In - Motion Alignment Method for a Low - cost IMU based GPS/INS System

Jeong-Won Kim*, Snag Heon Oh**, Dong-Hwan Hwang***
* Department of Electronics Engineering, Chungnam National University, Daejeon, Korea
(Tel : +82-42-821-7709; E-mail: kimjw@cnu.ac.kr)
** Department of Electronics Engineering, Chungnam National University, Daejeon, Korea
(Tel : +82-42-821-7709; E-mail: laborosh@cnu.ac.kr)
*** School of Electrical and Computer Engineering, Chungnam National University, Daejeon, Korea
(Tel : +82-42-821-5670; E-mail: dhhwang@cnu.ac.kr)

Abstract: When the low cost IMU is used, the result of the stationary self alignment is not suitable for navigation. In this paper, an in-motion alignment method is proposed to obtain an accurate initial attitude of a low cost IMU based GPS/INS integration system. To design Kalman filter for alignment, large heading error model is introduced. And then Kalman filter is designed to estimate initial attitude error as the indirect feedback filter. In order to assess performance of the alignment method, computer simulations are carried out. The simulation results show that initial attitude error rapidly reduces.

Keywords: Low cost IMU, Large heading error model, In-motion alignment, GPS/INS

1. INTRODUCTION

Due to recent development of the semiconductor technology, micro electro mechanical systems (MEMS) based inertial measurement unit (IMU) is being widely used in low cost navigation systems and guidance systems [1][2]. Usually, the MEMS IMU is integrated with non-inertial sensors to improve the performance of the navigation system. The integrated navigation systems of GPS with MEMS based attitude heading reference system (AHRS) and/or attitude determination GPS (AGPS) receiver with MEMS based IMU provide similar performance to the medium grade ring laser gyro (RLG) or fiber optic gyro (FOG) based inertial navigation system (INS) [3][4][5]. These systems measure the absolute attitude of a vehicle directly from the magnetometer in AHRS or AGPS receiver to estimate the inertial attitude error. However, when the attitude aiding information is not available as in MEMS IMU based GPS/INS system, the initial attitude of vehicle has to be determine at the start of the system using self alignment method. When the general stationary alignment algorithm is carried out using the inertial measurements from MEMS IMU with 10 mg accelerometer bias and 1°/sec gyro bias, the initial roll and pitch angle is within the misalignment angle of 1°. The heading error is larger than 360°. Although aided by GPS, that will not help the determination of attitude in stationary state.

In order to get initial attitude of a vehicle several methods can be considered as transfer alignment, in-motion alignment, stored-heading alignment etc. Generally, in-motion alignment is performed when a vehicle moves without initial attitude. It is known that initial attitude has large uncertainty. Especially, heading error is larger than level attitude error [6][8]. If the influence of large heading error is not considered in the algorithm, it cannot accurately estimate the initial error due to the mismatch of its error model. Many researches have been performed and it is shown that the in-motion alignment method using the large heading error model can rapidly estimate the heading error [6][8][10].

This paper proposes an initial alignment method for a low cost MEMS IMU based GPS/INS system. The proposed alignment method is based on the in-motion alignment scheme using the large heading error model. The GPS/INS system finds the roll and pitch angle using a coarse alignment algorithm in stationary and then estimates the heading error by using the proposed in-motion alignment method. A large heading error model and a alignment filter are presented. The computer simulation is carried out to demonstrate the validity of proposed method.

2. LARGE HEADING ERROR MODEL

The unknown initial heading is typically handled by modeling the heading error with two states; sine function of heading error and cosine function of heading error. The large heading error model for the in-motion alignment can be easily derived from the psi-angle error model for SDINS. The horizontal attitude error is small and heading error is a arbitrary value between -180 degree and 180 degree [6][7][8][10]. The large heading error model is described below.

