Fault Detection Using Propagator for Kalman Filter and Its Application to SDINS

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Abstract: In this paper, we propose a fault detection method for extended Kalman filter in decentralized filter structure. To detect a fault, a consistency between filter output and a monitoring signal is tested. State propagators are used to obtain the monitoring signal. However, the output of state propagator increases in magnitude and finally diverges as time runs. To solve such problem, two-propagator method was proposed for linear system. Two propagators are reset by Kalman filter output, alternatively, to avoid divergence. But a test statistics change abruptly at the reset instant in that method. Hence a N-step propagator method is proposed to fix up the problem. In the N-step propagator, only time propagations are performed from k-N+1 step to k step without measurement updates. A test statistics are defined by errors and its covariance between extended Kalman filter and N-step propagator. These fault detection methods are applied to integrated strapdown inertial navigation system (SDINS). By computer simulation, it is shown that the proposed methods detect a fault effectively.

Keywords: decentralized filter, extended Kalman filter, fault detection, integrated navigation system, SDINS

1. INTRODUCTION

In this paper, we describe a fault detection method for decentralized filter structure. There are two approaches to the process of measurements for state estimation in multi-sensor system. All sensor measurements are processed at one filter in centralized filter approach. On the other hand, each sensor measurement is processed at the dedicated filter (named as local filter) and filter outputs are combined to get a global state estimation in decentralized filter approach[1-3]. The later has a several advantages compared to the former, for example, less computational burden, more robust to sensor fault, and more flexible to the change of sensor components. Therefore decentralized filter structure is widely used for multi-sensor system such as inertial navigation system with simultaneous alternative navaid sensor subsystem[3].

Fault detection methods can be divided into two groups[4]. In using hardware redundancy, more than three sensors, which measure same physical quantities, are necessary, and measurements from sensors are compared to each other. Although this method can be simply implemented, it cost a great deal. In using analytic redundancy, sensory measurements are compared to the analytically calculated values based on system's model. Most of the recent studies of fault detection are related to model-based fault detection. In particular, fault detection in stochastic system uses the residual of Kalman filter[5]. Another method of fault detection in stochastic system is state chi-square test which is originally proposed by Kerr[6]. In that method, a state propagator was used to provide a reference system for fault detection. The state chi-square test was used to test the consistency of the state estimate of the Kalman filter and that of the state propagator. There are several previous studies using state chi-square test for fault detection[3,7-9]. Most of these results assumed a linear system and were concerned with a centralized filtering approach.

We propose two methods for fault detection in decentralized filter. The first method is extension of Ren Da[8] study to decentralized approach. The two propagators are reset by master filter output, alternatively, to avoid divergence. Another method is that a N-step propagator based on measurements till k-N step is used to get a reference signal Only time propagations are performed from k-N+1 step to k step without measurement updates in the N-step propagator. A test statistics for fault detection are defined by errors and its covariance between local filter output and the reference signal.

We are interested in fault detection of strapdown inertial navigation system (SDINS) with simultaneous alternative navaid sensor subsystem such as GPS, Position Fix, etc. Decentralized filter structure is used to get a navigation solution, and its structure is shortly discussed in the following section. The proposed fault detection methods in decentralized filter are applied to integrated SDINS, and its performance is examined by computer simulation.

In the following section 2, decentralized filter structure is reviewed, and fusion methods of local filter outputs in master filter are examined. In section 3, the proposed decentralized filter fault detection methods are discussed. In section 4, the proposed fault detection methods are applied to SDINS problem, and simulation results are discussed. Conclusion of this paper is offered in section 5.

2. DECENTRALIZED KALMAN FILETR

In case of integrated navigation system with various navaid sensors, decentralized filter is used for navigation solution owing to computational efficiency and robustness to sensor failure. Each local filter estimates the state vector with dedicated sensor measurements, and their estimated results are combined to estimate a global state vector[1,2]. In this section, we discuss the decentralized Kalman filter structure.

Consider the following linear discrete-time system:

$$x(k+1) = \Phi(k+1,k)x(k) + G(k)w(k) \tag{1}$$

$$z(k) = H(k)x(k) + v(k) \tag{2}$$

where $x(k) \in R^n$ and $z(k) \in R^m$ represent the state and the measurement vector, respectively. The process noise $w(k)$ and measurement noise $v(k)$ are assumed to be white noise processes with zero mean and to be mutually uncorrelated. The noise covariance kernels are $E\{\mathbf{w}(i)\mathbf{w}^T(j)\}=\mathbf{Q}(i)\delta_{ij}$ and $E\{\mathbf{v}(i)\mathbf{v}^T(j)\}=\mathbf{R}(i)\delta_{ij}$, respectively, where δ_{ij} the Kronecker-delta function.

