Improvement of Group Delay and Reduction of Computational Complexity in Linear Phase IIR Filters

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Abstract: A technique for realizing linear phase IIR filters has been proposed by Powell-Chau which gives a real-time implementation of H(z-1).H(z), where H(z) is a causal nonlinear phase IIR filter. Powell-Chau system is linear but not time-invariant system. Therefore, that system has group delay response that exhibits a minor sinusoidal variation superimposed on a constant value. In the signal processing, this oscillation seriously degrade the signal quality. Unfortunately, that system has a large sample delay of 4L and also more computational complexity. Proposed system is present a reduced computational complexity technique by moved the numerator polynomial of H(1/z) out to cascade with causal filter H(z) and remain only all-pole of H(1/z), then applied truncated infinite impulse response to finite with truncated IIR filtel $H_L(z)$ and L sample delay to subtract the output sequence from the top and bottom filter. Proposed system is linear time invariance and group delay response and total harmonic distortion are also improved.

Keywords: All-pass filter, elliptic IIR digital filter, Perfect linear phase IIR filters, Trancated IIR filter, Complementary magnitude response

1. INTRODUCTION

In Digital Signal Processing applications, the linear phase characteristic of digital filters is important. Although an accurate linear phase characteristic can be realized by FIR filter, it is desired to realize such a phase characteristic using an IIR filter because the computational complexity is greatly decreased. However, it is impossible to realize an accurate linear phase characteristic with IIR filter based on a time-invariant system and real-time process. To overcome this problem, the paper present a system analytic in time domain with state space representation and the condition of the system has perfect linear phase characteristic and to be linear time invariant. A new technique is based on an approximating the infinite impulse response of H(z) to a finite length (L). is also analyzed

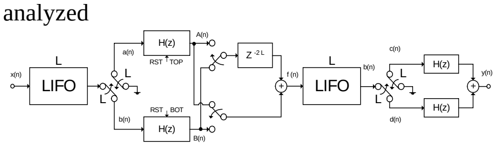

Fig. 1 The real-time implementation of linear phase IIR filter with overlap-add method

A technique for realizing linear phase IIR filter on real-time realization has been proposed by Powell and Chau Unfortunately, The filter is a time-variant system and the group delay response of the system has a minor amplitude and short period sinusoidal oscillation superimposed In this paper, the system is analyzed in time domain with state-space representation and a condition that the system has perfect linear phase characteristic and to be linear time-invariant system is described.

In their method, show in Fig. The input signal is divided into L-sample using two IIR Filter whose transfer function are

$$H_1(z) \cdot H_2(z) = H(z) \quad (1a)$$

$$H_1(z) = \frac{1}{B(z)} \quad (1b)$$

$$H_2(z) = A(z) \quad (1c)$$

$$H(z) = \frac{a_0 + a_1 z^{-1} + ... + a_7 z^{-7}}{1 - b_1 z^{-1} - b_2 z^{-2} - ... - b_7 z^{-7}} \quad (2)$$

$$H_1(z) = \frac{1}{1 - b_1 z^{-1} + ... + b_7 z^{-7}} \quad (3)$$

$$H_2(z) = a_0 + a_1 z^{-1} + ... + a_7 z^{-7} \quad (4)$$

$$H_{eq}(z) = H_1(z^{-1}) \cdot H_2(z) \quad (5)$$

this

$$H_2(z) = \frac{N_2(z)}{D_2(z)} \;, \quad H_1(z^{-1}) = \frac{N_1(z)}{D_1(z)} \quad (6)$$

$$H_{eq}(z) = \left\{ \frac{1}{D(z^{-1})} \right\} H_1(z) \cdot H_2(z) \quad (7)$$

The transfer function H(z) is usually of elliptic type. It has been shown that the proposed procedure we applied three filter H(z) and two $H_L(z)$, where $H_L(z)$ is the subfilter of H(z). The time chart of noncausal filter is illustrated in Fig.

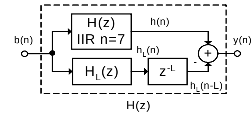

Figure 2. Approximating the infinite impulse response of H(z) truncated impulse response with a fixed length L is express as follows

$$h_L(n-L) = h(n), \quad n \geq L \qquad (8)$$
$$= 0 \qquad n < L$$

and the transfer function of IIR filter direct form II(n=2) can wrintten in a rational polynomial. Therefore, the output sequence of truncated impulse response in Fig. 2 yields,

$$y(n) = h(n) - h_L(n-L) \qquad (9)$$

When we applied a truncated impulse response in Fig. 2 to be a nonlinear phase IIR filters the (H(z)) of Powell and Chau system, becomes to linear time invariante system and perfect linear phase is obtained. Unfortunetely, this system has a large sample delay processing of 2L, in a some signal processing system application a long delay is not desired. Hence, those system can be improved by increasing a subfilter path of noncausal filter section from two subfilter H(z) paths to three subfilter H(z) paths as shown in Fig. 3. and time chart of proposed system is shown as Fig. 4 are respectively.