2.1 INS velocity error model

Velocity error equation of psi-angle error model is given by

$$\delta\dot{V}^c = f_t^p - f_t^c + \delta f^p + \delta g^c - (2\omega_{ie}^c + \omega_{ec}^c)\times\delta V^c \,. \tag{1}$$

where the subscript t denotes the true frame; the superscript p and c denote the platform frame and the computer frame, respectively. The symbol δv is the velocity error, δf the specific force error, δg the gravity error, and f_t the true specific force. The symbol ω_{ie} is the angular rate of the earth frame relative to the inertial frame, ω_{ec} the angular rate of the computer frame relative to the earth frame [12].
The misresolved specific force $f_t^p - f_t^c$ is written as

$$f_t^p - f_t^c = (I - C_p^c)\, f_t^p = (C_c^p - I)\, f_t^c \,. \tag{2}$$

where C_p^c is the direction cosine matrix (DCM) from the platform frame to the computer frame and C_c^p the direction cosine matrix from computer frame to the platform frame.
The DCM C_c^p is

$$C_c^p = \begin{bmatrix} c(\psi_y)c(\psi_z)-s(\psi_y)s(\psi_x)s(\psi_z) & c(\psi_y)s(\psi_z)+s(\psi_y)s(\psi_x)s(\psi_z) & -s(\psi_y)c(\psi_x) \\ -c(\psi_x)s(\psi_z) & c(\psi_x)c(\psi_z) & s(\psi_x) \\ s(\psi_y)c(\psi_z)+c(\psi_y)s(\psi_x)s(\psi_z) & s(\psi_y)s(\psi_z)-c(\psi_y)s(\psi_x)c(\psi_z) & c(\psi_y)c(\psi_y) \end{bmatrix}$$

where s denotes sine function and c cosine function; The ψ is misalignment of the platform frame with respect to the computer frame. The subscripts x, y, and z are roll, pitch, and yaw respectively. If level attitude error is small and vertical attitude error is large, the DCM C_c^p can be approximated with

$$\sin(\psi_x) \cong \psi_x, \sin(\psi_y) \cong \psi_y, \sin(\psi_z) = \sin(\psi_z)$$

$$\cos(\psi_x) \cong 1, \cos(\psi_y) \cong 1, \cos(\psi_z) = \cos(\psi_z)$$

$$C_c^p = \begin{bmatrix} c(\psi_z) & s(\psi_z) & -\psi_y \\ s(\psi_z) & -c(\psi_z) & \psi_x \\ \psi_y c(\psi_z) + \psi_x s(\psi_z) & \psi_y s(\psi_z) - \psi_x c(\psi_z) & 1 \end{bmatrix} \quad (3)$$

and misresolved specific force can be written

$$(C_c^p - I) f_t^c = \begin{bmatrix} & \vdots & f_{t(x)}^c \\ (f_t^c \times) & \vdots & f_{t(x)}^c \\ & \vdots & 0 \end{bmatrix} \begin{bmatrix} \psi_x \\ \psi_y \\ \sin(\psi_z) \\ \cos(\psi_z) - 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ f_z(f_t^c, \psi) \end{bmatrix}. \quad (4)$$

where $f_z(f_t^c, \psi) = f_{t(x)}^c(\psi_x \sin(\psi_z) + \psi_y \cos(\psi_z) - 1)$
$+ f_{t(y)}^c(\psi_y \sin(\psi_z) + \psi_x \cos(\psi_z) - 1).$

Substituting (4) into (1) yields

$$\frac{d}{dt}\begin{bmatrix} \delta V_x^c \\ \delta V_y^c \\ \delta V_z^c \end{bmatrix} = \begin{bmatrix} 0 & f_z^c & -f_y^c & f_x^c \\ -f_z^c & 0 & f_x^c & f_y^c \\ f_y^c & -f_x^c & 0 & 0 \end{bmatrix} \begin{bmatrix} \psi_x \\ \psi_y \\ \sin(\psi_z) \\ \cos(\psi_z)-1 \end{bmatrix} \quad (5)$$

$$+ \begin{bmatrix} 0 \\ 0 \\ f_x^c(\psi_x \sin(\psi_z) + \psi_y(\cos(\psi_z)-1)) + f_y^c(\psi_y \sin(\psi_z) + \psi_x(\cos(\psi_z)-1)) \end{bmatrix}$$

$$+ \begin{bmatrix} \delta f_x \\ \delta f_y \\ \delta f_z \end{bmatrix} + \begin{bmatrix} \delta g_x \\ \delta g_y \\ \delta g_z \end{bmatrix} + (2\omega_{ie}^c + \omega_{ec}^c) \times \delta V^c \ .$$

Equation (5) contains a nonlinear term in the vertical specific force error component. If vehicle is moving roughly straight and level, the nonlinear term can be considered either negligible or approximately random noise. A linear error model can then be obtained for implementation in a Kalman filter by simply ignoring the nonlinear term or replacing it with a random noise model[8][10].

