywords: Nonholonomic mobile robots, Obstacle avoidance, Tracking controller, Fuzzy logic

1. INTRODUCTION

In navigation or wandering task of mobile robots, obstacle avoidance is a very important issue, since collision with any obstacle means that the given task is failed. There are many researches in this field in recent decades. Among them, potential field approach is the most common and widely used method. And neural or fuzzy logic is used to avoid obstacles. Their primary concern is that how the mobile robot can reach its target position or follows its target path. But there are some tasks that the mobile robot must be in a specified position on a specified time like playing soccer. A robot can perform the task via trajectory tracking. So, we study on the obstacle avoidance in trajectory tracking.

Potential field approach is not a proper solution for performing trajectory tracking and obstacle avoidance simultaneously. And in trajectory tracking, the robot must have fast reaction to obstacles, but the velocity control base avoidance has slow reaction, so we use torque base avoidance function. We design the obstacle avoidance function with fuzzy logic, and its output as torque, which is applied to mobile robot. The trajectory tracking is performed via backstepping controller of which output is torque. In some situations, the obstacle avoidance function must have a prior rating than trajectory tracking, so it is need to determine the rating of each control input. The two control inputs, which are generated by tracking controller and obstacle avoidance function, are mixed via fuzzy logic, which determines the rating of each torque input.

In this paper, we design of fuzzy logic to obtain required effect and capability [5]. This fuzzy logic is organized with three sub fuzzy logic and these sub logics execute each operation independently

Finally, the organization of the paper is as follows. Section 2 describes the trajectory tracking controller of the nonholonomic wheeled mobile robot. In section 3, we design structure of fuzzy logic for obstacle avoidance. Section 4 is shown simulation results for static and moving obstacle. also, we performed simulation in case of multiple obstacles.

2. TRAJECTORY TRACKING

This section describes controller for reference trajectory tracking of nonholonomic wheeled mobile robot [1, 6]. A combined kinematic / torque control law of the tracking controller developed using backstepping. The controller based

on computed- torque controller.

Dynamic equation of a nonholonomic mobile robot is describes as follows,

$$\begin{aligned} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{V}_m(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{F}(\dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \boldsymbol{\tau}_d \\ = \mathbf{B}(\mathbf{q})\boldsymbol{\tau} + \mathbf{A}^T(\mathbf{q})\boldsymbol{\lambda} \end{aligned} \quad (1)$$

where $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{n \times n}$ is a symmetric, positive definite inertia matrix, $\mathbf{V}_m(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{n \times n}$ is the centripetal and coriolis matrix, $\mathbf{F}(\dot{\mathbf{q}}) \in \mathbb{R}^{n \times 1}$ is a vector of the surface frictional force, $\mathbf{G}(\mathbf{q}) \in \mathbb{R}^{n \times 1}$ is a vector of the gravitational force, $\boldsymbol{\tau}_d$ is the bounded unknown external disturbance, $\mathbf{B}(\mathbf{q}) \in \mathbb{R}^{m \times r}$ is the input transformation matrix, $\boldsymbol{\tau} \in \mathbb{R}^{m \times 1}$ is the input vector, $\mathbf{A}(\mathbf{q}) \in \mathbb{R}^{m \times n}$ is the matrix associated with the constraints, $\boldsymbol{\lambda} \in \mathbb{R}^{m \times 1}$ is the vector of constraint forces.

$\mathbf{q} = [x \ y \ \theta]^T$ is generalized coordinates.

And the velocity of mobile robot is defined as

$$\mathbf{v} = [\dot{v} \ \dot{\omega}]^T \quad (2)$$

Reference velocities of mobile robot can be presented as

$$\mathbf{v}_r = [v_r \ \omega_r]^T \quad (3)$$

In order to track reference trajectory, the tracking error vector is expressed as

$$\mathbf{e} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \quad (4)$$

where $[x_r \ y_r \ \theta_r]^T$ is the reference coordinates. and proposed also the control velocity as

$$\mathbf{v}_c = \begin{bmatrix} v_r \cos e_3 + k_1 e_1 \\ \omega_r + k_2 v_r e_2 + k_3 v_r \sin e_3 \end{bmatrix} \quad (5)$$

