On Design of Visual Servoing using an Uncalibrated Camera in 3D Space

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Abstract: In this paper we deal with visual servoing that can control a robot arm with a camera using information of images only, without estimating 3D position and rotation of the robot arm. Here it is assumed that the robot arm is calibrated and the camera is uncalibrated. We use a pinhole camera model as the camera one. The essential notion can be show, that is, epipolar geometry, epipole, epipolar equation, and epipolar constrain. These play an important role in designing visual servoing. For easy understanding of the proposed method we first show a design in case of the calibrated camera. The design is constructed by 4 steps and the directional motion of the robot arm is fixed only to a constant direction. This means that an estimated epipole denotes the direction, to which the robot arm translates in 3D space, on the image plane.

Keywords: visual servoing, calibrated, uncalibrated, epipolar geometry, planar projective transformation, estimation

1. Introduction

In this paper we consider problem about visual servoing[1]-[3] that can control a robot arm with a camera using information of images only, without estimating 3D position and rotation of the robot arm[4]-[8]. We propose a method of visual servoing. Here it is assumed that the robot arm is calibrated and the camera is uncalibrated and positions used here are expressed in two coordinate systems, the world coordinate system and the camera coordinate one, and a pinhole camera model is used as the camera one.

2. Models of Camera and Robot

2.1. Camera Model

Now consider a point $\mathbf{X} = [X, Y, Z]^{\mathrm{T}}$ and a camera C in the world coordinate system. C means either camera name or camera position in this paper. The camera coordinate system of C is obtained by rotating and translating the world coordinate system by $\hat{\mathbf{R}}$ and $\hat{\mathbf{T}}$, respectively. A point ${}^{c}\mathbf{X}$ describes camera coordinates of $\mathbf{X}$. The relation between $\mathbf{X}$ and ${}^{c}\mathbf{X}$ can be defined by

$$\mathbf{X} = \hat{\mathbf{R}}\,{}^{c}\mathbf{X} + \hat{\mathbf{T}}, \qquad (1)$$

where $\hat{\mathbf{R}}$ and $\hat{\mathbf{T}}$ are called *the extrinsic parameters* of the camera.

Furthermore suppose ${}^{c}\mathbf{X}$ is projected onto a point $\mathbf{m} = [u, v]^{T}$ on the image plane by the pinhole camera. It follows that

$$\tilde{\mathbf{m}} = \mathbf{A}\,{}^{c}\mathbf{X}, \qquad (2)$$

where $\mathbf{A}$ is a 3x3 upper triangular matrix with scale factor and the coordinates of the image center. This is called *the intrinsic parameter matrix*. And $\tilde{\mathbf{m}} = [m_1, m_2, m_3]^{\mathrm{T}}$ in homogeneous coordinates describes $\mathbf{m}$, That is, $u = m_1 / m_3$, $v = m_2 / m_3$, but m_3 is an arbitrary nonzero scalar.

As mentioned above, $\mathbf{X}$ is also projected onto a point $\mathbf{m}$ and we have

$$\lambda\tilde{\mathbf{m}} = \mathbf{P}\tilde{\mathbf{X}}, \qquad (3)$$

where

$$\mathbf{P} = \mathbf{A}[\mathbf{R}|\mathbf{T}], \quad \mathbf{R} = \hat{\mathbf{R}}^{\mathrm{T}}, \quad \mathbf{T} = -\hat{\mathbf{R}}^{\mathrm{T}}\hat{\mathbf{T}}.$$

Here $\tilde{\mathbf{X}}$ in homogeneous coordinates describes $\mathbf{X}$, and λ is an arbitrary nonzero scalar. We call Eq.(3) the *perspective camera model*, and the 3x4 matrix $\mathbf{P}$ is called the *perspective camera matrix*. The camera is *calibrated* if $\mathbf{P}$ is known, and this camera is called the calibrated camera.

2.1.1. Epipolar Geometry

Now consider two pinhole cameras C and C', and a point $\mathbf{X}_1$ in the world coordinates shown in Fig.1.