2.2 INS attitude error model

The angular rate of the platform frame with respect to the computer frame is given by

$$\omega_{cp}^p = (I - C_c^p)\omega_{ic}^c - \varepsilon^p \ . \quad (6)$$

Where DCM C_c^p is equal to (3).

$$\omega_{cp}^p = -\begin{bmatrix} & \vdots & \omega_{ic(x)}^c \\ \omega_{ic}^c \times & \vdots & \omega_{ic(y)}^c \\ & \vdots & 0 \end{bmatrix} \begin{bmatrix} \psi_x \\ \psi_x \\ \sin(\psi_z) \\ \cos(\psi_z)-1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ f_z(\omega_{ic}^c, \psi) \end{bmatrix} - \varepsilon^p \quad (7)$$

where $f_z(\omega_{ic}^c, \psi) = \omega_{ic(x)}^c(\psi_x \sin(\psi_z) + \psi_y \cos(\psi_z) - 1)$
$+ \omega_{ic(y)}^c(\psi_y \sin(\psi_z) + \psi_x \cos(\psi_z) - 1)$

In (7), it is clear that the $\sin(\psi_z)$ and $\cos(\psi_z)-1$ will have non – negligible term. Hence linearization is not straight – forward. Their rates of change will be small admitting the following approximation

$$\frac{d}{dt}\sin(\psi_z) \cong 0, \ \frac{d}{dt}(\cos(\psi_z)-1) \cong 0 \cdot \quad (8)$$

From (7) and (8) the attitude error model becomes the following for large heading error

$$\frac{d}{dt}\begin{bmatrix} \psi_x \\ \psi_y \\ \sin(\psi_z) \\ \cos(\psi_z)-1 \end{bmatrix} = -\begin{bmatrix} & \vdots & \omega_{ic(x)}^c \\ \omega_{ic}^c \times & \vdots & \omega_{ic(y)}^c \\ & \vdots & \\ \cdots & \cdots & \cdots \\ & 0_{1\times4} & \end{bmatrix} \begin{bmatrix} \psi_x \\ \psi_x \\ \sin(\psi_z) \\ \cos(\psi_z)-1 \end{bmatrix} \quad (9)$$

$$- \begin{bmatrix} 0_{2\times1} \\ \omega_x\psi_y - \omega_y\psi_y \\ \cdots \\ 0 \end{bmatrix} - \begin{bmatrix} \varepsilon^p \\ \cdots \\ 0 \end{bmatrix}.$$

If $\omega_x, \omega_y, \psi_y$ are very small, error model is as follow

$$\frac{d}{dt}\begin{bmatrix} \psi_x \\ \psi_x \\ \sin(\psi_z) \\ \cos(\psi_z)-1 \end{bmatrix} = -\begin{bmatrix} & \vdots & \omega_{ic(x)}^c \\ \omega_{ic}^c \times & \vdots & \omega_{ic(y)}^c \\ & \vdots & \\ \cdots & \cdots & \cdots \\ & 0_{1\times4} & \end{bmatrix} \begin{bmatrix} \psi_x \\ \psi_x \\ \sin(\psi_z) \\ \cos(\psi_z)-1 \end{bmatrix} \quad (10)$$

$$- \begin{bmatrix} \varepsilon^p \\ \cdots \\ 0 \end{bmatrix}.$$

2.3 INS position error model

The position error model is the same as the psi-angle error.

$$\delta\dot{R}^c = -\Omega_{ec}^c \delta R^c + \delta V^c \ . \quad (11)$$

where δR denotes position error, Ω_{ec} skew symmetric matrix of ω_{ec}.

3. KALMAN FILTER

In this section, a Kalman filter is presented for the in-motion alignment.