To determine the control input torque that makes the velocity of mobile robot converges to the reference velocity, dynamic equation of the mobile robot is derived as Eq. (6) from Eq. (1), and you can be informed about this equation from reference [1].

$$\bar{\mathbf{M}}(\mathbf{q})\dot{\mathbf{v}} + \bar{\mathbf{V}}_m(\mathbf{q}, \dot{\mathbf{q}})\mathbf{v} + \bar{\mathbf{F}}(\mathbf{v}) = \bar{\mathbf{B}}\boldsymbol{\tau}. \quad (6)$$

Let \mathbf{u} be an auxiliary input, then by applying the nonlinear feedback.

$$\boldsymbol{\tau} = \bar{\mathbf{B}}^{-1}(\mathbf{q})[\bar{\mathbf{M}}(\mathbf{q})\mathbf{u} + \bar{\mathbf{V}}_m(\mathbf{q}, \dot{\mathbf{q}})\mathbf{v} + \bar{\mathbf{F}}(\mathbf{v})]. \quad (7)$$

then the dynamic control can be converted into a kinematic control.

$$\dot{\mathbf{v}} = \mathbf{u}. \quad (8)$$

We determined the auxiliary control input as

$$\mathbf{u} = \dot{\mathbf{v}}_e + \mathbf{k}_4(\mathbf{v}_e - \mathbf{v}). \quad (9)$$

We applied the tracking controller in our research. This tracking controller uses together repulsive torque. And it is main controller of mobile robot. Also, this controller guarantees to be asymptotically stable by Lyapunov theory in [1].

3. OBSTACLE AVOIDANCE

Our goal is obstacle avoidance by controller based on reference trajectory tracking and using repulsive torque. Repulsive torque was generated by function which has information of obstacle position like shown in prior research [2]. Our repulsive torque is formed by fuzzy logic for more flexible performance than prior research. So, the structure of the proposed controller is represented like Fig. 1.

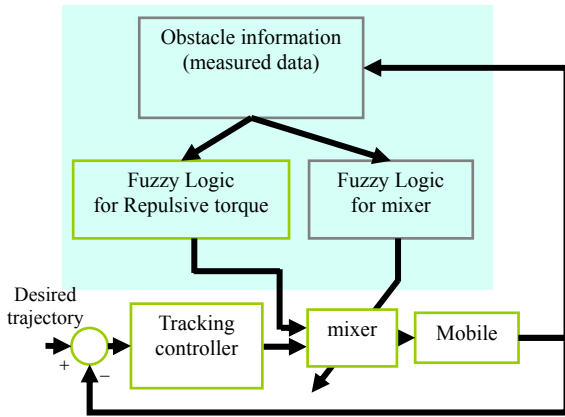


Fig. 1 Control structure.

The fuzzy logic we proposed is divided by three components.

- ✓ Steering control fuzzy logic for orientation.
- ✓ Fuzzy logic for speed control.
- ✓ Fuzzy logic for mixture above two components with control value produced in torque controller of reference trajectory tracking.

Fuzzy logic consisted by these three sub components has more flexibility than previous avoidance algorithm by function due to its structure built by linguistic form.

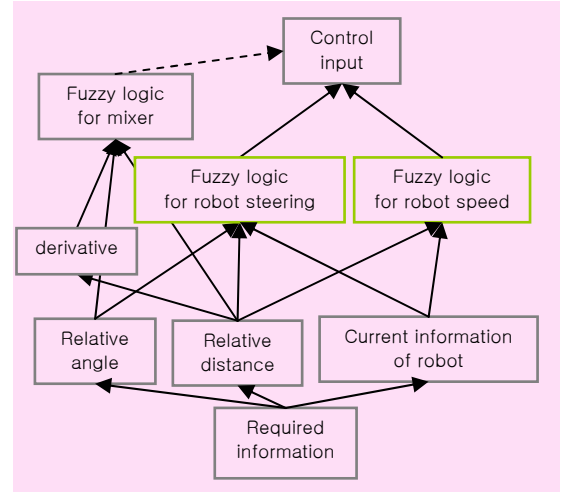


Fig. 2 Composition of fuzzy logic.

3.1 Fuzzy logic for repulsive torque

First of all we assume that information of obstacle is collected by low quality vision system and ultra sonic sensor. Therefore fuzzy logic consists of structure and variable as described in this section.