Σ_1 is a plane formed by three points, C , C', and $\mathbf{X}$, and is called *the epipolar plane*. $\mathbf{X}_1$ is projected onto points, $\mathbf{m}$, $\mathbf{m}'$ on the image planes by C and C', respectively. It follows from Eq.(3) that

$$\lambda\tilde{\mathbf{m}} = \mathbf{P}\tilde{\mathbf{X}} \qquad (4)$$

$$\lambda'\tilde{\mathbf{m}}' = \mathbf{P}'\tilde{\mathbf{X}}, \qquad (5)$$

where $\mathbf{P}$ and $\mathbf{P}'$ are the perspective camera matrix of

C and C', respectively. $\mathbf{l}_1$ and $\mathbf{l}'_1$ are called *the epipolar line*. All epipolar lines cross at a certain point $\mathbf{e}$ called the *epipole*. It is easy to know form Fig.1 that C' and C are projected onto the epipole $\mathbf{e}$, and $\mathbf{e}'$, respectively. Applying the *generalized inverse* in Eq.(4), the following equation can be obtained.

$$^{c'}\tilde{\mathbf{m}}'^{\mathrm{T}}\mathbf{F}^{c}\tilde{\mathbf{m}} = 0 \quad (6)$$

$$\mathbf{F} = [\mathbf{P}'\mathbf{d}]_{\times}\mathbf{P}'\mathbf{P}^{-}, \quad (7)$$

where

$$\mathbf{P}^{-} = \mathbf{P}^{\mathrm{T}}(\mathbf{P}\mathbf{P}^{\mathrm{T}})^{-1} \qquad \mathbf{Pd} = 0,$$

$\mathbf{F}$ is the skewed symmetry matrix so that the rank of $\mathbf{F}$ is 2. Eq.(6) is called *the epipolar equation* and $\mathbf{F}$ is called a fundamental matrix. The constrain Eq.(6) between $\mathbf{m}$ and $\mathbf{m}'$ is called the *epipolar constraint*.

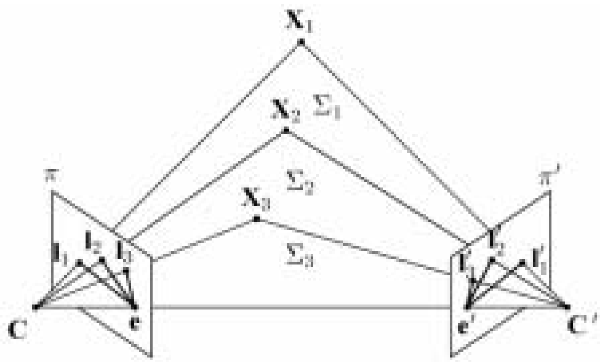


Fig.1 : Epipolar Geometry

2.2. Robot Model

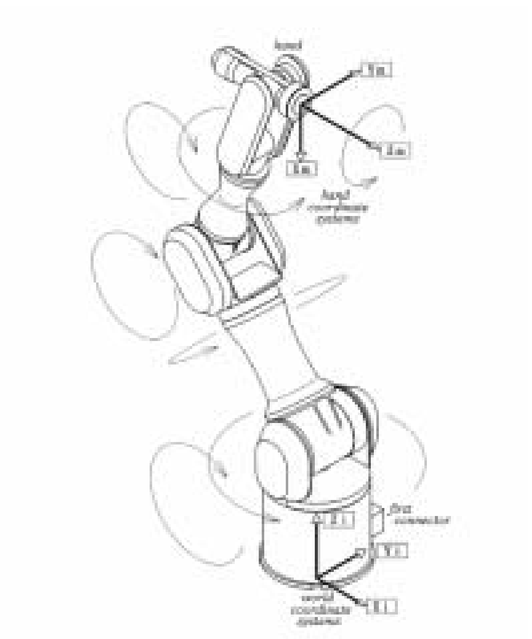


Fig2 : PA-10A-ARM

In this paper, suppose the calibrated robot is *MITUBISHI PA-10A-ARM*. Consequently, The robot arm performs rotation and translation by using the following[4]:

```
ERR pa_mov_YPR(ARM armno,
               REAL32 dYaw,
               REAL32 dPitch,
               REAL32 dRoll,
               WFP func );
ERR pa_mov_xyz( ARM armno,
               REAL32 dX,
               REAL32 dY,
               REAL32 dZ,
               WFP func );
```

