

# Design of I-PDA Controller Incorporating FFC for Flow Control Systems

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**Abstract:** In this paper, a design of I-PDA controller by CDM to control a flow process whose structure is first order lag plus a dead time is introduced. The factor of dead time of the process is approximated by second order Pade approximation in order to get a third order system. The simulations show that both of the transient and steady state specifications can be fulfilled. However, the transient response of the I-PDA control system still has long rise time. Then, a feedforward controller (FFC) with two adjustable parameters and one derivative time is introduced into I-PDA control system for improving the speed of the system response. It is shown, from the simulation results, that the performance of the I-PDA control system with suitable FFC has shorter rise time and no overshoot, while settling time remains almost the same. The performance comparison of the proposed control system with the PI control system with and without FFC is also made.

**Keywords:** Flow process, CDM, I-PDA controller, feedforward controller

## 1. INTRODUCTION

In the industry, most of the plants or processes are type 0 with three to five first-order lags or one first-order lag plus dead time. Type 1 plants with one or two first-order lags are also found in the industry [1]. Many types of industrial controller are designed both in time domain and frequency domain in order to meet the desired performances of the control system of the plants [2-3]. Controllers popularly employed to control the plants are PI, PID and I-PD.

On the other hand, the parameters design method for PIDA controller proposed by S. Jung and R.C. Dorf [4] to be utilized especially for third order plant has been reported in [5]. I-PDA controller scheme has also been reported in [6]. Both of controllers which are satisfying transient and steady state performances was designed based on coefficient diagram method (CDM) [7]; however, the transient response of the system generally still has long rise time. In order to improve the speed of transient response of the I-PDA control system, a feedforward controller (FFC) whose structure consist of proportional-derivative-acceleration control actions has also been reported [8].

In this paper, a design of I-PDA incorporating FFC for flow process is presented. The flow process is considered to be first order lag plus dead time [9]. As this paper is focused on I-PDA control system, the flow process should be a third order system. Therefore, the factor of dead time is first approximated to be a second order by utilizing Pade approximation. Using CDM concept, controller parameters can be designed from the standard stability index and the equivalent time constant respectively. The stability index and the equivalent time constant are defined based on the coefficient of the characteristic polynomials of the closed loop transfer function algebraically. Normally, the settling time  $t_s$  of the control system is first selected, then the equivalent time constant  $\tau$  can be obtained. However, in case of I-PDA controller for the flow process with approximated dead time factor, only stability index  $\gamma_i$  can be specified so that the equivalent time constant  $\tau$  can be found later.

Since the response of I-PDA control system is slow, the FFC whose structure is a phase lead structure with two parameters and one derivative time is added. The derivative time  $T_d$  of the FFC is chosen based on Zeigler-Nichols method. The two parameters  $\alpha$  and  $\beta$ , on the other hand, must be designed properly by utilizing the advantage of CDM. A

new parameter known as  $\sigma_f$  found from a percentage of time constant is then introduced. By varying  $\sigma_f$  and the standard stability index  $\gamma_i$  the best result can be obtained.

The simulation result using the actual flow process parameters in laboratory employing the I-PDA controller and FFC are shown. The value of  $\alpha$  and  $\beta$  are assigned from the  $\sigma_f$  equals to 60%, 80% and 100% of the equivalent time constant  $\tau$ . The response of I-PDA control system with FFC is then compared to the system without FFC when the  $\sigma_f$  equals to 60% of the equivalent time constant  $\tau$ . It is found that the rise time and settling time of I-PDA control with FFC are approximately 33% and 3% respectively faster than the system without FFC. Both of systems have no overshoot. The responses of I-PDA controller with FFC for  $\sigma_f$  varied from 60% to 100% of the equivalent time constant  $\tau$  are also shown. Therefore, when the values of the two parameters of the FFC are increased, the percentage of the equivalent time constant is increased. It means that the greater  $\sigma_f$  the faster rise time but the greater overshoot will occur. The effectiveness of the proposed controller is also going to be compared with PI control system designed by CDM either with or without FFC [10].