Fuzzy logic of speed control yields torques to control its speed adopting distance between obstacle and robot and its speed as input variables in Fig. 3, And Membership function and rule-base are shown in Fig. 4 and Table 1.

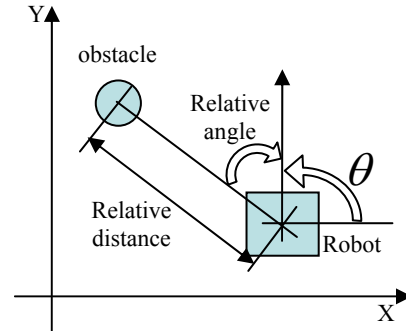


Fig. 3 Coordinate of robot.

We make robot avoid obstacle effectively adopting different speed down rate differently according to the distance between robot and obstacle. It has simple rule-base just like shown in Table 1. It is decelerating around obstacle regardless of relative angle. Here, method of defuzzification is the center of area (COA) in Eq. (10).

$$y = \frac{\sum_{i=1}^m y_i \mu_i}{\sum_{i=1}^m \mu_i}. \quad (10)$$

And the fuzzy inference engine is Mandani's method.

Fuzzy logic of steering control employs the angular velocity and angle between obstacle and robot as input variables in Fig. 3. This fuzzy logic produces steering control torque. Figure 5 indicates membership function and table 2 represents the part of rule-base. We let robot avoid obstacle effectively

converting steering rate to input torque according to the relative angle and distance.

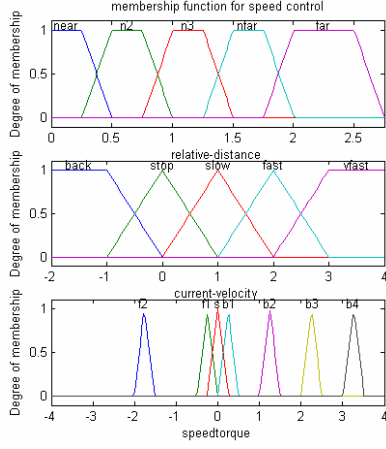


Fig. 4 Membership function of fuzzy logic (speed control).

Table 1 Rule-base of fuzzy logic (speed control).

	near	n2	n3	nfar	far
back	f1	f1	s	s	s
stop	f2	f1	s	s	s
slow	b3	b3	b2	b2	b2
fast	b4	b4	b3	b3	b3
vfast	b4	b4	b4	b3	b3

Inputs of fuzzy logic are composed of relative angle and relative distance. We determine membership function with respect to each input value. The mobile robot avoids obstacle effectively in accordance with error of angular velocity determined the extra input of fuzzy logic. i.e. the mobile robot can decide direction of movement quickly.

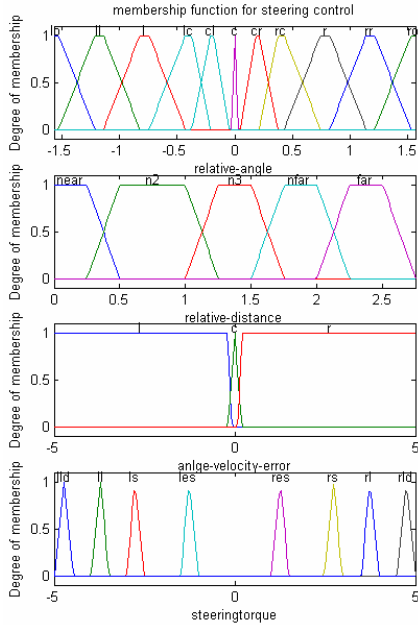


Fig. 5 Membership function of fuzzy logic (steering control).

Table 2 Rule-base of fuzzy logic (steering control at 'c' in third-input membership function).

c	lo	ll	l	lc	cl	c	cr	re	r	rr	ro
ne	le	ls	ll	ld	ld	ld	rd	rd	rl	rs	re
n1	le	ls	ls	ll	ld	ld	rd	rl	rs	rs	re
n2	le	ls	ls	ll	ll	rd	rl	rl	rs	rs	re
nf	le	ls	ls	ll	ll	rd	rl	rs	rs	rs	re
f	le	le	le	le	le	ll	re	re	re	re	re

* Where le, ld, rd, re correspond to les, lld, rld, res in Fig. 5, respectively.