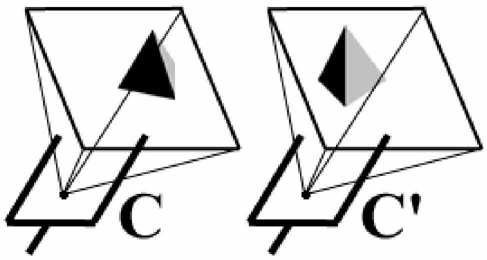
Where, parameters, “dYaw”, “dPitch”, and “dRoll” (in radians), are a rotation difference of a hand coordinate system with respect to the world coordinate system.

Another parameters, “dX”, “dY”, and “dZ” (in pixels), are a translation difference of an origin of the hand coordinate system with respect to the hand coordinate system. “armno” is ID of arm. “func” is the judgment of whether to wait until arm motions finishes.

In this paper, these functions only can be used to control the robot arm. Hence in designing visual servoing, it is not necessary to consider how to control the robot arm in details.

3. Statement of the Problem

Now consider the following two a priori images.



(1) Current Image (2) Final Image

Fig.3 A priori Images

Here, Fig.3(1) and Fig3(2) are images taken with position $\boldsymbol{X}_{final}$ and $\boldsymbol{R}_{final}$, and with $\boldsymbol{X}_{now}$ and $\boldsymbol{R}_{now}$, respectively. Here we suppose the following assumptions.

<Assumptions>

(1) $\boldsymbol{X}_{now}$, $\boldsymbol{R}_{now}$, $\boldsymbol{X}_{final}$ and $\boldsymbol{R}_{final}$ are unknown,

(2) Robot moves in the 3D space,

(3) Robot is calibrated, and

(4) Camera is uncalibrated.

Our aim of this study is to design Visual Servoing which satisfies the following equations.

$$X_{now} \to X_{final}, \quad R_{now} \to R_{final} .$$

Here Visual Servoing means to design a system to satisfy the above equations without estimating 3D information about the robot and the camera.

4. Design of Visual Servoing in case of the Calibrated Camera

Now using $\hat{\mathbf{R}}$, $\hat{\mathbf{T}}$, $\hat{\mathbf{R}}'$, $\hat{\mathbf{T}}'$, $\mathbf{A}$ and $\mathbf{A}'$, $\mathbf{P}'\mathbf{P}^{-}$ and $\mathbf{P}'\mathbf{d}$ in Eq.(7) can be described by

$$\mathbf{P}'\mathbf{P}^{-} = \mathbf{P}'\mathbf{P}\left(\mathbf{P}\mathbf{P}^{\mathrm{T}}\right)^{-1}$$

$$= \mathbf{A}'\mathbf{R}'^{\mathrm{T}}\left(\mathbf{I} + \mathbf{T}'\mathbf{T}^{\mathrm{T}}\right)\left(\mathbf{I} + \mathbf{T}\mathbf{T}^{\mathrm{T}}\right)^{-1}\mathbf{R}\mathbf{A}^{-1},$$

and we define as

$$\mathbf{Q} = \mathbf{T}' - \mathbf{T},$$

then the following equations can be obtained.

$$\mathbf{P}'\mathbf{P}^{-} = \mathbf{A}'\mathbf{R}'^{\mathrm{T}}\left(\mathbf{I} + \mathbf{Q}\mathbf{T}^{\mathrm{T}}\right)\mathbf{R}\mathbf{A}^{-1},$$

$$\mathbf{P}'\mathbf{d} = \mathbf{A}'\mathbf{R}'^{\mathrm{T}}\left(\left[\mathbf{I} \quad -\mathbf{T}\right] + \left[0 \quad -\mathbf{Q}\right]\right)\mathbf{d}.$$