For implementing decentralized filter, we assume that Eq. (2) can be divided into M block measurements, and they are not correlated with each other. Then, Eq. (2) can be represented by the following equations

$$z(k)=\{z_1^T(k)\ \ z_2^T(k)\ \cdots\ z_M^T(k)\}^T \tag{3}$$

$$H(k)=\{H_1^T(k)\ \ H_2^T(k)\ \cdots\ H_M^T(k)\}^T \tag{4}$$

$$R(k)=\text{block}\{R_1(k)\ \ R_1(k)\ \cdots\ R_M(k)\} \tag{5}$$

where $z_i(k)\in R^{m_i}$ and $m=\sum_{i=1}^{M} m_i$.

The measurement equation for each local filter is written as follows.

$$z_i(k)=H_i(k)x(k)+v_i(k),\quad i=1,2,\cdots,M \tag{6}$$

Each local filter, which is a standard Kalman filter, can be written by the following equations.

Time propagation:

$$\hat{x}_i(k+1|k)=\Phi_i(k+1,k)\hat{x}_i(k|k) \tag{7}$$

$$P_i(k+1|k)=\Phi_i(k+1,k)P_i(k|k)\Phi_i^T(k+1,k)+G_i(k)Q_i(k)G_i^T(k) \tag{8}$$

Measurement update:

$$\hat{x}_i(k|k)=[I-K_i(k)H_i(k)]\hat{x}_i(k|k-1)+K_i(k)z_i(k) \tag{9}$$

$$\begin{aligned}K_i(k)&=P_i(k|k-1)H_i^T(k)[H_i(k)P_i(k|k-1)H_i^T(k)+R_i(k)]^{-1}\\&=P_i(k|k)H_i^T(k)R_i^{-1}(k)\end{aligned} \tag{10}$$

$$P_i^{-1}(k|k)=P_i^{-1}(k|k-1)+H_i^T(k)R_i^{-1}(k)H_i(k) \tag{11}$$

where $P_i(k)$ is a covariance of the estimation error, and $K_i(k)$ is Kalman gain. For convenience, the covariance update is expressed by information form in Eq. (11).

If the assumed Eq. (3) and Eq. (11) are applied for the covariance fusion algorithm in master filter, then the covariance can be given as

$$\begin{aligned}P_m^{-1}(k|k)&=P_m^{-1}(k|k-1)+H^T(k)R^{-1}(k)H(k)\\&=P_m^{-1}(k|k-1)+\sum_{i=1}^{M}H_i^T(k)R_i^{-1}(k)H_i(k)\\&=P_m^{-1}(k|k-1)+\sum_{i=1}^{M}[P_i^{-1}(k|k)-P_i^{-1}(k|k-1)]\end{aligned} \tag{12}$$

where subscript 'm' stands for master filter, and second term on the right of Eq. (12) is the new information from measurements. From Eqs. (9)-(10), we can derive update equation for state vector as follows.

$$\begin{aligned}\hat{x}_m(k|k)&=[I-K(k)H(k)]\hat{x}_m(k|k-1)+K(k)z(k)\\&=P_m(k|k)P_m^{-1}(k|k-1)\hat{x}_m(k|k-1)+P_m(k|k)H^T(k)R^{-1}(k)z(k)\\&=P_m(k|k)P_m^{-1}(k|k-1)\hat{x}_m(k|k-1)+P_m(k|k)\sum_{i=1}^{M}H_i^T(k)R_i^{-1}(k)z_i(k)\\&=P_m(k|k)\{P_m^{-1}(k|k-1)\hat{x}_m(k|k-1)\\&\quad+\sum_{i=1}^{M}[P_i^{-1}(k|k)\hat{x}_i(k|k)-P_i^{-1}(k|k-1)\hat{x}_i(k|k-1)\}\end{aligned} \tag{13}$$

In some cases, master filter outputs, $\hat{x}_m$ and P_m, are fed back to local filters, so local filters are reset by more accurate information, for example, 'federated filter'. But, it is undesirable to feed back the information of master filter to local filters from the viewpoint of local filter fault detection. If any local filter has a fault, it is not detected immediately owing to inherent detection delay. So, if master filter information feed back to local filters, it is possible that all of local filters are contaminated by the undetected local filter fault. Therefore we do not feed the information of master filter to local filters.

3. FAULT DETECTION OF FILTER

A chi-square test, which tests the consistency between the checked signal and the reference signal, is widely used for fault detection in stochastic dynamic system[8]. Sometimes, the test statistics are calculated by using filter residual in chi-square test. In that case, it is almost impossible to detect a fault in actuator, although a fault in sensor may be easily detected. On the other hand, if the test statistics are calculated by using state vector in chi-square test, it may be possible to detect a fault in both of actuator and sensor.

For calculation of the test statistics by using state vector, the reference signal is necessary. We describe how to get the reference signal and how to detect a fault in decentralized Kalman filter by using that signal.

3.1 Fault detection with two propagators

Fault detection in decentralized filter with two state propagators is described in this section. The method is an extension of one proposed by Ren Da[8]. Ren Da proposed fault detection method with two propagators in Kalman filter. If only one state propagator is used to get a reference signal for consistency test, the signal increases in magnitude and finally diverges as time runs. To avoid such problem, the two-propagator method was proposed for Kalman filter fault detection. In that method, two propagators are reset by Klaman filter output, alternatively, to avoid divergence. However, if there are M local filter in decentralized filter, 2M state propagators are required in that method. So more simple structure to detect a fault is proposed for decentralized filter.