## 2. OVERVIEW OF I-PDA INCORPORATING FFC

The I-PDA control system with FFC as shown in Fig. 1 consists of a FFC, a feedback controller (FBC), an integral controller and a third-order plant.  $\alpha$ ,  $\beta$  and  $T_d$  are parameters of FFC.  $K_p$ ,  $K_d$ ,  $K_a$  are the proportional gain, derivative gain and acceleration gain of the FBC respectively.  $K_i$  is the integral gain of the integral controller.  $D(s)$  is the output step disturbance. The transfer function from  $R(s)$  to  $C(s)$  and from  $D(s)$  to  $C(s)$  are then given by

$$\frac{C(s)}{R(s)} = \frac{K_i G_p(s) [(\alpha T_d) s^2 + (\beta + K_i T_d) s + K_i]}{K_i (T_d s + 1) (s + G_p [s (K_p + K_d s + K_a s^2) + K_i])} \quad (1)$$

and

$$\frac{C(s)}{D(s)} = \frac{1}{1 + G_p(s) \left( \frac{K_i}{s} + K_p + K_d s + K_a s^2 \right)} \quad (2)$$

From (2), it is shown that the disturbance response is irrelevant to the FFC. It means that we have a freedom in selecting  $\alpha, \beta$  to improve transient specification while disturbance rejection and tracking capability performances are not affected.

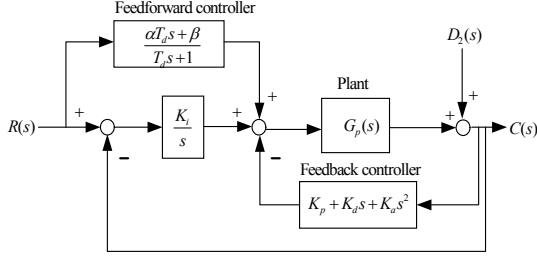


Fig. 1 Structure of I-PDA control system with FFC

### 3. CONTROL SYSTEM STRUCTURE

The proposed control system structure consisting of the CDM standard block diagram with the FFC and FBC is shown in Fig. 2.  $A_p(s)$  and  $B_p(s)$  are the polynomials of the plant  $G_p(s)$ .  $A_c(s)$ ,  $B_c(s)$  and  $B_d(s)$  are the polynomials of the CDM controller,  $B_{fb}(s)$  is the polynomials of the FBC.  $A_{ff}(s)$  and  $B_{ff}(s)$  are numerator and denominator of the FFC. The modified plant,  $G_p^*(s) = \frac{B_p^*(s)}{A_p^*(s)}$  and modified FFC,  $G_{ff}^* = \frac{B_{ff}^*(s)}{A_{ff}^*(s)}$ , can be obtained by simplifying block diagram as depicted on Fig. 3. Then the transfer function from  $R(s)$  to  $C(s)$  of Fig. 2 is

$$\frac{C(s)}{R(s)} = \frac{B_{ff}^*(s)B_p^*(s)}{A_{ff}^*(s)(A_c(s)A_p^*(s) + B_c(s)B_p^*(s))} \quad (3)$$

and the transfer function from  $D(s)$  to  $C(s)$  is

$$\frac{C(s)}{D(s)} = \frac{A_c(s)A_p^*(s)}{A_c(s)A_p^*(s) + B_c(s)B_p^*(s)} \quad (4)$$

Equation (3) will be employed in section 5 to assign the unknown gains of I-PDA controller and FFC that can give the desired system responses of the proposed control system.

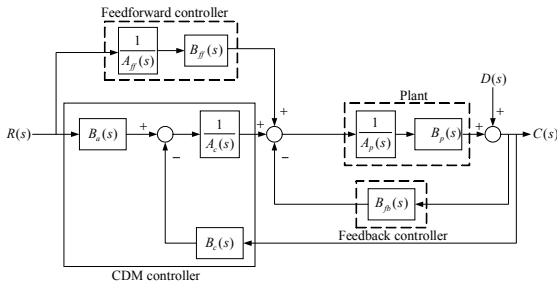


Fig. 2 SISO system with FFC and FBC

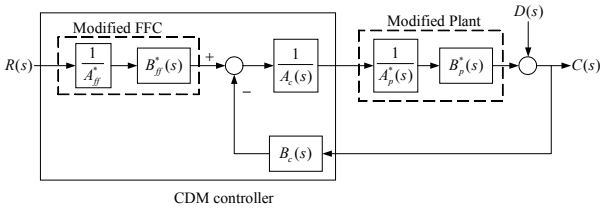


Fig. 3 Rearranged CDM block diagram

### 4. CONCEPT OF THE CDM

In this section, the concept of CDM is used to design the parameters of a controller so that the step response of the control system satisfies stability, fast response and robustness requirements [8]. Generally, the order of the controller designed by CDM is less than the order of the plant. From Fig. 3, the transfer function of the plant in the polynomial form in each block is