As $\mathbf{d}$ is a 4x1 vector and, the rank of $\mathbf{A}$ and $\mathbf{R}$ are full, $\mathbf{d}$ can satisfy the following equation.

$$\left[\mathbf{I} \quad -\mathbf{T}\right]\mathbf{d} = 0.$$

Using the above equation, $\mathbf{P}'\mathbf{d}$ can be rewritten as

$$\mathbf{P}'\mathbf{d} = d_4\mathbf{A}'\mathbf{R}'^{\mathrm{T}}\mathbf{Q} . \tag{8}$$

From the above result, it follows that

$$\lambda'\tilde{\mathbf{m}}' = \lambda\mathbf{A}'\mathbf{R}'^{\mathrm{T}}\left(\mathbf{I} + \mathbf{Q}\mathbf{T}^{\mathrm{T}}\right)\mathbf{R}\mathbf{A}^{-1}\tilde{\mathbf{m}}$$

$$- d_4\mathbf{A}'\mathbf{R}'^{\mathrm{T}}\mathbf{Q}.$$

Moreover we multiply the above equation by $\mathbf{R}'\mathbf{A}'^{-1}$ from the left,

$$\lambda'\mathbf{R}'\mathbf{A}'^{-1}\tilde{\mathbf{m}}' = \lambda\left(\mathbf{I} + \mathbf{Q}\mathbf{T}^{\mathrm{T}}\right)\mathbf{R}\mathbf{A}^{-1}\tilde{\mathbf{m}} - d_4\mathbf{Q}$$

can be obtained, and then we have

$$\mathbf{F} = \mathbf{A}'^{-\mathrm{T}}\mathbf{R}'^{\mathrm{T}}[\mathbf{Q}]_{\times}\left(\mathbf{I} + \mathbf{Q}\mathbf{T}^{\mathrm{T}}\right)\mathbf{R}\mathbf{A}^{-1}.$$

Hence from the fact that outer product of $\mathbf{Q}$ and itself is equal to zero, the following equation can be obtained.

$$\mathbf{F} = \mathbf{A}'^{-\mathrm{T}}\mathbf{R}'^{\mathrm{T}}[\mathbf{Q}]_{\times}\mathbf{R}\mathbf{A}^{-1}. \tag{9}$$

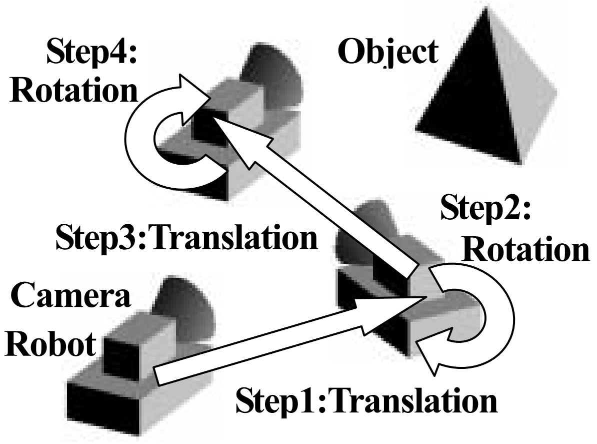


Fig.4 Outline figure of the proposed method

Step 1: Estimation of the Direction, to which the Robot Arm translates, on the Image Plane

In this study, the directional motion of the robot arm is fixed only to a constant direction. that is, it is the Z-axis here. This means that an estimated epipole is the direction, to which the robot arm translates in 3D space, on the image plane. This restriction can be easily controlled by using the function ERR pa_mov_xyz() of the robot arm. Under this restriction it is assumed that the camera translates form C to C', and its fundamental matrix is defined as $\mathbf{F}_1$.