Fig. 1 shows a block diagram of the proposed fault detection method with two state propagators in decentralized filter. In Fig. 1, local filters estimate state vector and its error covariance, and the consistency between the estimated states and the state propagator outputs are tested by fault detection algorithm before the estimated states are transferred to master filter to finally obtain the global state vector.

The state propagator can be written as

$$\hat{x}_s(k+1)=\Phi_s(k+1,k)\hat{x}_s(k) \tag{14}$$

$$P_s(k+1)=\Phi_s(k+1,k)P_s(k)\Phi^T(k+1,k)+G_s(k)Q(k)G_s^T(k) \tag{15}$$

where subscript 's' represents state propagator.

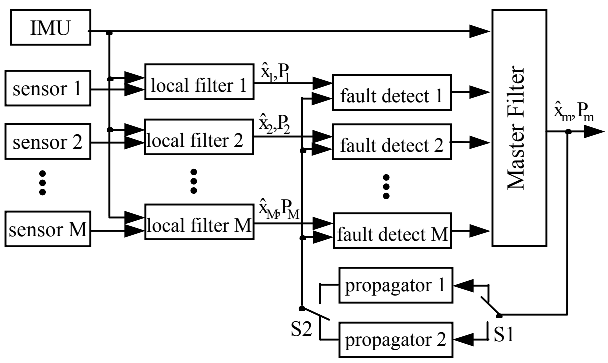

Fig. 1 Fault detection with two state propagators.

The two propagators do not update with sensor measurements, and they are reset by master filter output, alternatively with any time interval. The reset time interval is a design parameter, and it depends on the characteristics of the target system.

To derive the fault detection method with two propagators in decentralized filter, let us define error state vectors as

$$e_i(k) = x(k) - \hat{x}_i(k) \tag{16}$$

$$e_s(k) = x(k) - \hat{x}_s(k) \tag{17}$$

where $x(k)$ is true state vector, $e_i(k)$ is estimation error from i-th local filter, and $e_s(k)$ is estimation error from state propagator.

From Eq. (1), Eqs. (7)-(11), and Eq. (16), error equations for i-th local filter can be written as follows.

$$e_i(k|k)=[I-K_i(k)H_i(k)]\,e_i(k|k-1)-K_i(k)\nu_i(k) \tag{18}$$

$$e_i(k+1|k)=\Phi_i(k+1,k)e_i(k|k)+G_i(k)w_i(k) \tag{19}$$

Similarly, from Eq. (1), (14), and Eq. (17), error equations for state propagator can be derived as follows.

$$e_s(k|k)=e_s(k|k-1) \tag{20}$$

$$e_s(k+1|k)=\Phi_s(k+1,k)e_s(k|k)-G_s(k)w(k) \tag{21}$$

Let us consider the following variables to define the test statistic to detect a fault in any local filter.

$$\beta_i(k) = e_s(k) - e_i(k|k) \tag{22}$$

$$\begin{aligned} B_i(k) &= E\{\beta_i(k)\beta_i^T(k)\} \\ &= P_i(k) - P_{is}(k) - P_{is}^T(k) + P_s(k) \end{aligned} \tag{23}$$

where $P_{is}(k)$ the cross covariance between the i-th local filter and the state propagator.

Then, the test statistic is defined as follows.

$$\lambda_i(k) = \beta_i^T(k)B_i^{-1}(k)\beta_i(k) \tag{24}$$

The test statistic $\lambda_i(k)$ is chi-square distributed with n degree of freedom. The test for fault detection is

$$\begin{cases} \lambda_i(k) \ge \varepsilon_\beta & \text{fault} \\ \lambda_i(k) < \varepsilon_\beta & \text{no fault} \end{cases} \tag{25}$$

where the threshold ε_β is determined from tables of the chi-square distribution and function of false alarm rate (FAR).

If the interested system is linear and initial conditions of local filter and propagator are same, then Eq. (23) can be simplified as follows[7].

$$B_i(k) = P_s(k) - P_i(k) \tag{23-1}$$

However, if the system is nonlinear such as SDINS, the state transition matrix Φ_i and Φ_s are not identical. Therefore the cross covariance $P_{is}(k)$ is necessary to calculate the test statistics. Also, two propagators are periodically reset by master filter, the cross covariance between master filter and local filter, $P_{mi}(k)$, must be considered, before $P_{is}(k)$ is calculated. The cross covariance $P_{mi}(k)$ is calculated by following Eqs. (26)-(29) with Eqs, (18)-(19).