$$A_p^*(s) = p_k s^k + p_{k-1} s^{k-1} + \dots + p_0 \quad (4a)$$

$$B_p^*(s) = q_m s^m + q_{m-1} s^{m-1} + \dots + q_0 \quad (4b)$$

and the controller polynomials are

$$A_c(s) = l_\lambda s^\lambda + l_{\lambda-1} s^{\lambda-1} + \dots + l_0 \quad (5a)$$

$$B_c(s) = k_\lambda s^\lambda + k_{\lambda-1} s^{\lambda-1} + \dots + k_0 \quad (5b)$$

$$B_d(s) = k_0 \quad (5c)$$

where  $\lambda < k$  and  $m < k$ .  $B_d(s)$  is called as a pre-filter and has to be set to be  $k_0$  so that the step response with zero steady-state error is obtained. The characteristic polynomial of the closed-loop system without FFC structure as shown in Fig. 3 is given in the following forms

$$\begin{aligned} P(s) &= A_c(s)A_p^*(s) + B_c(s)B_p^*(s) \\ &= a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 \\ &= \sum_{i=0}^n a_i s^i \end{aligned} \quad (6)$$

where  $a_0, a_1, \dots, a_n$  are the coefficients of the characteristic polynomial. The stability index  $\gamma_i$ , the equivalent time constant  $\tau$  and the stability limit  $\gamma_i^*$  are respectively defined as follows

$$\gamma_i = \frac{a_i^2}{a_{i+1}a_{i-1}} \quad (7)$$

$$\tau = \frac{a_1}{a_0} \quad (8)$$

and

$$\gamma_i^* = \frac{1}{\gamma_{i+1}} + \frac{1}{\gamma_{i-1}} \quad ; \quad \gamma_0, \gamma_n = \infty \quad (9)$$

where  $i = 1, \dots, n-1$ . In order to meet the specifications, the equivalent time constant  $\tau$  and the stability index  $\gamma_i$  are chosen as follows

$$t_s = 2.5 \sim 3 \tau \quad (10)$$

$$\gamma_i > 1.5 \gamma_i^* \quad (11)$$

In general, settling time is selected to be  $t_s = 2.5 \tau$ , and the standard stability index is recommended as

$$\gamma_1 = 2.5, \gamma_2 = \gamma_3 = \dots = \gamma_{n-1} = 2. \quad (12)$$

The standard values stated in (12) can be used to design the controller if the following condition is satisfied

$$p_k / p_{k-1} > \tau / (\gamma_{n-1} \gamma_{n-2} \dots \gamma_1) \quad (13)$$

where  $p_k$  and  $p_{k-1}$  are the coefficients of the the plant at  $k$ th and  $(k-1)$ th. If the above condition is not satisfied, we can first increase  $\gamma_{n-1}$  then  $\gamma_{n-2}$  and so on, until (13) is satisfied. From (7)-(9), the coefficient  $a_i$  can be written by

$$a_i = a_0 \tau^i \frac{1}{\gamma_{i-1} \dots \gamma_2 \gamma_1^{i-1}} = a_0 \tau^i \prod_{j=1}^{i-1} \frac{1}{(\gamma_{i-j})^j}. \quad (14)$$

Then the characteristic polynomial to be used to design the parameters of a controller is expressed as

$$P(s) = a_0 \left[ \sum_{i=2}^n \left( \prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^j} \right) (\tau s)^i \right] + \tau s + 1. \quad (15)$$

By equating the characteristic polynomial (6) with a controller included to the characteristic polynomial (15) resulting from the known equivalent time constant  $\tau$  and stability index  $\gamma_i$ , the parameters of a controller are then obtained.

## 5. CONTROLLER DESIGN

Parameters of I-PDA controller will be designed based on the concept of CDM while parameters of FFC will be designed by using the advantage of CDM.

