GPS/INS Integration using Fuzzy-based Kalman Filtering

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Abstract: The integrated global position system (GPS) and inertial navigation system (INS) has been considered as a cost-effective way of providing an accurate and reliable navigation system for civil and military system. Even the integration of a navigation sensor as a supporting device requires the development of non-traditional approaches and algorithms. The objective of this paper is to assess the feasibility of integrated with GPS and INS information, to provide the navigation capability for long term accuracy of the integrated system. Advanced algorithms are used to integrate the GPS and INS sensor data. That is fuzzy inference system based Weighted Extended Kalman Filter(FWEKF) algorithm INS signal corrections to provided an accurate navigation system of the integrated GPS and INS. Repeatedly, these include INS error, calculated platform corrections using GPS outputs, velocity corrections, position correction and error model estimation for prediction. Therefore, the paper introduces the newly developed technology which is aimed at achieving high accuracy results with integrated system. Finally, in this paper are given the results of simulation tests of the integrated system and the results show very good performance

Keywords: GPS, INS, Kalman filter, Fuzzy control, Navigation.

1. INTRODUCTION

Over the past several years, a number of different GPS and INS integration techniques have been investigated and reported [1-11]. The integrated GPS and INS has been considered as a cost-effective way of providing an accurate and reliable navigation system for civil and military system. Early on, the majority of these systems were extremely expensive because, in part of the cost of high-quality, well-characterized sensors and typically the need for a stabilized sensor platform. This high cost limited such systems primarily to military, scientific, and commercial aircraft applications. Advances in material processing have made it possible to produce small, low-cost inertial sensors based on the MEMS technology. Although these low-cost sensors cannot be expected to meet the accuracy and precision specifications for all navigation applications, they do, alternatively, enable a new generation of low-cost, commercial navigation applications, especially when aided by other sensors, such as a GPS, that allow on-line calibration and error estimation.

GPS, which emerged as a powerful candidate of external aiding, is a satellite-based navigation system equipped with the receiver that provides the user with appropriate and accurate positioning information anywhere on the globe.

A pure INS integrates several differential equations containing inertial measurements to provide a navigation solution. As a result, small errors in the measurements can lead to large velocity and position errors if allowed to integrate without correction for long time periods. To correct for this problem, external aiding instrumentation can be used to periodically correct the navigation system errors.

In this paper we present high accuracy reliable GPS and INS integrated system by periodically correct the navigation system errors, which is based on Extended Kalman filtering(EKF) by Fuzzy inference system. This proposed method is based on a hybrid structure integrating fuzzy inference systems and weighted Kalman filtering techniques.

To determine the estimation of INS errors the system of the error equations are derived. The methods and algorithms of optimal filtering in INS software were developed. The integrated INS/GPS function diagram is depicted in Fig. 1.