3.2 Fuzzy logic for mixer

The Fuzzy logic for mixer is proposed for flexible combination of fuzzy logic and trajectory tracking. When a mobile robot detects obstacle, the fuzzy logic for mixer reduces prior rate of tracking control value. Therefore mobile robot avoid obstacles using control value of fuzzy logic for speed/steering. After the mobile robot passes obstacle, it returns the reference trajectory. At this time, tracking controller of mobile robot used position errors of robot. So recovery speed is as fast as position error of robot.

The proposed sub-Fuzzy logic has three inputs about the relative angle/relative distance and the derivative of relative distance. These inputs are used to generate the control gain for prior rate. Particularly the derivative of relative distance is important value because it decides the available possibility of repulsive torque.

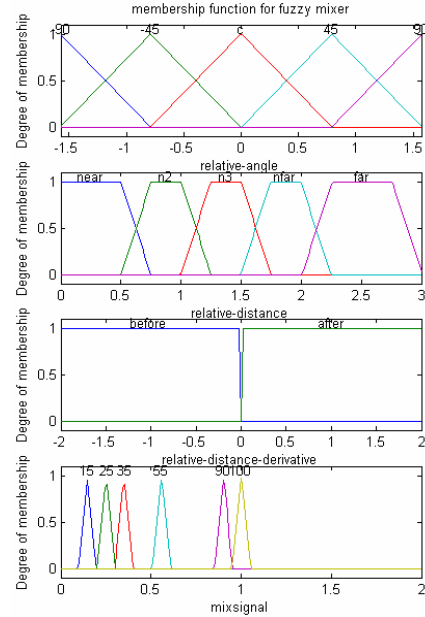


Fig. 6 Membership function of fuzzy logic (mixer).

Table 3 Rule-base of fuzzy logic (speed control at 'before' in third-input membership function).

before	-90	-45	c	45	90
near	15	15	15	15	15
n2	25	15	15	15	25
n3	35	25	25	25	35
nfar	55	35	35	35	55
far	90	55	55	55	90

Figure 6 indicates membership function. And table 3 is shown the section of rule-base.

4. SIMULATION RESULTS

4.1 Static obstacle

We execute computer simulations using proposed algorithm. First, we show simulation results for a static obstacle. When the obstacle exists on the reference trajectory of mobile robot, the mobile robot stops an operation to track the reference trajectory and avoids the obstacle by repulsive torque. But function of tracking controller doesn't stop completely.

The mobile robot avoids the obstacle during tracking reference trajectory of circle in Fig. 7. Here, the obstacle position is $(x, y) = (6, -0.2)$. And Table 4 shows the parameters of mobile robot for these simulations.

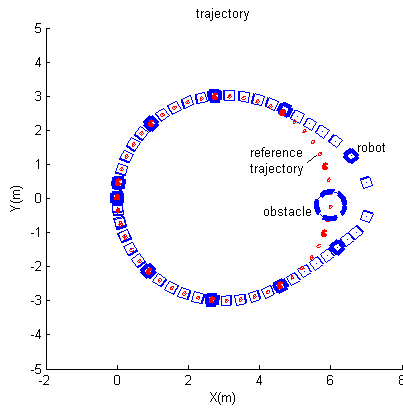


Fig. 7 Static obstacle avoidance in circle reference trajectory.

The mobile robot avoids obstacle during track straight line reference trajectory in Fig. 8(a). When the mobile robot detected obstacle, the value of the repulsive torque increases more than the value of tracking controller. Then, the mobile robot completes avoidance operation. And the mobile robot returns desired position in reference trajectory after passed obstacle.

The mobile robot avoids a obstacle during track curve line reference trajectory in Fig. 8(b).