2. TIME DIVISION ANALYSIS

Proposed system is analyzed in time domain with state-space representation for SISO (single input single output) model [3] and Fig. 4 shown the operation of noncausal filter section in block processing. Output sequence from LIFO is divided section by section with length M samples. It can be written as follows

$$c(nM) = \begin{cases} J \cdot u((n-1)L) & ; n > 0 \\ 0 & otherwise \end{cases} \qquad (10)$$

where block sequence input(u(n.L)) and output (c(m.L)) of LIFO can be written in vector as follows

$$X_p(n) = \left[X_{p1}(n)\, X_{p2}(n) \ldots X_{pm}(n)\right]^T \in R^{m \times 1} \qquad (11)$$

$$u(n \cdot L) = [u(n \cdot L) \ldots\ldots u((n+1)L-1)]^T \in \Re^{Mx1} \qquad (12)$$

$$c(n \cdot L) = [c(n \cdot L) \ldots\ldots c((n+1)L-1)]^T \in \Re^{Mx2} \qquad (13)$$

We show that Powell-Chau system does not have a complete linear phase characteristic by using the state space representation. As a result, the condition which becomes a complete linear phase is led. And we show that the system is a time-invariant system. The outline is described as follows. We lead the state space representation of the Fig

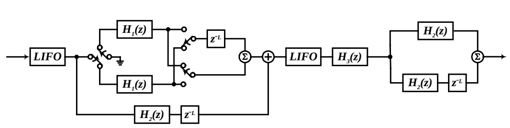

Fig. 3 Proposed system with time division multiple method and J is the MxM exchange matrix defined as

$$J = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \Re^{M \times M} \qquad (6)$$

The output block sequence of each subfilter paths can be written as follows

$$C_p(n.L) = \begin{cases} C(n.L) \;\; ; P = (n+1) \;\; ; n = (N+1)i + P \\ C(n.L) \;\; ; P = (n+1) \;\; ; n = (N+1)i \;\; ; i = 0,1,2\ldots \\ 0 \qquad ; otherwise \end{cases} \qquad (7)$$

Eq. (7) in a vector form is written as follows

$$c_P(nL) = [c_P(nL) \cdots c_P((nL)L-1)] \in \Re^{M\,x\,1} \qquad (8)$$

Where number path of IIR filters (p) = 1,2, N+1 , then, proposed system has N equal 2 and p = 3(for Powell and Chau system N=1 and p =2). H(z) is refered to each transfer function and M equal to block length when the input sequence is sectioned by LIFO and swiching(proposed system M = L/2). We can rewritten the state space equation for block processing as follows

$$X_p(n+1) = A \cdot X_p(n) + b \cdot a_p(n) \qquad (9)$$

$$Y_p(n) = C \cdot X_p(n) + d \cdot a_p(n) \qquad (10)$$

$$X_p(2(n+1)L) = B'.C_p(2.n.L) \qquad (11)$$

$$Y_p(2.n.L) = C'.X_p(2.n.L) + D'.C_p(2.n.L) \qquad (12)$$

If we resetting every 2L sample, the state vector yields

$$x_p((n+1)L(N+1) = 0 \qquad (13)$$

And the dimension of matrix and vector can be written as follows

$$A \in R^{MxM}, b \in R^{Mx1}, c \in R^{1xM}, d \in R^{1x1} \qquad (14)$$

$$X_P(n) = \left[X_{P1}(n) \ldots X_{PM}(n)\right]^T \in R^{Mx1} \qquad (15)$$

the output sequence of each subfilter

$$y_p((n'+P+n)L) = D' \cdot \begin{bmatrix} c_P((n'+p)L) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (16)$$

where $n' = (N+1)(n-1)$

each output sequence can be written in vector form as follows

$$y_P(nL) = [y_P(nL) \cdots y_P((nL)L-1)] \in \Re^{M\,x\,1} \qquad (17)$$

Therefore, general output sequence form of each subfilter can be written as follows