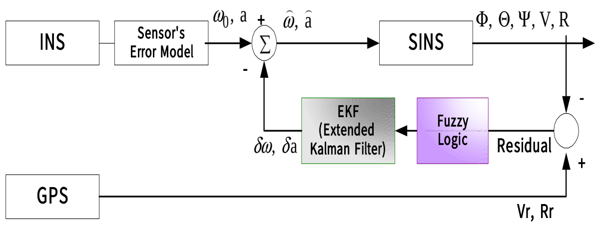

Fig. 1 INS/GPS Integration Block Diagram

In this paper developed the method of correction using error signal, which is difference between the computed and measured outputs of values of the vehicle's position and velocity. The error signal is multiplied by a set of filter gains and added at various stated in the inertial computation chain to reset those computation states [10-11]. The problem of finding the estimates of the vehicle's attitude, position and velocity is solved by the method of continuous correction of INS. The procedure of construction of the observer forming an estimation of attitude, position and velocity is proposed as the solution of the equations INS in which at the outputs of gyros and accelerometers add the corrected vectors determined as a function of the estimation errors. The error signals (difference between the computed and measured values of the vehicle's position and velocity) are used for the determination by discrete-time weighed extended Kalman filter these estimation errors. It was proposed to produce by FWEKF algorithm gyros and accelerometers signal corrections to provide long term accuracy of the integrated system. Therefore, In this paper are given the results of simulation tests of the integrated inertial navigation system and GPS[13]. The main purpose of this paper was to demonstrate the feasibility of using integrated system, along with new algorithms, to optimally process the data to achieve high position and velocity accuracies. The tests demonstrate that the integrated system shows promising results for accurate navigation, which allows the development of a cost-effective navigation system to support a wide spectrum of applications.

The remainder of this paper is organized as follows. Section 2 describes the strapdown algorithm and then Section 3 explains the Kalman filter. Section 4 summarizes the proposed fuzzy based Kalman filter. In order to show the effectiveness of this architecture, in section 5 an illustrative example is outlined. Finally, in section 6 the conclusions of this work are given.

2. STRAPDOWN INERTIAL NAVIGATION SYSTEMS

2.1 Correction of Attitude Equation

Kinematics equation used for rotational mechanization have the form :

$$\dot{\lambda}(t) = \frac{1}{2}\Phi(\omega)\lambda(t) \tag{1}$$

where $\lambda(t)$ is the quaternion that describes the rotation of vehicle into inertial frame, ω is the angular rate vector of the vehicle with respect to the inertial frame. Initial condition for Eq. (1) is defined by quaternion $\lambda(t_0)$, which represent the initial orientation of the body frame with respect to the inertial frame.[5]

For the present development, it is assumed that the available measurements of the angular rate vector(the gyro outputs) are expressed as the sum of the true value ω and error term ε_ω

$$\omega_0 = \omega + \varepsilon_0 \tag{2}$$

The procedure of construction of the observer forming of attitude by a quaternion $\delta\lambda(t)$ is proposed as the solution of the kinematics equation

$$\dot{\hat{\lambda}}(t) = \frac{1}{2}\Phi(\omega_0 + u_\omega)\hat{\lambda}(t) \tag{3}$$

In this equation the corrected vector u_ω of an angular rate will be defined as a function of the estimation error $\delta\lambda(t)$ which satisfies the addition formula of turns

$$\hat{\lambda}(t) = \Phi(\lambda(t))\delta\lambda(t) \tag{4}$$

2.2 Position & Velocity dynamic Equation

The algorithm for calculation of position $r(t)$ and velocity $v(t)$ vectors of the vehicle with respect to the navigation frame is based for $t_0 = 0$ on the following relation :

$$r(t) = A_\varphi(t) = \left[r_0 + t(v_0 + u_\varphi \times r_0) + r_\Delta(t)\right] \tag{5}$$

$$v(t) = A_\varphi(t)\left[v_0 + u_\varphi \times r_0 + v_\Delta(t)\right] - u_\varphi \times r(t) \tag{6}$$

where

$$r_\Delta(t) = \int_0^t\int_0^\tau a_g(\tau)d\tau^2 \ , \quad v_\Delta(t) = \int_0^t a_g(\tau)d\tau \tag{7}$$

$$a_g(t) = A^T(t)a(t) + A_\varphi^T(t)(g(t) - w_\varphi) \tag{8}$$

$$A_\varphi(t) = \cos\alpha I - \frac{\sin\alpha}{\Omega}u_\varphi \times + \frac{1-\cos\alpha}{\Omega^2}u_\varphi u_\varphi^T \tag{9}$$

$r_0 = r(t_0)$, $v_0 = v(t_0)$, $a(t)$ is the acceleration vector due to all non-gravitation forces, $g(t)$ is gravity vector.