3.1 Kalman filter state equation

The filter states are

$$x = [x_{nav}\ x_{sensor}]^T$$
$$x_{nav} = [\delta R_x\ \delta R_y\ \delta R_z\ \delta V_x\ \delta V_y\ \delta V_z\ \psi_x\ \psi_y\ \sin(\psi_z)\ \cos(\psi_z)-1]$$
$$x_{sensor} = [\delta f_x\ \delta f_x\ \delta f_x\ \delta\omega_x\ \delta\omega_y\ \delta\omega_z].$$

where δf and $\delta\omega$ is bias of accelerometer and gyroscope , respectively.
The state equation is

$$\dot{x}(t) = F(t)x(t) + w(t) \qquad w(t) \sim N(0,Q) \qquad (12)$$

where system matrix F can be written as (14).

3.2 Kalman filter measurement equation

When position and velocity information of GPS receiver is used, measurement equation can be written as

$$z(t) = H(t)x(t) + v(t) \qquad v(t) \sim N(0,R). \qquad (13)$$

Measurement matrix can be written as

$$H = \begin{bmatrix} I_{6\times6} & 0 \end{bmatrix}.$$

4. SIMULATION

This simulation is performed using Matlab and C-language. First of all, reference trajectory and sensor raw data are generated. Error characteristic of IMU is depicted form MEMS IMU of Crossbow Technology Inc. And proposed alignment algorithm is applied. Simulation compare two case: navigation with in-motion alignment, navigation without in-motion alignment.

4.1 Simulation environment

Simulation environment is as follow

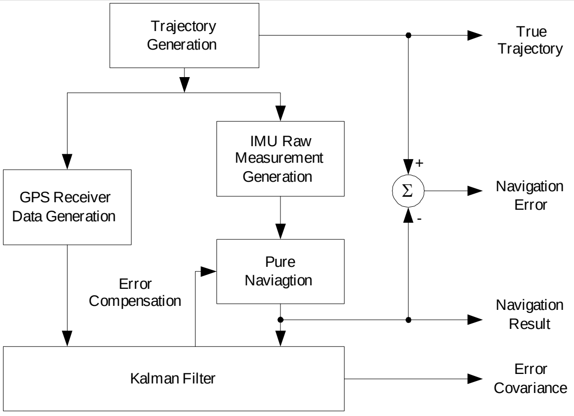

Fig 1. Block diagram of simulation

The vehicle trajectory is shown in Figure 2.