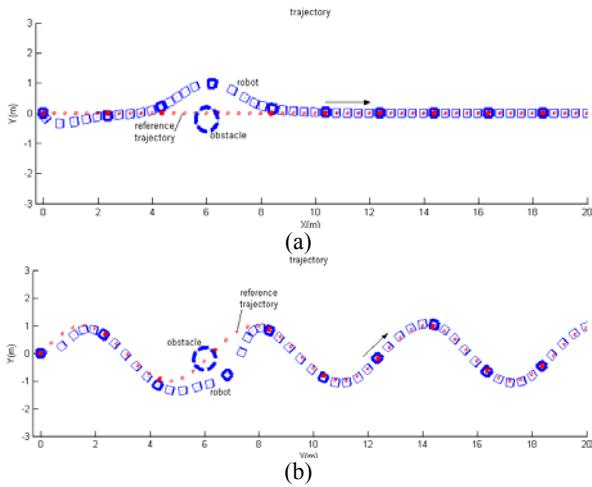


Fig. 8 Static obstacle avoidance in straight line and curve reference trajectory.

Table 4 Parameters of simulation.

Mass	10.6kg
Inertia	$0.875\text{kg}\cdot\text{m}^2$
distance between the driving wheels	0.15m
Wheel radius	0.08m
Initial position $[x_0, y_0, \theta_0]$	$[0, 0, -\pi/2]$
Desired velocity	1m/s

4.2 Moving obstacle

We show simulation results for a moving obstacle in Figs. 9, 10. When the moving obstacle crosses along arbitrary position above the reference trajectory of mobile robot, the mobile robot breaks operation to track the reference trajectory and avoids moving obstacle by repulsive torque. During avoidance operation, a control value of tracking controller decreases the decided rate by the proposed fuzzy logic for mixer. And the controller of the mobile robot increases the velocity during returning the desired reference trajectory.

Here, Speed of moving obstacle is 30~40% of the mobile robot.

The mobile robot avoids a moving obstacle during tracking the circle reference trajectory in Fig. 9.

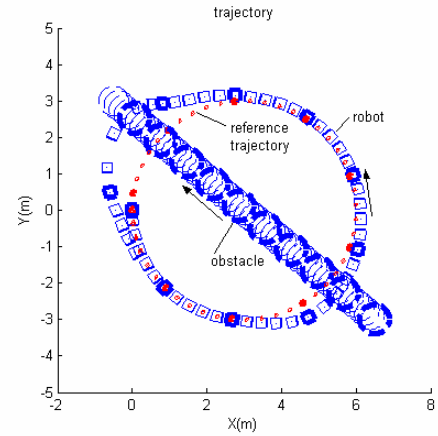


Fig. 9 Moving obstacle avoidance in circle reference trajectory.

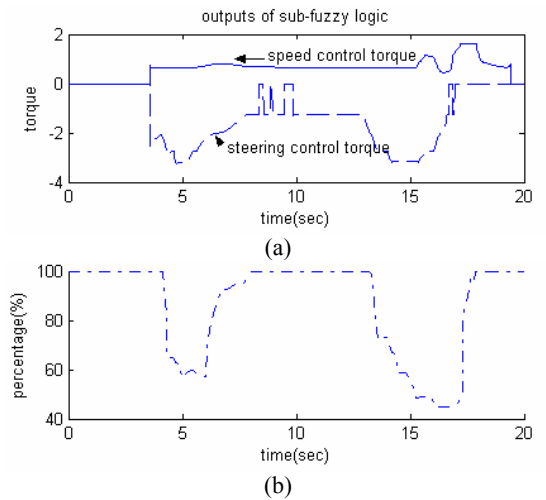


Fig. 10 Output of sub-fuzzy logic in circle reference trajectory.

Figure 10(a) shows outputs of fuzzy logic for steering and speed control during the driving operation in Fig. 9. It is equal to the repulsive torque. And Fig. 10(b) shows output of fuzzy logic for mixer during the driving operation in Fig. 9.

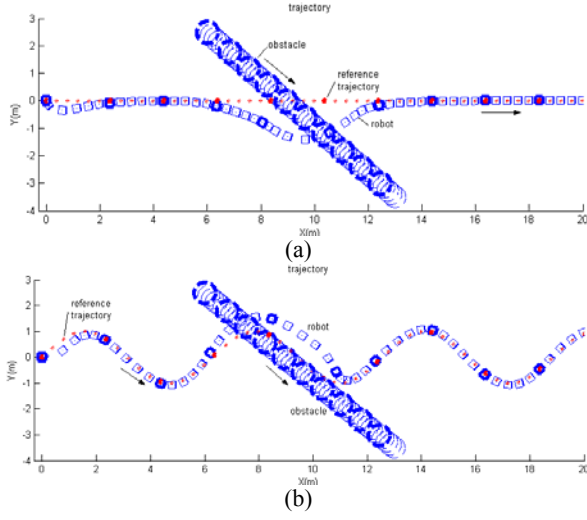


Fig. 11 Moving obstacle avoidance in straight line and curve reference trajectory.