As in the case of attitude estimate it is assumed that the gyro outputs have form Eq.(2) and that the measurements of acceleration are expressed as the sum of the true value a and error term ε_a

$$a_0 = a + \varepsilon_0 \tag{10}$$

The procedure of construction of the observer forming an estimation position $\hat{r}(t)$ and velocity $\hat{v}(t)$ is proposed as the solution of the equations

$$\dot{\hat{r}}(t) = \hat{v}(t) \tag{11}$$

$$\dot{\hat{v}}(t) = -2u_\varphi \times \hat{v}(t) - u_\varphi \times (u_\varphi \times \hat{r}(t)) + A_\varphi(t)[\hat{a}_g(t) + u_a(t)] \tag{12}$$

In Eqs. (11) ~ (12) the corrected vector u_a of acceleration will be defined as a function of the estimation error.

$$\begin{aligned} \delta r(t) &= \hat{r}(t) - r(t) \\ \delta p(t) &= \hat{p}(t) - p(t) \end{aligned} \tag{13}$$

where $\hat{p}(t) = \hat{v}(t) + u_\varphi \times \hat{r}(t)$.

3. KALMAN FILTER

3.1 Discrete error equation of corrected INS and Kalman Filter

The Kalman filter is characterized as an algorithm for computing the conditional mean and covariance of the probability distribution of the state a linear stochastic system with uncorrelated Gaussian process and measurement noise.

The application of the Kalman filtering in connection with INS consist of using error signal, which is difference between the computed and measured (supplied, for example, by the updating GPS) values of the vehicle's position and velocity. The error signal is multiplied by a set of filter gains and added in at various states in the inertial computation chain to correct or reset those computation states.

In term of small turn vector $\delta\theta(t) = 2\dfrac{\delta q(t)}{\delta\alpha(t)}$ the differential equation of angular error can be written in the first approximation as

$$\delta\dot{\theta}(t) = A^T(t)u_\omega(t) + \varepsilon_\theta \tag{14}$$

Now consider the technique of determination of the estimation error by Kalman filtering in case when we used GPS information about vehicle position and velocity.

Let $\tilde{r}_k$ and $\tilde{v}_k$ are vehicle position and velocity defined GPS data in moment $t = t_k$ and $\tilde{p}_k = \tilde{v}_k + u_\varphi \times \tilde{r}_k$. By setting the vectors $\hat{r}_k - \tilde{r}_k$ and $\hat{p}_k - \tilde{p}_k$ as the measurement update of plant defined difference equation the problem of the state estimate determination may be solution by the method of Kalman filtering.

So the state extrapolation error vector is formed by difference equation

$$\begin{bmatrix} \delta\hat{\theta}_k^- \\ \delta\hat{r}_k^- \\ \delta\hat{p}_k^- \end{bmatrix} = \begin{bmatrix} A_\varphi & 0 & 0 \\ A_\varphi V_{rk} & A_\varphi & \Delta\cdot A_\varphi \\ A_\varphi V_{vk} & 0 & A_\varphi \end{bmatrix} \begin{bmatrix} \delta\hat{\theta}_{k-1}^+ \\ \delta\hat{r}_{k-1}^+ \\ \delta\hat{p}_{k-1}^+ \end{bmatrix} + \begin{bmatrix} \Delta\cdot A_\varphi \bar{u}_{w,k-1} \\ \Delta^2/2\cdot A_\varphi \bar{u}_{a,k-1} \\ \Delta\cdot A_\varphi \bar{u}_{a,k-1} \end{bmatrix} \quad (15)$$

The state estimate observation update error vector is given by

$$\begin{bmatrix} \delta\hat{\theta}_k^+ \\ \delta\hat{r}_k^+ \\ \delta\hat{p}_k^+ \end{bmatrix} = \begin{bmatrix} \delta\hat{\theta}_k^- \\ \delta\hat{r}_k^- \\ \delta\hat{p}_k^- \end{bmatrix} + K_k \left(\begin{bmatrix} \hat{r}_k - \tilde{r}_k \\ \hat{p}_k - \tilde{p}_k \end{bmatrix} - \begin{bmatrix} \delta\hat{r}_k^- \\ \delta\hat{p}_k^- \end{bmatrix} \right) \quad (16)$$