$$e_m(k|k)= [I-K_m(k)H_m(k)]e_m(k|k-1)+K_m(k)\nu(k) \tag{26}$$

$$e_m(k+1|k) = \Phi_m(k+1,k)e_m(k|k) - G_m(k)w(k) \tag{27}$$

$$\begin{aligned} P_{mi}(k+1|k)&=E\{e_m(k+1|k)e_i^T(k+1|k)\} \\ &=\Phi_m(k+1,k)P_{mi}(k|k)\Phi_i^T(k+1,k)+G_m(k)Q(k)G_i^T(k)] \end{aligned} \tag{28}$$

$$\begin{aligned} P_{mi}(k|k) &= E\{e_m(k|k)e_i^T(k|k)\} \\ &=(I-K_mH_m)E\{e_m(k|k-1)e_i^T(k|k-1)\}(I-K_iH_i)^T \\ &\quad +K_mE\{\nu(k)\nu_i^T(k)\}K_i^T \\ &=P_m(k|k)P_m^{-1}(k|k-1)\overline{P}_{mi}(k|k)+P_m(k|k)H_i^TR_i^{-1}H_iP_i(k|k) \end{aligned} \tag{29}$$

where $\overline{P}_{mi}(k|k)=P_{mi}(k|k-1)P_i^{-1}(k|k-1)P_i(k|k)$ is calculated in the course of local filter update process. In Eq. (29), we use the assumption that measurements can be divided into M block measurements.

From Eq. (24), the cross covariance $P_{is}(k)$ is required, and the covariance is equal to $P_{mi}(k)$, which is verified in following section.

3.2 Fault detection with N-step propagator

In this section, another fault detection method in decentralized filter is discussed. The previous detection method in section 3.1 has disadvantage that the test statistic may change abruptly at the reset time of the propagator. The fact may be a cause of increasing false alarm rate. Also the previous detection method is a little complicated because of using 'two' propagators. So we propose the N-step propagator, which is applicable to nonlinear system, to overcome such disadvantages. The function of the N-step propagator is to generate a monitoring signal which is the same as the two state propagators in section 3.1.

Let us define the following nonlinear state propagation equation to derive the N-step propagator.

$$\dot{\hat{x}}(t) = f(\hat{x}(t)) \tag{30}$$

where the propagated time interval is $t_{k-1} \le t < t_k$ with initial condition $\hat{x}(t_{k-1}) = \hat{x}(k-1|k-1)$. From Eq. (30), the propagated state vector at $t=t_k$ can be approximated as follows.

$$\hat{x}(k|k-1) = f(\hat{x}(k-1|k-1))\Delta T + \hat{x}(k-1|k-1) \tag{31}$$

where ΔT is measurement updated interval and $\hat{x}(k|k-1)$ is the state vector based on measurements $Z_{k-1}=\{z_1, z_2, \cdots, z_{k-1}\}$.

Then, the state vector $\hat{\hat{x}}(k)$ based on measurement set $Z_{k-2}=\{z_1, z_2, \cdots, z_{k-2}\}$ can be approximately derived as

$$\begin{aligned} \hat{\hat{x}}(k) &= f(\hat{x}(k-1|k-2))\Delta T + \hat{x}(k-1|k-2) \\ &= f(\hat{x}(k-1|k-1) - \delta\hat{x}_{k-1})\Delta T + \hat{x}(k-1|k-1) - \delta\hat{x}_{k-1} \\ &= f(\hat{x}(k-1|k-1))\Delta T + \hat{x}(k-1|k-1) - [I + F_{k-1}\Delta T]\delta\hat{x}_{k-1} \\ &= \hat{x}(k|k-1) - \Phi(k,k-1)\delta\hat{x}_{k-1} \end{aligned} \tag{32}$$

where $\delta\hat{x}_{k-1}$ is the estimated results from extended Kalman filter with measurement z_{k-1}, and $\Phi(k,k-1) = (I+F_{k-1}\Delta T)$, $F_{k-1} = \left.\dfrac{\partial f(x)}{\partial x}\right|_{x=\hat{x}(k-1|k-1)}$.

Lemma 1 In nonlinear system, the propagated state vector $\hat{\hat{x}}(k)$ based on measurement set $Z_{k-N-1}=\{z_1, z_2, \cdots, z_{k-N-1}\}$ can be approximately written as

$$\hat{\hat{x}}(k) = \hat{x}(k|k-1) - \sum_{j=k-N}^{k-1}\Phi(k,j)\delta\hat{x}_j \tag{33}$$

where $\hat{x}(k|k-1)$ is the estimated state obtained from extended Kalman filter.

Proof: From Eq. (32), the predicted state $\hat{\hat{x}}(k-N+1)$ at k-N+1 step without using measurements $\{z_{k-N}, z_{k-N+1}\}$ can be approximated as

$$\hat{\hat{x}}(k-N+1)=\hat{x}(k-N+1|k-N+1)-\delta\hat{x}_{k-N+1}-\Phi(k-N+1,k-N)\delta\hat{x}_{k-N} \quad (34)$$

where $\Phi(k-N+1,k-N)$ is similarly defined as Eq. (32).