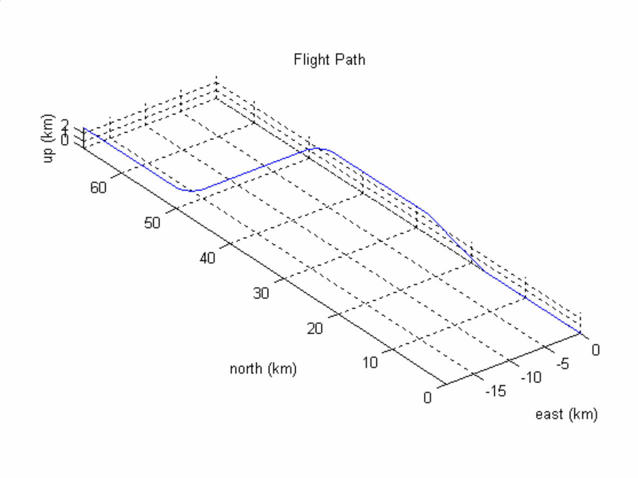

Fig. 2 Vehicle trajectory

$$F = \begin{bmatrix} F_{11} & F_{12} \\ 0 & 0 \end{bmatrix}$$

$$F_{11} = \begin{bmatrix}
0 & -\frac{V_E \tan L}{R_l + h} & \frac{V_N}{R_m + h} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{V_E \tan L}{R_l + h} & 0 & \frac{V_E}{R_l + h} & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-\frac{V_N}{R_m + h} & -\frac{V_E}{R_l + h} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\frac{g_n}{R_l + h} & 0 & 0 & 0 & -2\Omega\sin L - \frac{V_E \tan L}{R_l + h} & \frac{V_N}{R_m + h} & 0 & -f_D & f_E & f_N \\
0 & \frac{g_n}{R_e + h} & 0 & 2\Omega\sin L + \frac{V_E \tan L}{R_l + h} & 0 & 2\Omega\cos L + \frac{V_E}{R_l + h} & f_D & 0 & -f_N & f_E \\
0 & 0 & \frac{2g_n}{R_e + h} & -\frac{V_N}{R_m + h} & -2\Omega\cos L - \frac{V_E}{R_l + h} & 0 & -f_E & f_N & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Omega\sin L - \frac{V_E \tan L}{R_l + h} & \frac{V_N}{R_m + h} & -\Omega\cos L - \frac{V_E}{R_l + h} \\
0 & 0 & 0 & 0 & 0 & 0 & \Omega\sin L + \frac{V_E \tan L}{R_l + h} & 0 & \Omega\cos L + \frac{V_E}{R_l + h} & \frac{V_N}{R_m + h} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{V_N}{R_m + h} & -\Omega\cos L - \frac{V_E}{R_l + h} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 0 \\ \hat{C}_b^n & 0 \\ 0 & -\hat{C}_b^n \end{bmatrix} \qquad (14)$$

Reference profile of velocity, attitude is as follow

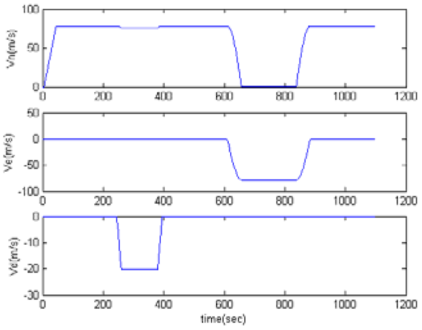

Fig 3. Reference velocity

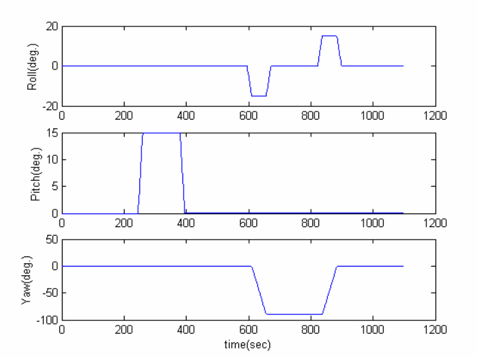

Fig 4. Reference attitude.

Table 1. shows the error of low cost IMU and GPS for simulation.

Table 1 Characteristic of IMU error

Sensor	Bias	Scale factor error	Random walk
Gyroscope	3600 deg/hr	10000 ppm	2.25deg/√hr
Accelerometer	10 mg	10000 ppm	0.15 g/√hr

Table 2. Characteristic of GPS error

Item		Error
Position	Horizontal	10 m
	Vertical	15 m
Velocity		0.2 m/s

The sampling frequency of the IMU is 100 Hz, the GPS 1 Hz.

The entire operation of navigation system is as follow

(1) Leveling mode

This mode is short term and stationary process. Only horizontal coarse alignment is performed. Also this process roughly estimate sensor bias error

(2) In – motion alignment mode

This mode is moving base process. Until covariance of heading error reaches the selected threshold, the alignment is being performed.

(3) Navigation mode

After in-motion alignment mode, Navigation mode start.

Progress of navigation system operation is as follow

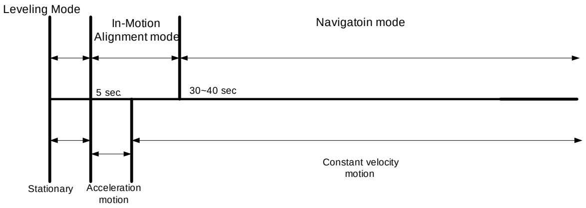

Fig 5. Navigation system operation process.

In order to perform in-motion alignment simulation, initial attitude error of vehicle was given by

- Roll error: 1 degree.
- Pitch error: 1 degree.
- Heading error: 45 degree, 90degree, and 120 degree.

4.2 Simulation Results

Figure 6, 7, and 8 show the heading error of in-motion alignment process.