### 5.1 I-PDA controller design

Since denominator of the transfer function of closed-loop system excluding FFC is a fifth order, therefore, the characteristic polynomial is given by

$$P(s) = a_0 \left[ \frac{\tau^5}{\gamma_4 \gamma_3 \gamma_2 \gamma_1^3} s^5 + \frac{\tau^4}{\gamma_3 \gamma_2 \gamma_1^2} s^4 + \frac{\tau^3}{\gamma_2 \gamma_1^2} s^3 + \frac{\tau^2}{\gamma_1} s^2 + \tau s + 1 \right]. \quad (16)$$

From the modified plant, its polynomials are

$$B_p^*(s) = B_p(s) \quad (17)$$

and

$$A_p^*(s) = A_p(s) + B_p(s)[K_p + K_d s + K_a s^2], \quad (18)$$

and the polynomials of the integral controller are defined by

$$B_c(s) = k_o \quad (19)$$

and

$$A_c(s) = s \quad (20)$$

where  $k_o = K_i$ . By using CDM concept previously described, parameters of I-PDA controller  $K_p$ ,  $K_d$ ,  $K_a$  and  $K_i$  can be designed from the following procedures:

- 1) Based on the standard stability index, choose  $\gamma_1=2.5$ ,  $\gamma_2=\gamma_3=\gamma_4=2$ .
- 2) Since settling time  $t_s$  can not be specified, set the time constant  $\tau$  as a variable that will be found later.
- 3) Derive the characteristic polynomials from (3) with the polynomials of the plant including feedback and integral controller polynomial, then equates it to characteristic polynomials obtained by (16). Thus the parameters of the I-PDA controller and time constant  $\tau$  can be obtained.
- 4) Set the pre-filter  $B_a(s)=k_o$ .

### 5.2 FFC design

The structure of the proposed FFC as shown in Fig. 1 is a phase lead structure with the following transfer function

$$\frac{B_{ff}(s)}{A_{ff}(s)} = \frac{\alpha T_d s + \beta}{T_d s + 1}. \quad (21)$$

The value of  $\alpha$  and  $\beta$  must be properly selected and  $T_d$  is the derivative time obtained from reaction curve of the flow process. By rearranging the structure of the control system as shown in Fig. 1 into the control system as shown in Fig. 3, the polynomial forms of the pre-filter are

$$B_f(s) = B_a(s)A_{ff}(s) + A_c(s)B_{ff}(s) = m_2 s^2 + m_1 s + m_0 \quad (22)$$

and

$$A_f(s) = A_{ff}(s) = T_d s + 1 \quad (23)$$

where  $m_2 = \alpha T_d$ ,  $m_1 = (\beta + k_o T_d)$  and  $m_0 = k_o$ . The proper values of  $\alpha$  and  $\beta$  can be assigned from the following procedures:

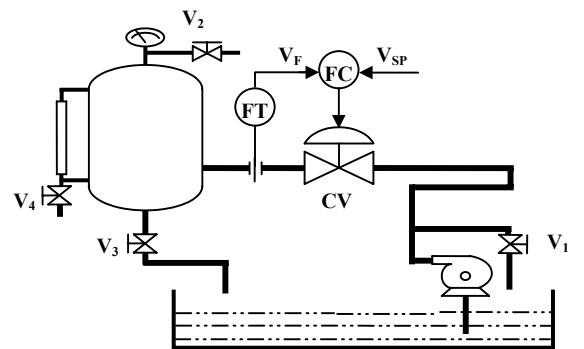
- 1) Let  $m_1 / m_0 = \sigma_f$  be 60% of the equivalent time constant  $\tau$
- 2) Find  $m_1$  from the known values of  $m_0$  and  $\sigma_f$
- 3) Find  $m_2$  from  $m_2 / m_1 = \sigma_f / \gamma_1$  where  $\gamma_1 = 2.5$
- 4) Find  $\alpha$  and  $\beta$  from  $m_2 = \alpha T_d$  and  $m_1 = (\beta + k_o T_d)$

## 6. SIMULATION RESULTS

In this section, MATLAB simulation will be performed to analyze the responses of the control system. As in [9] the process to be controlled is a flow process in laboratory which is illustrated in Fig. 4. By applying the reference flow rate of 36.3 % of the maximum flow rate, the transfer function of the flow process can be given by

$$G_p(s) = \frac{B_p(s)}{A_p(s)} = \frac{K_p}{Ts + 1} e^{-sL} = \frac{1.3774}{1.3333s + 1} e^{-Ls} \quad (24)$$

where  $T$ ,  $L$  and  $K_p$  are the time constant, dead time and gain of the process respectively.