Now since $\mathbf{S} = 0$, $\mathbf{F}_1$ can be obtained as

$$\mathbf{F}_1 = \left(\mathbf{A}'^{-1}\right)^{\mathrm{T}}\left[\hat{\mathbf{R}}^{\mathrm{T}}\mathbf{Q}\right]_{\times}\mathbf{A}^{-1}. \tag{10}$$

Since $\left(\mathbf{A}'^{-1}\right)^{\mathrm{T}}\mathbf{F}_1\mathbf{A}^{-1}$ is the skewed symmetrical matrix, the following relation can be gained.

$$\mathbf{F}_1^{\mathrm{T}} = -\mathbf{F}_1, \tag{11}$$

where the epipoles, $\mathbf{e}_1$ and $\mathbf{e}_1'$, satisfy the following relation from the definition.

$$\mathbf{F}_1\tilde{\mathbf{e}}_1 = 0 \tag{12}$$

$$\mathbf{F}_1^{\mathrm{T}}\tilde{\mathbf{e}}_1' = 0 \tag{13}$$

Here the epipole $\mathbf{e}_1 = \mathbf{e}_1'$ is defined as $\mathbf{m}_v$. Thus the estimated epipole $\mathbf{m}_v$ on the image plane is the direction to which the robot arm translates in 3D space.

Step 2: Rotating to the Direction of the Final Position X_{final}

Here we attempt to control the camera in the direction of the final position X_{final} by rotating the robot arm at the position C'. For this reason, the motion of the robot arm is restricted only to its rotation at C'. Several images can be taken during rotating the robot arm, and it is assumed that $\mathbf{F}_2$ is the fundamental matrix obtained from each image of them and the final image. By using $\mathbf{F}_2$, the epipole $\mathbf{e}_2$ can be calculated as before.

Now the difference $\Delta \boldsymbol{m}$ between $\mathbf{e}_2$ and $\mathbf{m}_v$ is defined by

$$\Delta \boldsymbol{m} = (\boldsymbol{e}_u - \boldsymbol{m}_u)^2 + (\boldsymbol{e}_v - \boldsymbol{m}_v)^2 . \qquad (14)$$

It is clear that if our aim of this step is acquired, then $\Delta \boldsymbol{m} = 0$. Hence we can use this $\Delta \boldsymbol{m}$ as the performance criteria to control the rotation of the robot arm.

Step 3: Guiding to the final position X_{final}

We could rotate the robot arm to the final position in step 2. Hence our aim of this step is that the robot arm is translated to the final position keeping the rotation fixed in step 2.

Now consider the case that the robot arm arrives at the final position, that is, $\mathbf{Q} = 0$ and

$$\lambda'^{c'}\tilde{\mathbf{m}}' = \lambda \mathbf{H}_3{}^{c}\tilde{\mathbf{m}} \qquad (15)$$

$$\mathbf{H} = \mathbf{A}'\hat{\mathbf{R}}'^{\mathrm{T}}\hat{\mathbf{R}}\mathbf{A}^{-1}, \qquad (16)$$

where $\mathbf{H}$ is called *the planar projective transformation matrix.*

The relation (15) can be used to check where the robot arm is on the final position or not. It is assumed that the feature points $\mathbf{X}_i\,(\boldsymbol{i} = 1,2,\cdots,\boldsymbol{n})$ is projected onto $\mathbf{m}_i$ of the image plane $\boldsymbol{C}_1$ and onto $\mathbf{m}'_i$ of the final image plane, and $\hat{\mathbf{m}}'_i$ are points estimated using Eq.(15).

Hence the following difference $\Delta \boldsymbol{H}$ can be used as the performance criteria.

$$\Delta \boldsymbol{H} = \sum_{i=1}^{n} (\boldsymbol{u}'_i - \hat{\boldsymbol{u}}'_i)^2 + (\boldsymbol{v}'_i - \hat{\boldsymbol{v}}'_i)^2 , \qquad (17)$$

that is, if $\Delta \boldsymbol{H} = 0$, then we stop translating the robot arm and it means that our aim can be obtained.