The predicted state $\hat{\hat{x}}(k-N+2)$ at k-N+2 without using measurements $\{z_{k-N},z_{k-N+1}\}$ can be approximated as

$$\begin{aligned}
\hat{\hat{x}}(k-N+2) &= f[\hat{\hat{x}}(k-N+1)]\Delta T+\hat{\hat{x}}(k-N+1)\\
&= f[\hat{x}(k-N+1|k-N+1)-\delta\hat{x}_{k-N+1}-\Phi(k-N+1,k-N)\delta\hat{x}_{k-N}]\Delta T\\
&\quad +\hat{x}(k-N+1|k-N+1)-\delta\hat{x}_{k-N+1}-\Phi(k-N+1,k-N)\delta\hat{x}_{k-N}\\
&= f[\hat{x}(k-N+1|k-N+1)]\Delta T+\hat{x}(k-N+1|k-N+1)\\
&\quad -\Phi(k-N+2,k-N+1)[\delta\hat{x}_{k-N+1}-\Phi(k-N+1,k-N)\delta\hat{x}_{k-N}]\\
&= \hat{x}(k-N+2|k-N+2)-\delta\hat{x}_{k-N+2}\\
&\quad -\Phi(k-N+2,k-N+1)\delta\hat{x}_{k-N+1}-\Phi(k-N+2,k-N)\delta\hat{x}_{k-N}
\end{aligned} \quad (35)$$

If the procedure mentioned above is repeated, then the propagated state $\hat{\hat{x}}(k-1)$ at k-1 without using measurements $\{z_{k-N},z_{k-N+1},\cdots,z_{k-2}\}$ can be written as follows.

$$\hat{\hat{x}}(k-1)=\hat{x}(k-1|k-1)-\delta\hat{x}_{k-1}-\sum_{j=k-N}^{k-2}\Phi(k-1,j)\delta\hat{x}_j \quad (36)$$

Therefore, the propagated state $\hat{\hat{x}}(k)$ at k step without using measurements $\{z_{k-N},z_{k-N+1},\cdots,z_{k-1}\}$ can be approximated by Eq. (33). ▲

Fig. 2 represents the basic concept of the N-step propagator mentioned above lemma, and the propagated state $\hat{\hat{x}}(k)$ can be applied for the monitoring signal instead of two state propagator outputs.

For the test statistic calculated by Eq. (24), the covariance of the propagated state $\hat{\hat{x}}(k)$ is needed, and the following lemma is about that covariance.

Lemma 2 The covariance of the propagated state $\hat{\hat{x}}(k)$ given by Eq. (33) can written as

$$P_{\hat{\hat{x}}_k}=P(k|k-1)+\sum_{j=k-N+1}^{k-1}\Phi(k,j)\{P(j|j-1)-P(j|j)\}\Phi^T(k,j) \quad (37)$$

where P(k|k-1) is the covariance of $\hat{x}(k|k-1)$.

Proof: Let define the propagator error $\bar{e}(k|k-1)$ as follows.

$$\begin{aligned}
\bar{e}(k) &= x(k|k-1)-\hat{\hat{x}}(k)\\
&= e(k|k-1)+\sum_{j=k-N}^{k-1}\Phi(k,j)\delta\hat{x}_j
\end{aligned} \quad (38)$$

Then, the covariance $P_{\hat{\hat{x}}_k}$ can be written as follows.

$$\begin{aligned}
P_{\hat{\hat{x}}_k} &= E\{\bar{e}(k)\bar{e}^T(k)\}\\
&= E\{[\hat{e}(k|k-1)+\sum_{j=k-N+1}^{k-1}\Phi(k,j)\delta\hat{x}_j][\hat{e}(k|k-1)+\sum_{j=k-N+1}^{k-1}\Phi(k,j)\delta\hat{x}_j]^T\}
\end{aligned} \quad (39)$$

In Eq. (39), $E\{\delta\hat{x}_{k-j}\hat{e}^T(k|k-1)\}\ (j=k-N+1,\cdots,k-1)$ is equal to zero by orthogonal projection lemma[10]. Therefore, the covariance equation can be derived as follows.

$$\begin{aligned}
P_{\hat{\hat{x}}_k} &= P(k|k-1)+\sum_{j=k-N+1}^{k-1}\Phi(k,j)E\{\delta\hat{x}_j\delta\hat{x}_j^T\}\Phi^T(k,j)\\
&= P(k|k-1)+\sum_{j=k-N+1}^{k-1}\Phi(k,j)\{P(j|j-1)-P(j|j)\}\Phi^T(k,j)
\end{aligned} \quad (40)$$

▲

Now, the cross the covariance $P_{si}(k)$ between N-step propagator and i-th local filter in Eq. (24) is considered. From

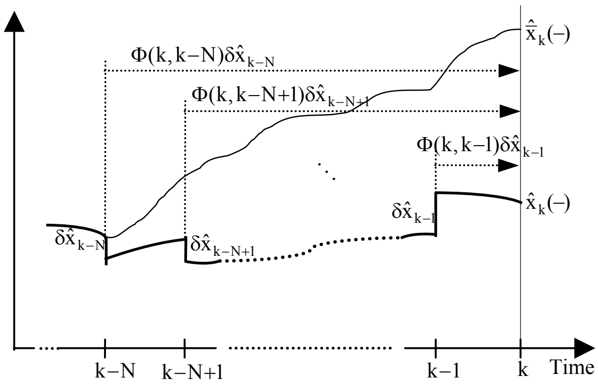

Fig. 2 Basic concept of N-step propagator.