CV: Control valve      V1: Load disturbance valve  
FT: Flow transmitter      FC: I-PDA controller

Fig. 4 Flow process structure

In order to be able to employ the CDM technique for designing the proposed control system, the process with

second-order Pade approximation on dead time factor is then described as a third-order system as follows

$$G_p(s) = \frac{B_p(s)}{A_p(s)} = \frac{1.3774}{1.3333s+1} \cdot \frac{8-4Ls+(Ls)^2}{8+4Ls+(Ls)^2} \quad (25)$$

When  $L = 1$  second, the above equation is rewritten as

$$G_p(s) = \frac{B_p(s)}{A_p(s)} = \frac{1.3774s^2 - 5.5096s + 11.0192}{1.3333s^3 + 6.5096s^2 + 15.0192s + 8} \quad (26)$$

It is seen from equation (26) that the process contains 2 zeros. The performances of the proposed system will be exhibited by simulation. Comparison with PI control system proposed by [10] will also be done concurrently.

### 6.1. System performance of the flow process

According to design procedure stated previously, 50% of the maximum step input will be applied. The stability index  $\gamma_1 = 2.5, \gamma_2 = \gamma_3 = \gamma_4 = 2$  as described in (12) are specified first. From the design procedures in the subsection 5.1, the I-PDA controller gains  $K_p, K_d, K_a$  and  $K_i$ , and the equivalent time constant  $\tau$  can be obtained simultaneously. Here the value of  $K_p, K_d, K_a$  and  $K_i$  are 1.20748, 0.518364, 0.112653 and 0.722707, and  $\tau = 2.17534$ . In case of FFC is added, the design procedure stated in subsection 5.2 is used.

The time responses and the response of control signals obtained by simulation are depicted in Fig. 5 and Fig. 6 respectively. It is seen from Fig. 5 that the I-PDA control system with FFC will have faster response with zero overshoot by choosing the suitable  $\sigma_f$ . It is also seen from Fig. 6 that the control signal of I-PDA control system with FFC during the transient state is larger and is accepted. Performance comparison of the I-PDA and PI control systems with and without FFC is also summarized in Table 1.

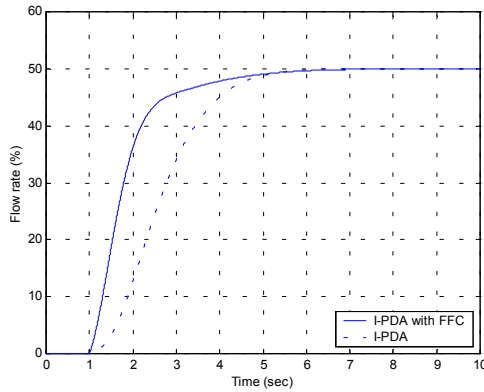


Fig.5 System responses.

From Table 1, it is seen that the speed of the system response for the flow process control system using the I-PDA controller designed by CDM with FFC is faster than the system without FFC. It is also shown that the I-PDA control system has better performances than the PI control system either with or without FFC. One can observe from the Table 1 that all of the control systems have no overshoots  $P_o$  and no steady-state errors  $E_{ss}$ .

Table 1 System performance comparison

Controller	$t_r$ (sec)	$P_o$ (%)	$t_s$ (sec)	$E_{ss}$ (%)
I-PDA with FFC	1.60	0.00	4.93	0.00
I-PDA	2.37	0.00	5.09	0.00
PI with FFC	2.20	0.00	5.35	0.00
PI	3.20	0.00	6.19	0.00

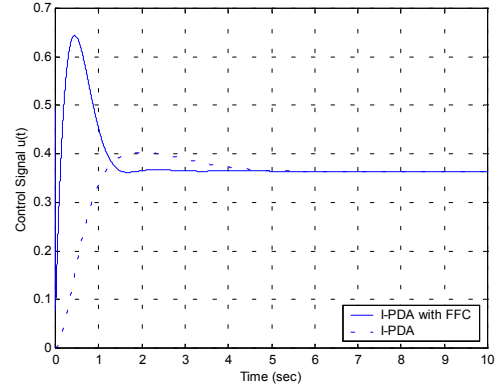


Fig. 6 Control signals.