$$y_P((n'+p+q)M) = D_q \cdot c((n'+p)M) \tag{18}$$

$$y_P((n'+p+q)M) = D_q \cdot J \cdot c((n'+p-1)M) \tag{19}$$

where q=0,1,N and M=2 and all D matrix can be written as follows

$$C_p(2.n.L) = \begin{bmatrix} C_p(2.n.L + P.L - 2.L) \\ C_p(2.n.L + P.L - 2.L + 1) \\ \vdots \\ C_p(2.n.L + P.L - 1) \end{bmatrix} \in R^{m(n+1)\times 1} \tag{20}$$

$$Y_p(2.n.L) = \begin{bmatrix} Y_p(2.n.L + P.L - 2.L) \\ Y_p(2.n.L + P.L - 2.L + 1) \\ \vdots \\ Y_p(2.n.L + P.L - 1) \end{bmatrix} \in R^{2.L\times 1 = m(n+1)\times 1} \tag{21}$$

$$X_p(2(n+1)L) = \begin{bmatrix} X_{p1}((2.n+P)L) \\ X_{p2}((2.n+P)L) \\ \vdots \\ X_{pm}((2.n+P)L) \end{bmatrix} \in R^{m\times 1} \tag{22}$$

$$D_q = D_0 ; D_1$$

$$D_0 = \begin{bmatrix} d & 0 & & & \cdots & 0 & 0 \\ C.b & d & & & \cdots & 0 & 0 \\ C.A.b & C.b & d & & \cdots & 0 & 0 \\ C.A^2.b & C.A.b & C.b & d & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ C.A^{L-3}.b & C.A^{L-4}.b & & & \cdots & d & 0 \\ C.A^{L-2}.b & C.A^{L-3}.b & C.A^{L-4}.b & & \cdots & C.b & d \end{bmatrix} \tag{23}$$

$$D_1 = \begin{bmatrix} C.A^{L-1}.b & C.A^{L-2}.b & \cdots & C.A.b & C.b \\ C.A^{L}.b & C.A^{L-1}.b & \cdots & C.A^2.b & C.A.b \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ C.A^{-3}.b & C.A^{-4}.b & \cdots & C.A^{L-1}.b & C.A^{L-2}.b \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{24}$$

the output sequence of noncausal filter is

$$d(nM) = \begin{cases} \sum_{q=1}^{N} J \cdot D_q \cdot J \cdot u((n-(2N+2-q))L) \\ 0 \qquad ; \qquad n \le N+1, N=2 \end{cases} \tag{25}$$

at the causal filter section, output from noncausal filter section b(nL) is divided into two paths, first path applied to the top IIR filter

$$d_1(n.L) = \begin{cases} \begin{bmatrix} d(n.L) \\ d((n+1)L) \end{bmatrix} & ;\ n = odd \\ 0 & ; otherwise \end{cases} \tag{26}$$

and other is applied to trancated IIR filter and shifted by L version

$$d_2(n.L) = \begin{cases} \begin{bmatrix} d(n.L) \\ d((n+1)L) \end{bmatrix} & ;\ n = even \\ 0 & ; otherwise \end{cases} \tag{27}$$

or in the vector form is

$$d_i(nL) = \left[d_i(nL) \cdots d_i((n+1)L-1)\right]^T \in \Re^{L\,x2} \tag{28}$$

when T = matrix Transpose and output of the H(z) filter are

$$y_1((2n-1)NL) = D_L \begin{bmatrix} d((2n-1(NL)) \\ \vdots \\ d((2n-1(N+N-1)L) \end{bmatrix} \tag{29}$$

$$y_1((2n-1)NL) = D_T \begin{bmatrix} d((2n-1(NL)) \\ \vdots \\ d((2n-1(N+N-1)L) \end{bmatrix} \tag{30}$$

$$y_1((2n-1)NL) = D_L \begin{bmatrix} d((2n-1(NL)) \\ \vdots \\ d((2n-1(N+N-1)L) \end{bmatrix} \tag{31}$$

$$y_2((2n-1)NL) = D_T \begin{bmatrix} d((2n-1(NL)) \\ \vdots \\ d((2n-1(N+N-1)L) \end{bmatrix} \tag{32}$$

where D_L and D_T can be written in a matrix form as follows

$$D_L = \begin{bmatrix} d & 0 & \cdots & 0 & 0 \\ cb & d & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ cA^{L-3}b & cA^{L-4}b & \cdots & d & 0 \\ cA^{L-2}b & cA^{L-3}b & \cdots & cb & d \end{bmatrix} \in \Re^{LxL} \tag{33}$$