the definition of cross covariance $P_{si}(k)$ can be written as

$$\begin{aligned}
P_{si}(k) &= E\{\bar{e}(k)[\hat{e}_i(k|k)]^T\}\\
&= E\{[\hat{e}_m(k|k-1)+\sum_{j=k-N+1}^{k-1}\Phi_m(k,j)\delta\hat{x}_j^m][\hat{e}_i(k|k)]^T\}\\
&= E\{[\hat{e}_m(k|k-1)][\hat{e}_i(k|k)]^T\}+E\{[\sum_{j=k-N+1}^{k-1}\Phi_m(k,j)\delta\hat{x}_j^m][\hat{e}_i(k|k)]^T\}
\end{aligned} \quad (41)$$

where subscript or superscript 'm' represents master filter and 'i' does i-th local filter.

In Eq. (41), the second term in right side equals to zero by orthogonal projection lemma. Therefore the cross covariance $P_{si}(k)$ is derived as follows.

$$\begin{aligned}
P_{si}(k) &= E\{[\hat{e}_m(k|k-1)][\hat{e}_i(k|k)]^T\}\\
&= P_{mi}(k|k-1)[I-K_i(k)H_i(k)]^T\\
&= \bar{P}_{mi}(k)
\end{aligned} \quad (42)$$

where $P_{mi}(k|k-1)$ is the cross covariance between master filter and i-th local filter, and $K_i(k)$ Kalman gain of i-th local filter. $\bar{P}_{mi}(k)$ is calculated in i-th local filter update, and so the covariance $P_{si}(k)$ can be easily calculated.

From Eq. (9), (33), (37), and (42), the test statistic in Eq. (24) can be computed, and it is used for fault detection of i-th Kalman filter in decentralized filter structure.

4. APPLICATION TO SDINS

The proposed fault detection methods for decentralized Kalman filter are applied to SDINS.with two navaid sensors, global position system (GPS) and position fix. The performance of the methods is examined by computer simulation.

4.1 SDINS model

The dynamic equations of the position, velocity, and attitude of SDINS in navigation frame are given as follows[11]:

$$\dot{L}=\frac{v_N}{R_m+h} \quad (43)$$

$$\dot{l}=\frac{v_E}{(R_t+h)\cos L} \quad (44)$$

$$\dot{h}=-v_D \quad (45)$$

$$\dot{\mathbf{v}}^n=\mathbf{C}_b^n\mathbf{f}^b-(2\boldsymbol{\omega}_{ie}^n+\boldsymbol{\omega}_{en}^n)\times\mathbf{v}^n+\mathbf{g}^n \quad (46)$$

$$\dot{\mathbf{q}} = \frac{1}{2}\mathbf{q} * \boldsymbol{\omega}_{nb}^{b} \quad (47)$$

where L , l , and h represent latitude, longitude, and altitude, respectively. The scripts *i, e, n*, and *b* denote inertial, earth, navigation, and body frame, respectively. $\mathbf{v}^n = [v_N \ v_E \ v_D]^{\mathrm{T}}$ is a velocity vector, $\mathbf{g}^n$ is a gravity vector in navigation frame, $\mathbf{q} = [q_0 \ q_1 \ q_2 \ q_3]^{\mathrm{T}}$ is a quaternion, R_m is a meridian radius of curvature, R_t is a transverse radius of curvature, $\mathbf{f}^b$ is an accelerometer output, and $*$ represents a quaternion multiplication. The transformation matrix from body frame to navigation frame, $\mathbf{C}_b^n$, can be obtained using the quaternion as follows:

$$\mathbf{C}_b^n = \begin{bmatrix} q_0^2+q_1^2-q_2^2-q_3^2 & 2(q_1q_2-q_0q_3) & 2(q_1q_3+q_0q_2) \\ 2(q_0q_3+q_1q_2) & q_0^2-q_1^2+q_2^2-q_3^2 & 2(q_2q_3-q_0q_1) \\ 2(q_1q_3-q_0q_2) & 2(q_0q_1+q_2q_3) & q_0^2-q_1^2-q_2^2+q_3^2 \end{bmatrix} \quad (48)$$

The angular rates $\boldsymbol{\omega}_{ie}^n$, $\boldsymbol{\omega}_{en}^n$, and $\boldsymbol{\omega}_{nb}^b$ represent the turn rate of earth expressed in navigation frame, the turn rate of navigation frame with respect to the earth frame, the turn rate of body frame with respect to the navigation frame, respectively. These rates are expressed as

$$\boldsymbol{\omega}_{ie}^n = [\Omega_N \ \ 0 \ \ \Omega_D]^{\mathrm{T}} = [\Omega\cos L \ \ 0 \ \ -\Omega\sin L]^{\mathrm{T}} \quad (49)$$

$$\boldsymbol{\omega}_{en}^n = [\rho_N \ \ \rho_E \ \ \rho_D]^{\mathrm{T}} = [\dot{l}\cos L \ \ -\dot{L} \ \ -\dot{l}\sin L]^{\mathrm{T}} \quad (50)$$

$$\boldsymbol{\omega}_{nb}^b = \boldsymbol{\omega}_{ib}^b - \mathbf{C}_n^b(\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) \quad (51)$$

where Ω is the earth rotation rate, $\mathbf{C}_n^b$ is the transpose of $\mathbf{C}_b^n$, and $\boldsymbol{\omega}_{ib}^b$ is a gyro output. Navigation solutions for SDINS can be obtained by integrating Eqs. (43)–(47) with given initial conditions.