where K_k is the Kalman gain matrix,

$$\bar{u}_{\omega,k} = -c_\omega \delta\hat{\theta}_k^+ \quad (17)$$

$$\bar{u}_{a,k} = -c_r \delta\hat{r}_k^+ - c_p \delta\hat{p}_k^+ \quad (18)$$

Let's remark, than values of corrected vectors are given by

$$u_\omega(t_k) = A(\hat{\lambda}(t_k))\bar{u}_{\omega,k} \quad (19)$$

$$\bar{u}_a(t_k) = A(\hat{\lambda}(t_k))\bar{u}_{a,k} \quad (20)$$

3.2 Weighted Extended Kalman Filter

An extended Kalman filter is used to fuse the measurements from the GPS and INS. To prevent divergence by keeping the filter from discounting measurements for large k, the exponential data weighting is used.

Let us set the model covariance matrices as

$$R_k = R\alpha^{-2(k+1)} \quad (21)$$

$$Q_k = Q\alpha^{-2(k+1)} \quad (22)$$

By defining the weighted covariance

$$P_k^{\alpha-} = P_k^- \alpha^{2k} \quad (23)$$

The Kalman gain can be computed as :

$$K_k = P_k^{\alpha-} H_k^T (H_k P_K^{\alpha-} H_k^T + R/\alpha^2)^{-1} \quad (24)$$

The updated estimation on the states can be computed as :

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - \hat{z}_k) \quad (25)$$

Computing the α *priori* covariance matrix :

$$P_{k+1}^{\alpha-} = \alpha^2 \Phi_k P_k^\alpha \Phi^T + Q \quad (26)$$

Computing the α *postpriori* covariance matrix gives

$$P_k^\alpha = (I - K_k H_k) P_k^{\alpha-} \quad (27)$$

In Eq. (25), the term $z_k - \hat{z}_k = r$ is called residual or innovation vector.

4. A HYBRID KALMAN FILTER-FUZZY LOGIC ARCHITECTURE

4.1 Fuzzy logic Controller

Fuzzy logic is simple, easy to understand and reflects human type thinking. Its architecture is very well suited for implementing heuristic knowledge of the knowledge gained through experience [12].

If the Kalman filter is based on a complete and perfectly tuned model, the residuals should be a zero-mean white noise process. If the residuals are not white noise, this is an indication of a poor design and the filter does not perform optimally, either diverging or converging to a large bound.

There were two inputs and one output to the fuzzy logic architecture, and 9 rules were used. The purpose of our fuzzy logic controller in this paper is to detect the bias in the measurements and prevent divergence of the extended Kalman filter. .

The covariance of residuals P_r relates to Q and R. The covariance of the residual is given by :

$$P_r = H_k(\Phi P_{k-1}\Phi + Q)H_k^T + R \quad (28)$$

A Takagi-Sugeno fuzzy system is used to detect the divergence of EKF and adapt the filter.

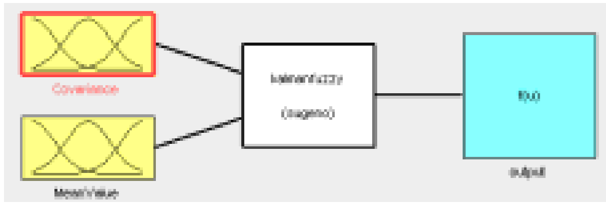

Fig. 2 Fuzzy Inference System

The main inputs were the covariance of the residuals and the mean of residuals for fuzzy inference engines. The output is a linear combinations of the input variable covariance and then α is computed as the weighted average of the output y_i

$$\alpha = \frac{\sum_{i=1}^{n} w^i y^i}{\sum_{i=1}^{n} w^i} \quad (29)$$

The weighted w_i are computed as

$$w^i = \prod_{j=1}^{k} \mu_{A_j^i(x_j)} \quad (30)$$

As an input to fuzzy logic controller, the covariance of the residuals and mean values of residuals are used in order to decide the degree of divergence.