### 6.2. Step response with the variation of $\sigma_f$

In this subsection,  $\sigma_f$  will be varied for 60%, 80% and 100% of  $\tau$  in order to observe its effect. The performance of the step responses due to the variation of  $\sigma_f$  can be summarized in Table 2.

Table 2 I-PDA step responses due to the variation of  $\sigma_f$ .

$\sigma_f$	$\alpha$	$\beta$	$t_r$ (sec)	$P_o$ (%)	$t_s$ (sec)	$E_{ss}$ (%)
100% of $\tau$	2.735	1.21	0.43	42.3	6.22	0.0
80% of $\tau$	1.750	0.896	0.67	10.1	5.57	0.0
60% of $\tau$	0.985	0.582	1.60	0.00	4.93	0.0

It is shown that when the percentage of the equivalent time constant  $\tau$  greater than 60% and closed to 100% is used for designing FFC, faster rise time but higher overshoot will be obtained. The same effect also occurred in PI control system with FFC (see Table 3 for details). From Table 2 and 3, it is shown that I-PDA control system with FFC exhibits rise time and settling time better than PI control system with FFC. However, in case of higher value of  $\sigma_f$  the worse overshoot will be resulted.

Table 3 PI step responses due to the variation of  $\sigma_f$  [10].

$\sigma_f$	$\alpha$	$\beta$	$t_r$ (sec)	$P_o$ (%)	$t_s$ (sec)	$E_{ss}$ (%)
100% of $\tau$	2.27	0.83	0.59	30.1	8.01	0.0
80% of $\tau$	1.45	0.63	1.06	5.96	7.25	0.0
60% of $\tau$	0.81	0.42	2.20	0.00	5.35	0.0

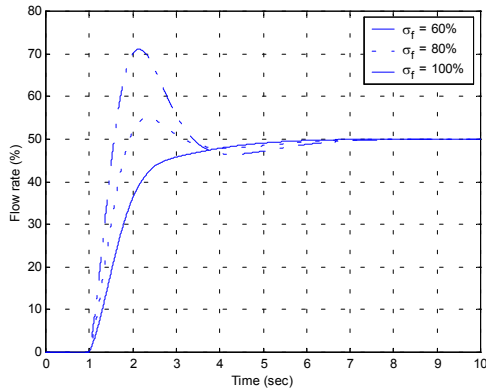


Fig. 7 Step responses due to the variation of  $\sigma_f$

The coefficient diagram is depicted in Fig. 8. Variation of the coefficient  $m_i$  of  $B_{ff}(s)$  due to the variation of  $\sigma_f$  is also shown. It is seen that all of the values of coefficient  $m_i$  are smaller than the coefficient  $a_i$  of the characteristic polynomial  $P(s)$  obtained from the flow process and the I-PDA controller designed by CDM. The slope at the left end of the curve  $m_i$  corresponding to the values of  $\sigma_f$ , as well as  $\tau$ , can be used to measure the speed response of the proposed flow control system. When the values of  $\sigma_f$  is larger, the faster response of the system is obtained (see Table 2).

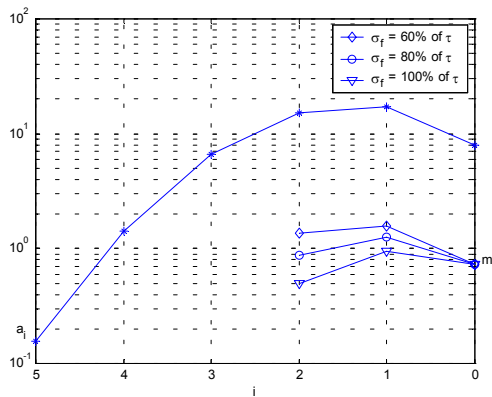


Fig. 8 Comparison of  $m_i$  with coefficient  $a_i$ .

## 7. CONCLUSIONS

The control system using the I-PDA controller designed by CDM technique with the proposed FFC for controlling the flow process has been proposed in this paper. It has been shown that the gains for the I-PDA controller can be properly and easily designed by employing the concept of CDM. After applying FFC which is also designed based on the advantage of CDM, the faster response of the control system can be achieved. The proper selection of  $\sigma_f$  is considered based on the tradeoff between the speed and overshoot of the response.

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