Compensator Design for Linear Systems with Random Delay.

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Abstract: Modern control systems often use a communication network to send measurement and control signals between nodes. Communication delays can be time varying. The length of the time delays is often hard to predict and are modeled as being random. This paper proposes a combined controller used to compensate network time delay by estimating the delay with the interacting multiple model (IMM). The network delay is modeled as a Markov chain and 3 modes representing heavy, medium, and low network loads are used in the IMM. The proposed method is applied to an optimal control system with double integrators and the results are compared with the existing control methods.

Keywords: Random delay, Combined controller,

1. INTRODUCTION

Modern control systems often use a communication network to send measurement and control signal between nodes. A common communication network reduces the cost of cabling, and offers modularity and flexibility in systems design. Communication delays in such network can vary in a random fashion. The reason for this can be e. g. interrupt driven events, data dependent computation times, use of dynamic schedules, collision or varying network load. Random varying distributed delays, induced by a computer communication network, may degrade stability and performance of systems because timely transfer of sensor and control signal from one device to another is not guaranteed. Then length of the time delays are hard to predict.

This paper proposes a combined controller used to compensate network time delay by estimating the delay with the interacting multiple model (IMM). The network delay is modeled as a Markov chain and 3 modes representing high, medium, and low network loads are used in the IMM. IMM approach consists of filters corresponding to the modes, a mode probability evaluator, and a combined controller utilizing the estimates of the filters. With the assumption that the mode switching is governed by an underlying Markov chain, the mixer uses the mode probabilities and the mode switching probabilities to compute a combined estimate and combined control input. The proposed method is applied to an optimal control system with double integrators and the results are compared with the existing control methods.

2. MODELING OF SYSTEM DELAY

In a real communication system the transfer time will usually be correlated with the last transfer delay. For example, the network load is one of the factors affecting the delay. The network load is typically varying at a slower than the sampling period in a control system. Time varying network load can be modeled as a Markov chain, in which transition between the states is governed by transition probabilities. In this paper, to get a simple network model it is assumed that the network has three states, one for low network load, one for medium network load, and one for high network load. In Fig. 1 the transition between different states in the communication network is modeled as a Markov chain. Besides every state in the Markov chain we have a corresponding delay distribution modeling for the network state. The model could typically look like the probability distribution in Fig. 2.

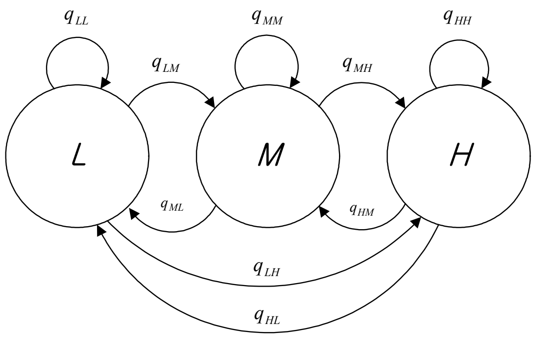

Fig. 1 Markov chain modeling of the state in a communication network.

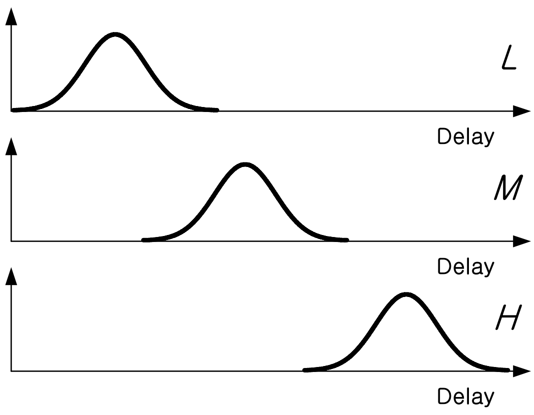

Fig. 2 The delay distribution corresponding to the state of the Markov chain in Fig. 1.

3. PROBLEM FORMULATION

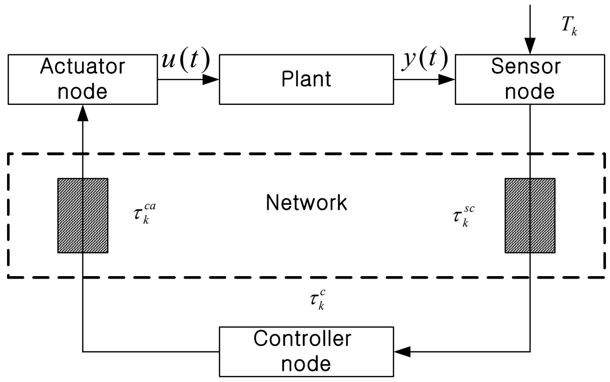

Fig. 3 Distributed digital control system with induced delays, τ^{sc}, τ^{c} and τ^{ca}.