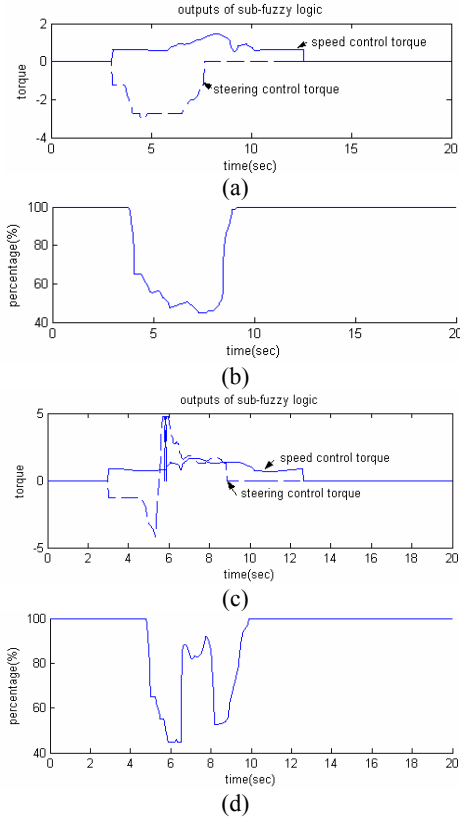


Fig. 12 Output of sub-fuzzy logic in straight line and curve reference trajectory.

The mobile robot avoids moving obstacle during tracking a straight line reference trajectory in Fig. 11(a). As shown in Fig. 11, velocity of the mobile robot increase for some time. Because possession rate of tracking controller expand suddenly by fuzzy logic for mixer.

The mobile robot avoids moving obstacle during track curve line reference trajectory in Fig. 11(b).

Figure 12(a) shows outputs of fuzzy logic for steering and speed control during the driving operation in Fig. 11(a). It is equal to the repulsive torque. And Fig. 12(b) shows output of fuzzy logic for mixer during the driving operation in Fig. 11(a).

Figure 12(c) shows outputs of fuzzy logic for steering and speed control during the driving operation in Fig. 11(b). It is equal to the repulsive torque. And Fig. 12(d) shows output of fuzzy logic for mixer during the driving operation in Fig. 11(b).

4.3 Multi-static & moving obstacles

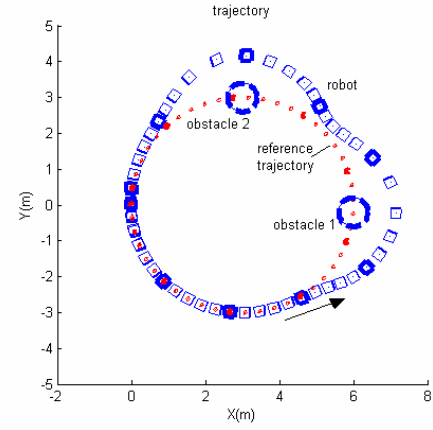


Fig. 13 Multi-static obstacle avoidance in circle reference trajectory.

Finally, we show simulation results for multiple obstacles. When the multiple obstacles block arbitrary positions above the reference trajectory of mobile robot, the mobile robot breaks operation to track the reference trajectory and avoids multiple obstacles by repulsive torque. The mobile robot avoids multiple obstacles during track circle reference trajectory in Fig. 13. Here, obstacle position is $(x_1, y_1) = (6, 0.2)$, $(x_2, y_2) = (3, 3)$ and $(x_1, y_1) = (6, 0.2)$, $(x_2, y_2) = (9.8, -0.3)$ in each situation.