Step 4: Rotating to the final rotation R_{final}.

Here we attempt to control the camera to have the final rotation $\boldsymbol{R}_{final}$ by rotating the robot arm at $\boldsymbol{X}_{final}$ in such a way that can be done in step2..

In this case the epipole is trivial and we can't define $\Delta \boldsymbol{m}$ as the performance criteria. If the current rotation coincides with $\boldsymbol{R}_{final}$, then *the planar projective transformation matrix* $\mathbf{H}$ becomes identity matrix $\mathbf{I}$.

The following difference $\Delta \boldsymbol{I}$ between $\mathbf{H}$ and $\mathbf{I}$ can be used as the performance criteria.

$$\Delta \boldsymbol{I} = \sum_{i=1}^{3}\sum_{j=1}^{3} (\delta_{ij} - \boldsymbol{h}_{ij})^2 \qquad (18)$$

It is clear that $\Delta \boldsymbol{I} = 0$ means to establish our aim.

5. Design of Visual Servoing in case of the Uncalibrated Camera

As the camera is uncalibrated, we can't use straightly the previous method. To use the previous method needs to estimate $\mathbf{F}$ and $\mathbf{H}$. So we will show how to estimate these matrixes below.

5.1 Estimation of the Fundamental Matrix F

The epipolar equation (6) can be written in the following vector form.

$$\omega_1^T \boldsymbol{f}_1 = 0 \qquad (19)$$

where

$$\omega_1^T = (\, \boldsymbol{uu'}\ \boldsymbol{vu'}\ \boldsymbol{u'}\ \boldsymbol{uv'}\ \boldsymbol{vv'}\ \boldsymbol{v'}\ \boldsymbol{u}\ \boldsymbol{v}\ 1\,)$$
$$\boldsymbol{f}_1^T = (\, \boldsymbol{f}_{11}\ \boldsymbol{f}_{12}\ \boldsymbol{f}_{13}\ \boldsymbol{f}_{21}\ \boldsymbol{f}_{22}\ \boldsymbol{f}_{23}\ \boldsymbol{f}_{31}\ \boldsymbol{f}_{32}\ \boldsymbol{f}_{33}\,)$$

Here we estimate $\mathbf{F}$ with $\boldsymbol{f}_{21} = 1$. Therefore, the above equation (19) can be rewritten as

$$\omega^T(\boldsymbol{u},\boldsymbol{v})\,\boldsymbol{f} = -\boldsymbol{uv}' \qquad (20)$$

where

$$\omega^T(\boldsymbol{u},\boldsymbol{v}) = (\, \boldsymbol{uu'}\ \boldsymbol{vu'}\ \boldsymbol{u'}\ \boldsymbol{vv'}\ \boldsymbol{v'}\ \boldsymbol{u}\ \boldsymbol{v}\ 1\,)$$
$$\boldsymbol{f}^T = (\, \boldsymbol{f}_{11}\ \boldsymbol{f}_{12}\ \boldsymbol{f}_{13}\ \boldsymbol{f}_{22}\ \boldsymbol{f}_{23}\ \boldsymbol{f}_{31}\ \boldsymbol{f}_{32}\ \boldsymbol{f}_{33}\,).$$

Suppose that $\boldsymbol{u}_i, \boldsymbol{v}_i\ (\boldsymbol{i} = 1,2,\cdots,\boldsymbol{n}), \boldsymbol{n} \geq 8$ are observed on the image plane as the feature points. Then Eq.(20) can be described as follows.

$$\Omega\,\mathbf{f} = -\mathbf{g} \qquad (21)$$

where

$$\Omega^T = (\, \omega(\boldsymbol{u}_1,\boldsymbol{v}_1)\ \omega(\boldsymbol{u}_2,\boldsymbol{v}_2)\cdots\omega(\boldsymbol{u}_n,\boldsymbol{v}_n)\,)$$
$$\boldsymbol{f}^T = (\, \boldsymbol{f}_{11}\ \boldsymbol{f}_{12}\ \boldsymbol{f}_{13}\ \boldsymbol{f}_{22}\ \boldsymbol{f}_{23}\ \boldsymbol{f}_{31}\ \boldsymbol{f}_{32}\ \boldsymbol{f}_{33}\,)$$
$$\boldsymbol{g}^T = (\, \boldsymbol{u}_1\boldsymbol{v}'_1\ \boldsymbol{u}_2\boldsymbol{v}'_2\ \cdots\ \boldsymbol{u}_n\boldsymbol{v}'_n\,)$$