4.1.1 Error models of SDINS and inertial sensors

The error model of SDINS can be obtained by the perturbation method under several assumptions[11].

$$\delta\dot{L} = \frac{\rho_E}{R_m+h}\delta h + \frac{1}{R_m+h}\delta v_N \quad (52)$$

$$\delta\dot{l} = \rho_N \sec L \tan L \ \delta L - \frac{\rho_N \sec L}{R_t+h}\delta h + \frac{\sec L}{R_t+h}\delta v_E \quad (53)$$

$$\delta\dot{h} = -\delta v_D \quad (54)$$

$$\delta\dot{\mathbf{v}}^n = [\mathbf{C}_b^n\mathbf{f}^b]\times\boldsymbol{\varphi} - (2\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n)\times\delta\mathbf{v}^n + \mathbf{C}_b^n\delta\mathbf{f}^b + \mathbf{v}^n\times(2\delta\boldsymbol{\omega}_{ie}^n + \delta\boldsymbol{\omega}_{en}^n) \quad (55)$$

$$\dot{\boldsymbol{\varphi}} = -\boldsymbol{\omega}_{in}^n \times \boldsymbol{\varphi} - \mathbf{C}_b^n\delta\boldsymbol{\omega}_{ib}^b + \delta\boldsymbol{\omega}_{in}^n \quad (56)$$

where δL, δl, and δh are errors of latitude, longitude, and altitude, respectively, $\delta\mathbf{v}^n = [\delta v_N \ \delta v_E \ \delta v_D]^{\mathrm{T}}$ is a velocity error vector in navigation frame, $\boldsymbol{\varphi} = [\varphi_N \ \varphi_E \ \varphi_D]^{\mathrm{T}}$ is a tilt angle vector that is approximately equal to the Euler-angle error under small angle assumption, and $\delta\boldsymbol{\omega}_{ie}^n$, $\delta\boldsymbol{\omega}_{en}^n$, and $\delta\boldsymbol{\omega}_{in}^n$ are defined as

$$\delta\boldsymbol{\omega}_{ie}^n = [-\Omega\sin L \ \delta L \quad 0 \quad -\Omega\cos L \ \delta L]^{\mathrm{T}} \quad (57)$$

$$\delta\boldsymbol{\omega}_{en}^n \approx \begin{bmatrix} -\frac{\rho_N}{R_t+h}\delta h + \frac{1}{R_t+h}\delta v_E \\ -\frac{\rho_E}{R_m+h}\delta h - \frac{1}{R_m+h}\delta v_N \\ -\rho_N\sec^2 L \ \delta L - \frac{\rho_D}{R_t+h}\delta h + \frac{\rho_D}{v_E}\delta v_E \end{bmatrix} \quad (58)$$

$$\delta\boldsymbol{\omega}_{in}^n = \delta\boldsymbol{\omega}_{ie}^n + \delta\boldsymbol{\omega}_{en}^n . \quad (59)$$

In Eqs. (55) and (56), $\delta\mathbf{f}^b$ is an accelerometer error vector and $\delta\boldsymbol{\omega}_{ib}^b$ is a gyro error vector. These inertial sensor errors may be simply modeled as a sum of random constant bias and white noise.

4.1.2 Error models of measurements

The two aided sensors, GPS and position fix, are used to compensate for the navigation errors of SDINS. Errors of aided sensors are simply modeled as additive white gaussian noise in our concern.

We assume that GPS measurements have velocity and position information and position fix have position information in navigation frame

4.2 Simulation

The proposed fault detection methods are simulated to verify those performances. The filter model used in the simulation has sixteen state variables: three position errors, three velocity errors, four quaternion errors, and six gyro and accelerometer biases. As discussed in the previous section, measurement models include variables of velocity and position.

In the simulation, it is assumed that GPS measurements are sampled with a frequency of 1Hz and position fix measurements can be obtained at 0.5Hz. The reference trajectory for the simulation is that the initial position of the vehicle is assumed to be 37deg in latitude, 127deg in longitude, 0m in altitude. The speed is constant at 250m/s with the exception of the initial stage. The vehicle changes its attitude with turning rates of 30deg/s in yaw at 100 sec. In Table 1, the standard deviations of sensor errors are shown.