In Fig. 3 the control system is illustrated in a block diagram. It is assumed that the sensor node is sampled regularly at a constant sampling period T. The communication delays τ^{sc} and τ^{ca} are randomly varying. All time delays are independent over the full horizon and their probability distributions are known a priori. The controller node is assumed to be event driven, i.e. upon arrival to the controller node the control signal is calculated and sent via the network to the actuator node. The total time delay is always less than one sampling period($\tau^{sc}+\tau^{ca}<T$). The computation time, τ^{c}, is included in τ^{ca}. The actuator node is assumed to be event driven, i.e. the control signal will be used as soon as the actuator arrives.

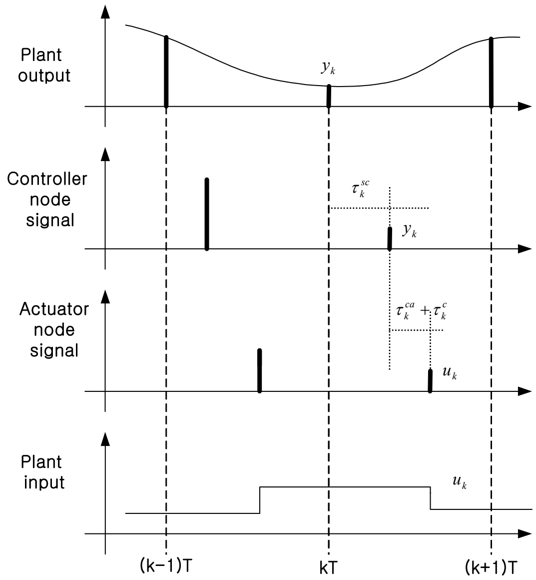

Fig. 4 Timing of signals in the control system with time delays.

Fig. 4 describes the effect of delay in the control system.

3.1 The Markov communication network

In order to analyze the influence of the delay on the system, a random variable denoted as τ_k is introduced. τ_k is a random variable with probability distribution given by the state of a Markov chain. For instance τ_k can be a vector with the delays in the loop, i.e. $\tau_k=[\tau_k^{sc},\tau_k^{ca}]^T$. The Markov chain has the state $\gamma_k\in\{1,\cdots,s\}$ when τ_k is generated. The Markov chain then makes a transition between k and $k+1$. The transition matrix for the Markov chain is $Q=\{q_{ij}\}$, $i,j\in\{1,\cdots,s\}$, where

$$q_{ij}=\Pr(\gamma_{k+1}=j\mid\gamma_k=i)\,. \tag{1}$$

The Markov state probability is denoted as

$$\pi_i(k)=\Pr(\gamma_k=i) \tag{2}$$

then the Markov state distribution vector is expressed as

$$\pi(k)=\begin{bmatrix}\pi_1(k) & \pi_2(k) & \cdots & \pi_s(k)\end{bmatrix}. \tag{3}$$

The probability distribution for γ_k is given by the recursion.

$$\pi(k+1)=\pi(k)Q\,,\quad \pi(0)=\pi_0\,, \tag{4}$$

where π_0 is the probability distribution for γ_0.

3.2 Discrete Time System with Delay

The controlled process is assumed to be of the form

$$\dot{x}(t)=Ax(t)+Bu(t)+v(t) \tag{5}$$

where $x(t)$ is the state vector, $u(t)$ is the input. Discretizing Eq. (5) at the sampling instance results in [4]

$$x_{k+1}=\Phi x_k+\Gamma_0(\tau_k^{sc},\tau_k^{ca})u_k+\Gamma_1(\tau_k^{sc},\tau_k^{ca})u_{k-1}+v_k \tag{6}$$

where

$$\Phi=e^{AT}\,,$$

$$\Gamma_0(\tau_k^{sc},\tau_k^{ca})=\int_0^{T-\tau_k^{sc}-\tau_k^{ca}}e^{As}ds B\,,$$

$$\Gamma_1(\tau_k^{sc},\tau_k^{ca})=\int_{T-\tau_k^{sc}-\tau_k^{ca}}^{T}e^{As}ds B\,.$$

The output equation is

$$y_k=Cx_k+w_k\,. \tag{7}$$

The random noise sequences v_k and w_k are uncorrelated white noises with zero mean and covariance R_1 and R_2 respectively.