$$D_T = \begin{bmatrix} cA^{L-1}b & cA^{L-2}b & \cdots & cA\ b & cA\ b \\ cA^{L}b & cA^{L-1}b & \cdots & cA^2b & cAb \\ \vdots & \vdots & & \vdots & \vdots \\ cA^{2L+3}b & cA^{2L+4}b & \cdots & cA^{L-1}b & cA^{L-2}b \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \Re^{L\,xL} \tag{34}$$

and the final sequence output of the entire system is the summation of each subfilter output sequence

$$y(nL) = y_1(nL) + y_2(nL) \tag{35}$$

or in vector form is as follows

$$y_i(nL) = \left[y_i(nL) \cdots y_i((n+1)L-1)\right]^T \in \Re^{L\,x\,1}; i = 1,2 \tag{36}$$

we can written entire output sequence in summation form as follows

$$y(nL) = \sum_{q=l}^{N} \cdot D_e \sum_{q=k}^{N} J \cdot D_R \cdot J \cdot u((n-e-(2N+2-k))L) \tag{37}$$

Unfortunately, entire output sequence of Eq. 32 in case of N=2 shows that system is linear but time variante when we applied is impulse sequence as input sequence. Therefore, this system is not LTI (linear-time invariante) and its is show that impulse response is not symmetry around 3L and perfect linear phase characteristic is not obtained. For the symmetry impulse response, we choose accusatory truncated as follows

At the noncausal filter

$$cA^R b = 0 \quad ; R \geq L \tag{38}$$

At the causal filter

$$cA^R b = 0 \quad ; R \geq L \tag{39}$$

then we applied Eq. 40 to the D matrixs of noncausal and cuasal filter sections, we can rewriten D2 and DT are follows

$$D_{2T} = \begin{bmatrix} 0 & cA^{MN-2}b & \cdots & cA^{M+1}b & cA^{M}b \\ 0 & 0 & \cdots & cA^{M+2}b & cA^{M+1}b \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & cA^{2M-2}b \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \Re^{M\,xM} \tag{40}$$

$$D_{TT} = \begin{bmatrix} 0 & cA^{L-2}b & \cdots & cAb & cb \\ 0 & 0 & \cdots & cA^{2}b & cAb \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & cA^{L-2}b \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \Re^{L\,x1} \tag{41}$$

From Fig.7) shows an impulse response of proposed system which is symmetry around 4L and first sample and output length appear at 2L therefore, our system can reduced sample delay by L.

3. A REDUCE OF COMPUTTATIONAL COMPLEXITY METHOD

Filter specification pole-zero plot of prototype and pole-zero of noncausal and causal filter of proposed system

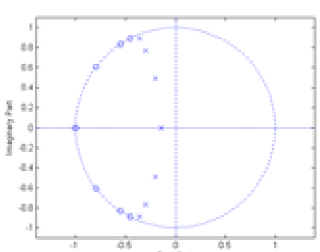

Fig. 4 comparison of group delay variation

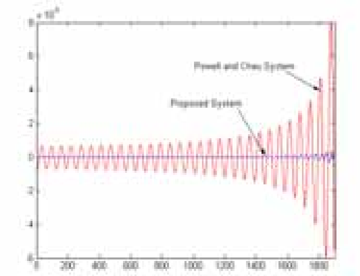

Fig. 5 Group delay variation of Powell and Chau system and proposed system

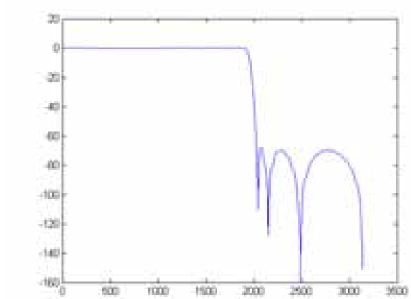

Fig. 6 Magnitude Response of proposed system

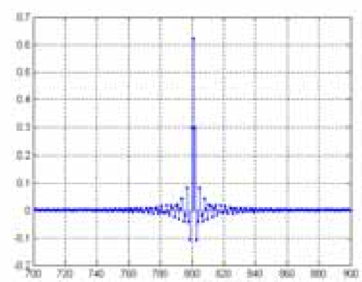

Fig. 7 Impulse Response of proposed system sample delay is 4L

In general, the residual impulse response is governed dominantly by the poles of the transfer function nearest to the unit circle in the z plane. Therefore, $H_L(z)$ can be approximated by the 2nd-order function , $H_L(z)$ using the pair of complex conjugate poles nearest to the unit circle. Denoting the conjugate poles as Z1 and Z1, the numerator of $H_L(z)$ are determined by equating the first three impulse response samples of $H_L(z)$