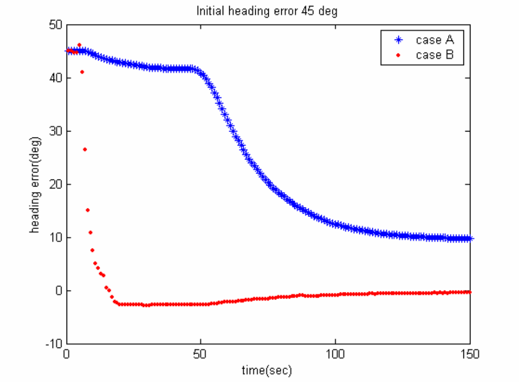

Fig 6. Case of initial heading error 45 degree

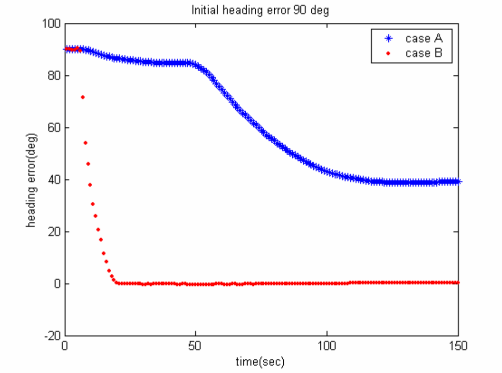

Fig 7. Case of initial heading error 90 degree

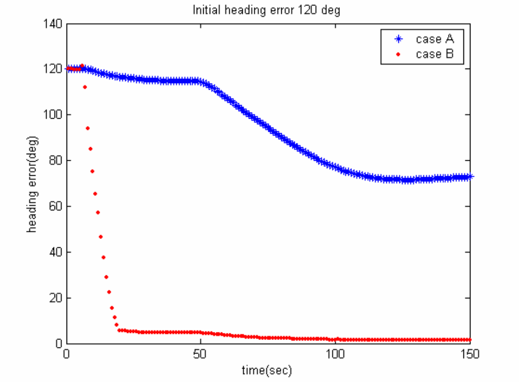

Fig 8. Case of initial heading error 120 degree

Case B is result of in-motion alignment. Case A is result of navigation without in-motion alignment. In case of B, large heading error rapidly reduces within 50 second. But case of A, heading error still large.

Figure 9 shows the attitude error of entire operation. In this case, initial heading error is 40 degree.

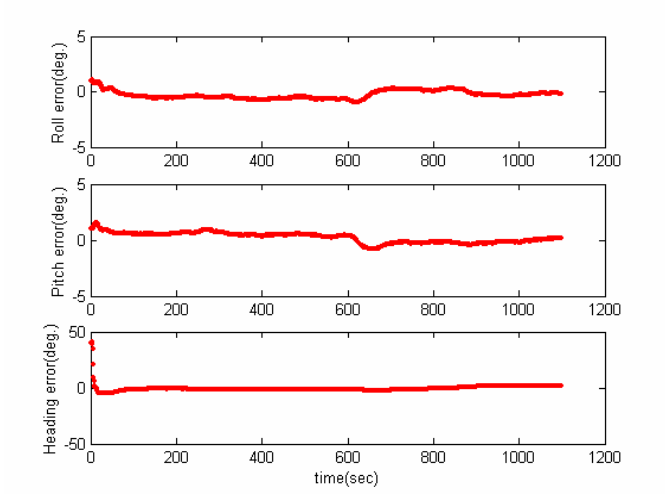

Fig 9. Entire operation attitude error

Table 3. Simulation result

Attitude error	mean	standard deviation
Roll error	-0.3033	0.329
Pitch error	0.1954	0.4511
Heading error	-0.7584	3.358

Simulation result show that proposed in-motion algorithm rapidly estimate initial heading error and this provide initial attitude for reliable navigation.

5. CONCLUSION

In this paper, an initial alignment algorithm is proposed for a low cost IMU based GPS/INS integration system. The proposed alignment method is based on the in-motion alignment scheme using a large heading error model. Although low cost system can not provide initial attitude, the proposed alignment algorithm can determine initial attitude on a moving base. Simulation results show that the proposed alignment method provide more accurate initial attitude. This alignment method becomes very useful in GPS/INS system. Also it can be used for other type of aided-navigation system using a velocity sensor.

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