Table 1. Noise characteristics of Sensors noise

	Characteristic	Magnitude (1σ)
Gyro	Bias White noise	3 deg/h 0.35 deg/h
Accelerometer	Bias White noise	1 mg 50 μg
GPS Velocity Position	White noise White noise	0.2 m/s 10 m
Position measure	White noise	5 m

The position and velocity are considered for the test statistics; hence the test statistic has a chi-square distributed with 6 degree of freedom. The used false alarm (FAR) rate is 0.1%. We assume two kinds of faults, GPS jamming and position measure bias in longitude at 120 sec. For the assumed GPS jamming, the measurement noise strengths is increased to 5 times at the fault time, and the magnitude of the added bias in longitude data for position measurement fault is 5 sigma.

Fig. 3 shows the test statistics for the two state propagators method discussed in section 3.1, and Fig. 4 shows the test statistics for the N-step propagators method discussed in section 3.2, when GPS jamming is assumed as fault model. The fault in GPS filter is detected at 124 sec in both methods. But, as previously discussed, test statistics in Fig. 3 change more abruptly than those in Fig. 4. The reset interval of two state propagators is 20 sec, so the test statistics in Fig. 3 change at every 10 sec, in particular, in case of position measure filter.

Figs. 5 and 6 represent the test statistics and longitudinal errors, respectively, in the case of the position measure bias in longitude at 120 sec. The N-step propagator is used as a fault detection method for this case. The fault is normally detected at 122 sec. In Fig. 6, it is noted that the master filter is more accurate than other two filters, which is the basics of decentralized filter, before the fault exists.

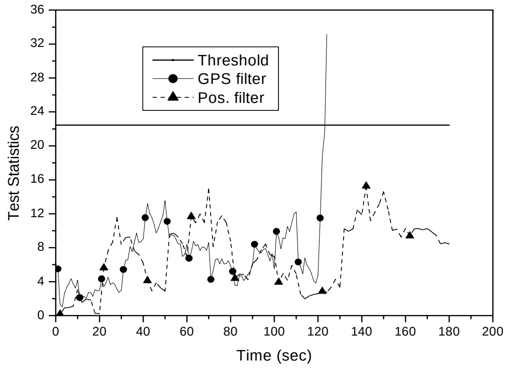

Fig. 3 Test statistics for two propagators with GPS fault

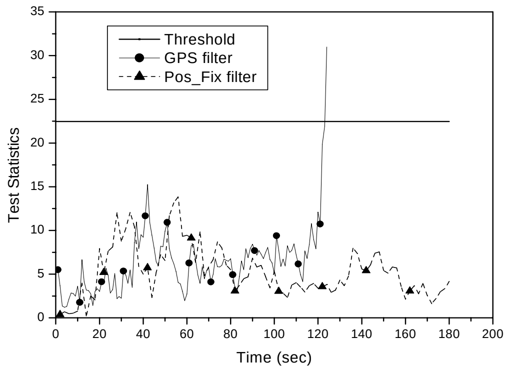

Fig. 4 Test statistics for N-step propagator with GPS fault

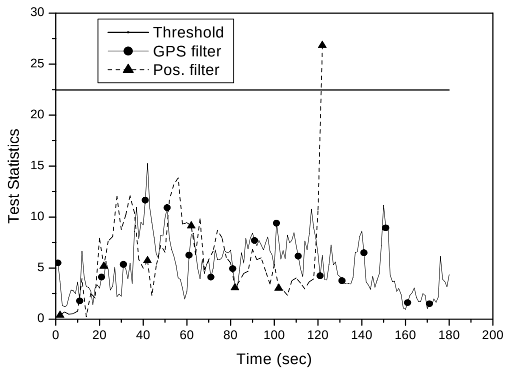

Fig. 5 Test statistics for N-step propagator with position measurement fault

5. CONCLUSION

In this paper, we are proposed the fault detection methods in decentralized filter structure. The proposed methods are the two propagators method and the N-step propagator method. The previously studies of a fault detection for Kalman filter are concerned about a centralized filter structure. But, the proposed methods are concerned about a decentralized filter structure and nonlinear system.

The proposed fault detection methods are applied to SDINS with two aided sensors, and its performance is examined by computer simulation. In the simulation, the proposed methods

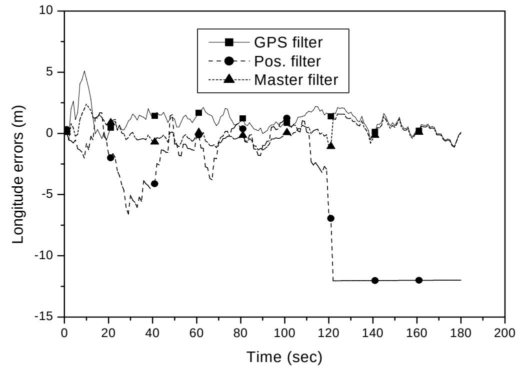

Fig. 6 Longitude errors for N-step propagator with GPS fault

detect a fault well. In case of two propagators method, the test statistics change abruptly at the propagator reset time, which may be a cause of increasing false alarm rate. On the other hand, in the N-step propagator method, the test statistics do not abruptly change.

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