4. SIMULATION

In general, the residual impulse response [4] is governed dominantly by the poles of the transfer function nearest to the unit circle in the z plane. Therefore, $H_L(z)$ can be approximated by the 2^{nd} order function $\tilde{H}_L(z)$ using the pair of complex conjugate poles nearest to the unit circle. Denoting the conjugate poles as Z_1 and $\overline{Z}_1$, the numerator of $\tilde{H}_L(z)$ are determined by equating the first three impulse response written samples of $\widetilde{H}_L(z)$ with the residual response as follows.

$$\widetilde{H}_L(z) = \frac{e_0 z^2 + e_1 z + e_2}{(z - z_1)(z - \bar{z}_1)} = \frac{e_0 + e_1 z^{-1} + e_2 z^{-2}}{1 + f_1 z^{-1} + f_2 z^{-2}} \qquad (38)$$

Table I: coefficience of $\widetilde{H}_L(z)$

e_0	h(L)	1.825 x 10-5
e_1	b_2 x h(L-1)	3.8879 x 10-6
e_2	0	0
f_1	-	0.7085589
f_2	-	0.9175521

An elliptic function H(z) is designed according to the following specification :
Normalized frequency edge frequency Fp=0.3,
Normalized stop band edge frequency Fs=0.325,
Pass band ripple δ_p = 0.005 dB, Stop band ripple δ_s = 35dB. The design H(z) of n=7 is expressed as a sum of two all pass sub functions [5] as described in table II

$$H(z) = \frac{1}{2}\left[\frac{a_0 + z^{-1}}{1 + a_0 z^{-1}} \cdot \frac{b_0 + b_1 z^{-1} + z^{-2}}{1 + b_1 z^{-1} + b_0 z^{-2}} + \frac{c_0 + c_1 z^{-1} + z^{-2}}{1 + c_1 z^{-1} + c_0 z^{-2}} \cdot \frac{d_0 + d_1 z^{-1} + z^{-2}}{1 + d_1 z^{-1} + d_0 z^{-2}}\right] \qquad (39)$$

Table II: coefficience of H(z)
(elliptic IIR filter n=7)

a_0=0.1404000	
b_0=0.6832507	b_1=0.6008522
c_0=0.2868453	c_1=0.4101568
d_0=0.9175524	d_1=0.7085589

Table III Computational complexity

	Multiplication	additions
(a) Proposed system with 2th-order $H_L(Z)$	37	45
(b) Proposed system withtime division multiple method direct$H_L(z)$	41	49
(c) Limited-cycle-free Powell-Chau system	28	62
(d) FIR	73	145

The numbers of multiplications and additions per sample for various systems of L=200 are shown in table III. (b) is the proposed system as show in Fig. 3 which use three H(z) and two $\widetilde{H}_L(z)$. The numbers of multiplication and additions per sample are 41 and 49 respectively, under those of the Table III (b) and less than of FIR Table III(c)

5. CONCLUTION

The proposed system in time domain with state-space equation representation is analyzed, and a condition that the system has perfect linear phase characteristic and to be linear time-invariant system is shown. Simulated result of group delay variation response is differential small than the Powell and Chau system, when we applied a perfect linear phase condition to three subfilter H(z) and with the computational complexity reduced by time division multiple method.

REFERENCE

[1] S. R. Powell and P. M. Chau, " A technique for realizing linear phase IIR filters," IEEE Trans. Signal Processing, Vol. 39, No. 11, pp. 2425-2435, Nov. 1991.

[2] B. Djokic, M. Popavic and M. Lutovac, "A new improvement to the Powell and Chau linear phase IIR filters," IEEE Trans. Signal Processing, Vol. 46, No. 6, pp. 1685-1688, June 1998.

[3] P. M. Derusso, R. J. Roy and C. M. Close,"State Variables for Engineers," John Wiley and Sons, New York, 1965.

[4] S. Varasumanta D.Sookcharoenphol, U. Sriteraviroj and K. Janjiraphongvej al.,"Truncating of infinite impulse response of linear phase IIR filters and their group delay effect", The Proceedings of 41st Kasetsart University Annual Conference, 2003.

[5] P. P. Vaidyanathan, S. K. Mitra and Y. Neuvo, "A new approch to the realization of low sensitivity IIR filters," IEEE Trans. ASSP., Vol. 34, No. 2, pp. 350-361, Apr. 1986.

[6] D. Sookcharoenphol*, K. Janjitrapongvej*, S. Varasumanta* and S. Tomiyama** An Analysis of Linear Phase IIR Filters and a Condition for Perfect Linear Phase