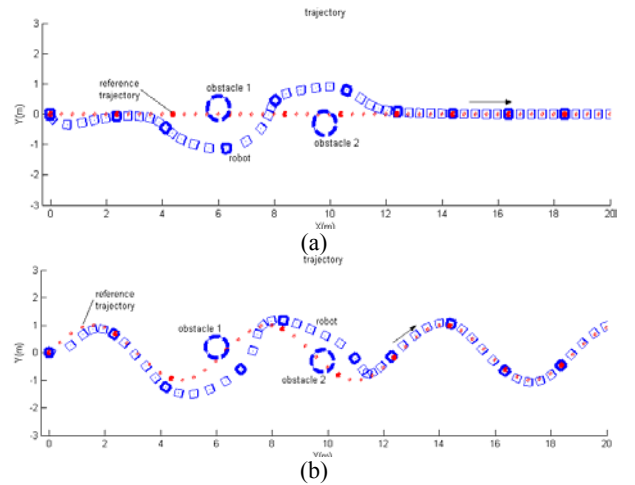


Fig. 14 Multi-static obstacle avoidance in straight line and curve reference trajectory.

The mobile robot avoids multiple obstacles during track straight line reference trajectory in Fig. 14(a). When the mobile robot detects multi-obstacles, value of the repulsive torque increases more than value of tracking controller. And the mobile robot returns desired position in reference trajectory during disappear adjacent obstacle. The mobile robot avoids multi-obstacles during track curve line reference trajectory in Fig. 14(b).

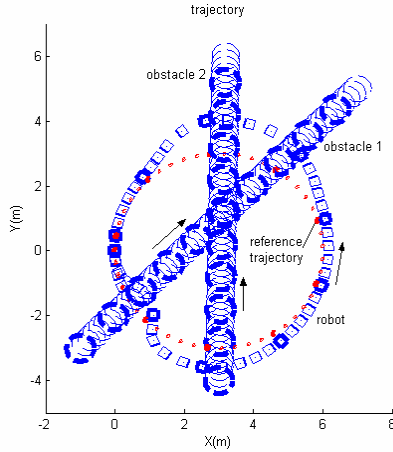


Fig. 15 Multi-Moving obstacle avoidance in circle reference trajectory.

The mobile robot avoids multi-moving obstacle during track circle reference trajectory in Fig. 15. The mobile robot avoids multi-moving obstacle during track straight line reference trajectory in Fig. 15(a) and curve line reference trajectory in Fig. 15(b). As shown in Figs. 15, 16, Velocity of the mobile robot increases for some time. Because, possession rate of tracking controller expand suddenly by fuzzy logic for mixer.

As shown in results of section 4, proposed algorithm has useful performance for static obstacle and moving obstacle. Also, it provides suitable function for multiple obstacles.

In addition, Fuzzy logic for repulsive torque provided low performance without fuzzy logic for mixer. But proposed algorithm has fast reversion ability by fuzzy logic for mixer. i.e. the mobile robot returns rapidly to reference trajectory.

5. CONCLUSIONS

We applied three sub-fuzzy logics for obstacle avoidance in the nonholonomic wheeled mobile robot with trajectory tracking controller. Proposed algorithm provides good performance in on-time based works.

And obstacle avoidance capability of mobile robot was improved by tuning of fuzzy logic in proposed algorithm.

Proposed algorithm was designed effectively using fuzzy logic for single obstacle avoidance. But, it provided suitable performance in multiple obstacle avoidance.

Our future work is to implementation actual nonholonomic wheeled mobile robot with low quality vision system.

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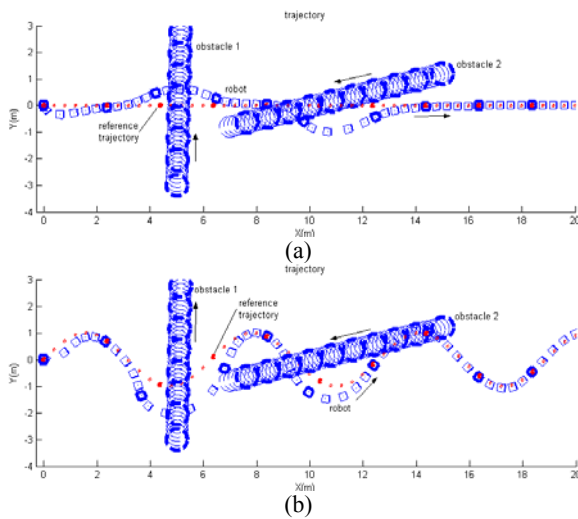


Fig. 16 Multi-Moving obstacle avoidance in straight line and curve reference trajectory.