$$residual = \frac{1}{n}\sum_{j=l-n}^{l} r_j \quad (31)$$

$$\hat{P}_r = \frac{1}{n}\sum_{j=l-n+l}^{l} r_j r_j^T \quad (32)$$

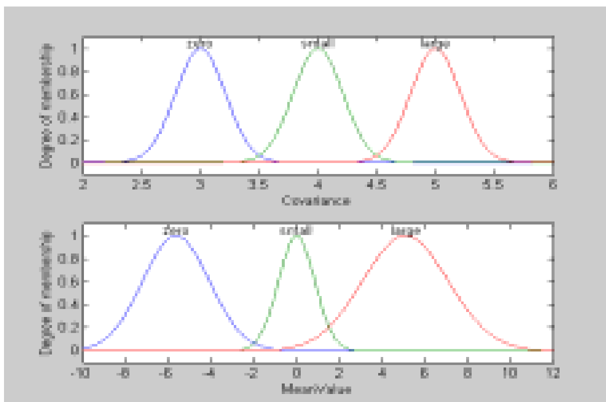

Fig. 3 Covariance of the residuals and mean value of residuals membership function

The membership functions were Gaussian to allow smooth transition between rules. The fuzzy operators have max/min used for or/and, the product method is used for implication, and the centroid method is used for defuzzification [15].

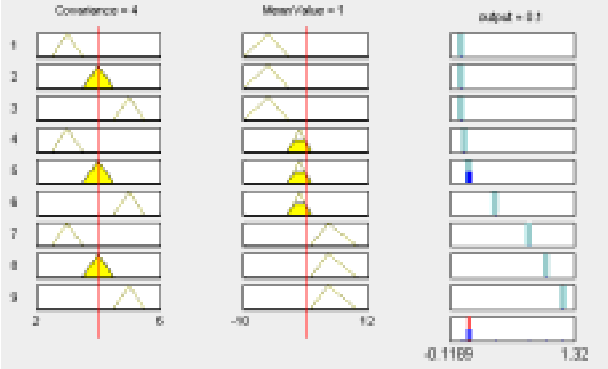

Fig. 4 Fuzzy Inference System

Table. 1 Rule Table for α .

α		Covariance		
		Zero	Small	Large
Mean Value	Zero	Z	S	S
	Small	S	L	S
	Large	Z	NS	NS

S : Small, L : Large, Z : Zero, NS : Negative Small

5. SIMULATION RESULT

In order to demonstrate the accuracy of the system, a series of tests were conducted in both EKF and FWEKF. The test procedures and results are described below.

The GPS/INS integration was simulated during the process system initialization & alignment, and GPS and INS mode after INS alignment. The initial position and velocity of INS were adjusted to 0 and kept during INS alignment.

Fig 5 shows initial alignment process for the INS during the first 20 seconds and integrated GPS and INS navigation beginning after alignment during 20 seconds. The actual value of R for each sensor has been assumed unknown, but its starting value in all sensors was selected as 0.001. The true position, velocity and acceleration in navigation frame is given as follows.

$$r=\frac{3}{4}\begin{bmatrix}0 & t^2-c^2\sin^2(\frac{t}{c}) & 0\end{bmatrix}^T,\ v=\frac{3}{2}\begin{bmatrix}0 & t-\frac{c}{2}\sin(\frac{2\cdot t}{c}) & 0\end{bmatrix}^T \quad (33)$$

$$a=3\begin{bmatrix}0 & \sin^2(\frac{t}{c}) & 0\end{bmatrix}^T$$

where, $c=20/\pi$.