4. OPTIMAL CONTROLLER

In this section we solve the optimal control problem to minimize the cost function

$$J_N=x_N^TQ_Nx_N+E\left\{\sum_{k=0}^{N-1}\begin{bmatrix}x_k\\u_k\end{bmatrix}^T Q\begin{bmatrix}x_k\\u_k\end{bmatrix}\right\}, \tag{8}$$

where

$$Q=\begin{bmatrix}Q_{11} & Q_{12}\\ Q_{12}^T & Q_{22}\end{bmatrix}$$

is symmetric, positive semi-definite, and of which Q_{22} is positive definite. Given the plant Eq. (6), with noise free measurement of the state vector x_k, and knowledge of the Markov state γ_k. The control law that minimizes the cost function Eq. (8) is given by

$$u_k^* = -L_k(\tau_k^{sc}, \gamma_k)\begin{bmatrix} x_k \\ u_{k-1}^* \end{bmatrix} \tag{9}$$

where, for $\gamma_k = i$, $i = 1,2,\cdots,s$, we have

$$L_k(\tau_k^{sc}, i) = (Q_{22} + \tilde{S}_i^{22}(k+1))^{-1}\left[Q_{12}^T + \tilde{S}_i^{21}(k+1) \quad \tilde{S}_i^{23}(k+1)\right]$$

$$\tilde{S}_i(k+1) = \underset{\tau_k^{ca}}{E}\left\{G^T \sum_{j=1}^{s} q_{ij} S_j(k+1) G \mid \tau_k^{sc}, \gamma_k = i\right\}$$

$$G = \begin{bmatrix} \Phi & \Gamma_0(\tau_k^{sc}, \tau_k^{ca}) & \Gamma_1(\tau_k^{sc}, \tau_k^{ca}) \\ 0 & I & 0 \end{bmatrix}$$

$$S_i(k) = \underset{\tau_k^{sc}}{E}\left\{F_1^T Q F_1 + F_2^T \tilde{S}_i(k+1) F_2 \mid \gamma_k = i\right\}$$

$$F_1 = \begin{bmatrix} I & 0 \\ -L_x(\tau_k^{sc}, \gamma_k) & -L_u(\tau_k^{sc}, \gamma_k) \end{bmatrix}$$

$$F_2 = \begin{bmatrix} I & 0 \\ -L_x(\tau_k^{sc}, \gamma_k) & -L_u(\tau_k^{sc}, \gamma_k) \\ 0 & I \end{bmatrix}$$

$$S_i(N) = \begin{bmatrix} Q_N & 0 \\ 0 & 0 \end{bmatrix}$$

$\tilde{S}_i^{ab}(k)$ is block (a,b) of the symmetric matrix $\tilde{S}_i(k)$, and Q_{ab} is block (a,b) of Q.

5. COMBINED CONTROLLER

The Interacting Multiple Model(IMM)[5] is known for its cost-effectiveness regarding computation complexity and performance. The IMM consists of 4 steps ; interaction, prediction, measurement update, and combination as depicted in Fig. 5. In this section we propose a combined controller.
The three modes representing low, medium, and high network loads as depicted in Fig. 2 are employed as the states of the Markov chain, and the IMM utilizes them as the hypotheses upon which Kalman filters are based as shown in Fig. 5. The three hypothesis, H_{k+1}^1, H_{k+1}^2 and H_{k+1}^3, corresponding to the delay are used in this study. For example, H_{k+1}^1 represents the low network load case, H_{k+1}^2, the medium network load case and H_{k+1}^3, the high network load case. At the sampling time k, interacting states and covariance are represented as follows

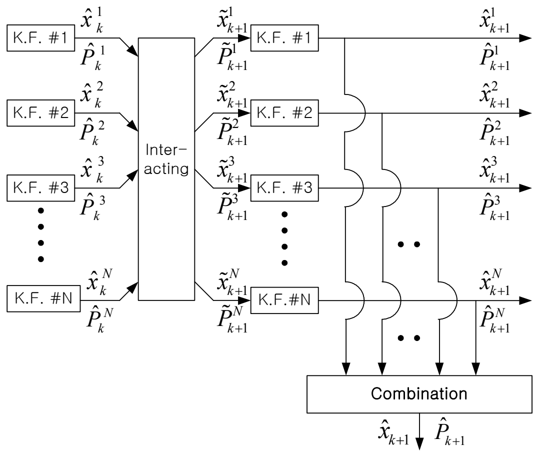

Fig. 5 Block diagram of Interacting Multiple Model.