If $\det \Omega^T\Omega \neq 0$, $\mathbf{f}$ can be determined uniquely by

$$\mathbf{f} = -(\Omega^T \Omega)^{-1} \mathbf{g} . \quad (22)$$

Thus we can estimate $\mathbf{F}$.

5.2.Estiomation of the Planar Projective Transformation Matrix H

Eq.(15) can be written by

$$\lambda_1 \boldsymbol{m}_1' = \boldsymbol{h}_{11}\boldsymbol{m}_1 + \boldsymbol{h}_{12}\boldsymbol{m}_2 + \boldsymbol{h}_{13}\boldsymbol{m}_3$$

$$\lambda_2 \boldsymbol{m}_2' = \boldsymbol{h}_{21}\boldsymbol{m}_1 + \boldsymbol{h}_{22}\boldsymbol{m}_2 + \boldsymbol{h}_{23}\boldsymbol{m}_3$$

$$\lambda_3 \boldsymbol{m}_3' = \boldsymbol{h}_{31}\boldsymbol{m}_1 + \boldsymbol{h}_{32}\boldsymbol{m}_2 + \boldsymbol{h}_{33}\boldsymbol{m}_3 .$$

Here $(\boldsymbol{m}_1, \boldsymbol{m}_2, \boldsymbol{m}_3) \rightarrow (\boldsymbol{u}, \boldsymbol{v}, 1)$ in form of homogeneous coordinate, then

$$\lambda \boldsymbol{u}'(\boldsymbol{h}_{31}\boldsymbol{u} + \boldsymbol{h}_{32}\boldsymbol{v} + \boldsymbol{h}_{33}) = \boldsymbol{h}_{11}\boldsymbol{u} + \boldsymbol{h}_{12}\boldsymbol{v} + \boldsymbol{h}_{13}$$

$$\lambda \boldsymbol{v}'(\boldsymbol{h}_{31}\boldsymbol{u} + \boldsymbol{h}_{32}\boldsymbol{v} + \boldsymbol{h}_{33}) = \boldsymbol{h}_{21}\boldsymbol{u} + \boldsymbol{h}_{22}\boldsymbol{v} + \boldsymbol{h}_{23}$$

can be obtained. Furthermore, this equations can be written as

$$\sigma_1 \boldsymbol{h}_1 = 0 \quad (23)$$

where

$$\sigma_1 = \begin{pmatrix} \boldsymbol{u} & \boldsymbol{v} & 1 & 0 & 0 & 0 & -\boldsymbol{uu'} & -\boldsymbol{vu'} & -\boldsymbol{u'} \\ 0 & 0 & 0 & \boldsymbol{u} & \boldsymbol{v} & 1 & -\boldsymbol{uv'} & -\boldsymbol{vv'} & -\boldsymbol{v'} \end{pmatrix}$$

$$\boldsymbol{h}_1^T = (\boldsymbol{h}_{11}\ \boldsymbol{h}_{12}\ \boldsymbol{h}_{13}\ \boldsymbol{h}_{21}\ \boldsymbol{h}_{22}\ \boldsymbol{h}_{23}\ \boldsymbol{h}_{31}\ \boldsymbol{h}_{32}\ \boldsymbol{h}_{33})$$