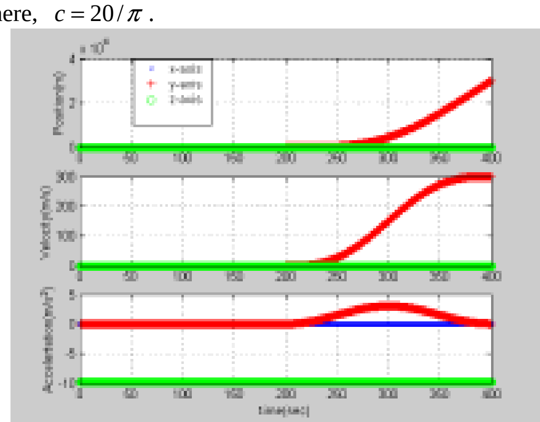

Fig. 5 Reference Input

Since the outputs of INS and GPS include the measurement noise, the standard deviation of the measurement noise can be selected under the following conditions. The first test case demonstrates with initial conditions $\sigma_\omega=\sigma_a=\sigma_r=\sigma_v=0$. The second test case demonstrates with initial conditions $\sigma_\omega=0.0003 rad/s$, $\sigma_a=0.003 m/s^2$, $\sigma_r=10.0m$, $\sigma_v=0.2m/s$. The next Figs. 6~8 compare the accuracy of the EKF and FWEKF.

The prime aim was to generate an optimal set of filter parameters to provide accurate estimation. Plots shown in Fig. 6 illustrate the convergence for the errors in position and velocity. The position and velocity error converges to less than 0.005m; 0.02m/s in 10 sec. Plots shown in Fig. 7 illustrate the convergence for the errors in position and velocity using EKF. The position error converges to less than 0.5m in 20 sec. Again, the velocity converges 10 sec into simulation. Velocity and position errors mostly evident during continuous alignment are reduced dramatically due to the continuous velocity and position update, as well as the misalignment estimation. The FWEKF algorithm simply has less error to correct out than the EKF algorithm. The FWEKF algorithm would have eventually corrected the error during initial alignment of integrated system. Fig. 9 shows the graph for the change of covariance of the residual time history. The value of $\hat{P}_r$ increases for the first 5 seconds and then keeps the constant value, which given the input of the fuzzy inference system and the proper α can be generated. Then alpha gives the updated correction for Q and R of weighted extended Kalman filter and the updated filter gains give a motivation to error reduction. Finally, The residual of measurement reduced.

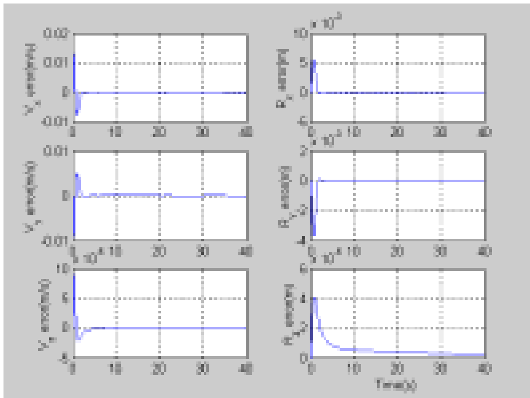

Fig. 6 Velocity & Position error
(data without noises+EKF))

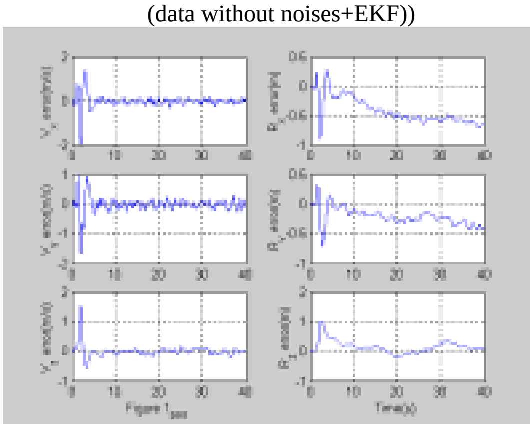

Fig. 7 Velocity & Position error
(data with noises +EKF)

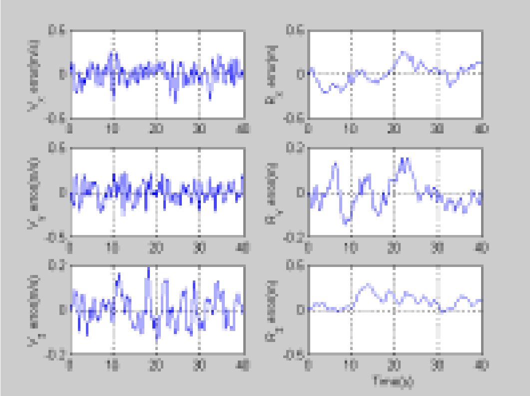

Fig. 8 Velocity & Position error
(data with noises +FWEKF)

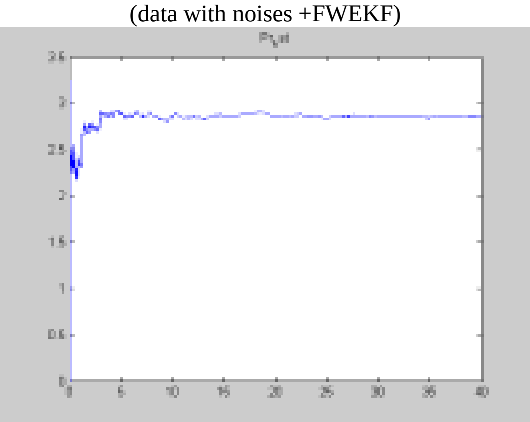

Fig. 9 Covariance of Residual

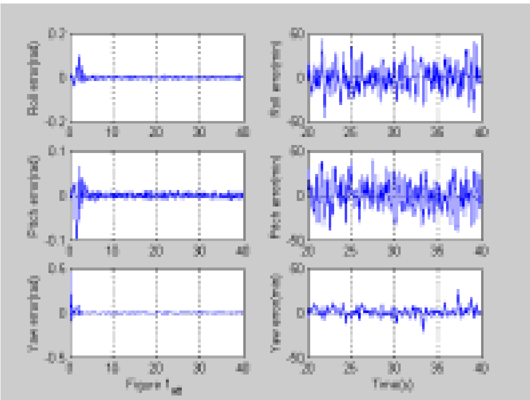

Fig. 10 Attitude error
(data with noises +EKF)

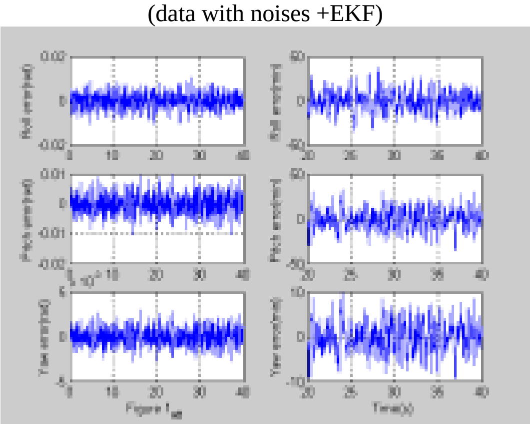

Fig. 11 Attitude error
(data with noises +FWEKF)

6. CONCLUSION

In this approach the problem of finding the estimates of the vehicle's attitude, position and velocity is solved by the method of continuous correction of INS. The Kalman filter has been modified using the fuzzy inference system and compared with the performance of regular extended Kalman filter. In this paper, a FWEKF was been developed to detect and prevent the EKF from divergence. If the filter does not perform well, it will apply an appropriate weighting factor α to improve the accuracy of an EKF. In the fuzzy inference system, 9 rules are used, therefore, little computational time is needed. The main purpose of this paper was to demonstrate the feasibility of using fuzzy inference system to optimally process the data to achieve high attitude, position and velocity accuracies. The results of simulations show that the fuzzy inference is an efficient and stable method. In the future works, the real-time implementation will be considered as well.

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