Here we estimate $\mathbf{H}$ with $\boldsymbol{h}_{33} = 1$. Suppose that $\boldsymbol{u}_i, \boldsymbol{v}_i\ (\boldsymbol{i} = 1, 2, \cdots, \boldsymbol{n}), \boldsymbol{n} \geq 4$ are observed on the image plane as the feature points. Then Eq.(23) can be described as follows

$$\Sigma\, \boldsymbol{h} = \boldsymbol{g}' \quad (24)$$

where

$$\Sigma = (\ \sigma(\boldsymbol{u}_1, \boldsymbol{v}_1)^T\ \sigma(\boldsymbol{u}_2, \boldsymbol{v}_2)^T \cdots \sigma(\boldsymbol{u}_n, \boldsymbol{v}_n)^T\ )^T$$

$$\boldsymbol{g}' = (\ \boldsymbol{u}_1, \boldsymbol{v}_1, \boldsymbol{u}_2, \boldsymbol{v}_2, \cdots \boldsymbol{u}_n, \boldsymbol{v}_n\ )^T$$

$$\sigma(\boldsymbol{u}, \boldsymbol{v}) = \begin{pmatrix} \boldsymbol{u} & \boldsymbol{v} & 1 & 0 & 0 & 0 & -\boldsymbol{uu'} & -\boldsymbol{vu'} \\ 0 & 0 & 0 & \boldsymbol{u} & \boldsymbol{v} & 1 & -\boldsymbol{uv'} & -\boldsymbol{vv'} \end{pmatrix}$$

$$\boldsymbol{h}^T = (\boldsymbol{h}_{11}\ \boldsymbol{h}_{12}\ \boldsymbol{h}_{13}\ \boldsymbol{h}_{21}\ \boldsymbol{h}_{22}\ \boldsymbol{h}_{23}\ \boldsymbol{h}_{31}\ \boldsymbol{h}_{32})$$

If $\det \Sigma^T \Sigma \neq 0$, $\mathbf{h}$ can be determined uniquely by

$$\boldsymbol{h} = (\Sigma^T \Sigma)^{-1} \boldsymbol{g}' .$$

Thus we can estimate $\mathbf{h}$.

6. Numerical Simulation

Simulation Condition:

C_0 : the current image

$$\hat{\mathbf{R}} = \mathbf{R}_{X0,Y0,Z0} \quad \hat{\mathbf{T}} = [0 \quad 60 \quad -80]^T .$$

C_5 ;the final image

$$\hat{\mathbf{R}}' = \mathbf{R}_{X20,Y20,Z0} \quad \hat{\mathbf{T}}' = [40 \quad 30 \quad 30]^T .$$

.

Step 1:

The epipole $\mathbf{m}_v$ was obtained as

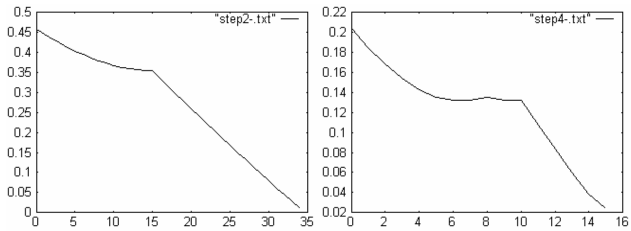
$$\mathbf{m}_v = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix} .$$

Step 2:

The responses of the performance criteria $\Delta \boldsymbol{m}$ are shown inFig.5 (1). The error about $\Delta \boldsymbol{m}$ is 0.32 pixel.

Step 3:

The error about $\Delta \boldsymbol{H}$ is 0.01 pixel.



(1) $\Delta \boldsymbol{m}$ (2) $\Delta \boldsymbol{I}$

Fig.5 Simulation results:

STEP-4:

The responses of the performance criteria $\Delta \boldsymbol{I}$ are shown in Fig.5 (2). The error about $\Delta \boldsymbol{I}$ is 0.02 pixel.

The result of the simulation is as follows.

$$\hat{\mathbf{R}} = \mathbf{R}_{X20.2,Y20.1,Z0.15}$$

$$\hat{\mathbf{T}} = [38.74 \quad 30.87 \quad 29.68]^T$$

It is obvious that the results of the numerical simulation can be satisfied.

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