<interacting step>

$$\tilde{x}_{k+1}^i = \frac{\sum_{j=1}^{3} \hat{x}_k^j q_{ij} \Pr(H_k^j \mid Z_k)}{\sum_{j=1}^{3} q_{ij} \Pr(H_k^j \mid Z_k)}, \quad \{i = 1,2,3\} \tag{10}$$

$$\tilde{P}_{k+1}^i = \frac{\sum_{j=1}^{3} (\hat{P}_k^j + \hat{x}_{k+1}^j \hat{x}_{k+1}^{j\,T}) q_{ij} \Pr(H_k^j \mid Z_k)}{\sum_{j=1}^{3} q_{ij} \Pr(H_k^j \mid Z_k)} - \tilde{x}_{k+1}^i \tilde{x}_{k+1}^{i\,T}. \tag{11}$$

where $q_{ij} = \Pr(\gamma_{k+1} = j \mid \gamma_k = i)$.

After $\tilde{x}_{k+1}^i$ and $\tilde{P}_{k+1}^i$ obtained by interacting, $\overline{x}_{k+1}^i$ and $\overline{P}_{k+1}^i$ can be obtained from the prediction step of filter.
<prediction step>

$$\overline{x}_{k+1}^i = \Phi \tilde{x}_{k+1}^i + \Gamma_0\{E(\tau_k^i)\} u_k^* + \Gamma_1\{E(\tau_k^i)\} u_{k-1}^* \tag{12}$$

$$\overline{P}_{k+1}^i = \Phi \tilde{P}_{k+1}^i \Phi^T \tag{13}$$

where $E(\tau_k^i)$ is the mean value of the i th delay mode, u_k^* and u_{k-1}^* are optimal control inputs at sampling period k and $k-1$. After the prediction step, each filter receives measurement z_{k+1} and each filter expressed in Eq. (7) calculates the mode probability by using the residual of each filter.
<mode probability calculation>

$$\Pr(H_{k+1}^i \mid Z_{k+1}) = \frac{f(z_{k+1} \mid H_{k+1}^i, Z_k) \sum_{j=1}^{3} q_{ij} \Pr(H_k^j \mid Z_k)}{\sum_{l=1}^{3} f(z_{k+1} \mid H_{k+1}^l, Z_k) \sum_{m=1}^{3} q_{lm} \Pr(H_k^m \mid Z_k)} \tag{14}$$

Updated states, updated covariance and control input are calculate in the update step based on each hypothesis.
<update step>

$$\hat{x}^i_{k+1} = \overline{x}^i_{k+1} + K^i_{k+1}(z_{k+1} - C\overline{x}^i_{k+1}) \quad (15)$$

$$\hat{P}^i_{k+1} = (I - K^i_{k+1}C)\overline{P}^i_{k+1}(I - K^i_{k+1}C)^T + K^i_{k+1}R_{k+1}{K^i_{k+1}}^T \quad (16)$$

$$K^i_{k+1} = \overline{P}^i_{k+1}C^T(C\overline{P}^i_{k+1}C^T + R_{k+1})^{-1} \quad (17)$$

$$u^i_{k+1} = -L(\tau^i)\begin{bmatrix} \hat{x}^i_{k+1} \\ {u_k}^* \end{bmatrix} \quad (18)$$

where $L(\tau^i)$ is optimal control gain for the hypothesis H^i_{k+1}. Finally, one can obtain the combined state estimate and the combined covariance as follows
<combination step>

$$\hat{x}_{k+1} = \sum_{i=1}^{3} \hat{x}^i_{k+1} \Pr(H^i_{k+1} \mid Z_{k+1}) \quad (19)$$

$$\hat{P}_{k+1} = \sum_{i=1}^{3} (\hat{P}^i_{k+1} + (\hat{x}^i_{k+1} - \hat{x})(\hat{x}^i_{k+1} - \hat{x})^T) \Pr(H^i_{k+1} \mid Z_{k+1}) \quad (20)$$

In this paper, we propose a combined controller to control the system with random delay. The proposed controller is obtained as follows.
<combined control>

$${u_{k+1}}^* = \sum_{i=1}^{3} \Pr(H^i_{k+1} \mid Z_{k+1}) u^i_{k+1} \quad (21)$$

Note that ${u_{k+1}}^*$ is the combined control input composed of control inputs derived based on each hypothesis and mode probabilities.

6. SIMULATION RESULT

In this section, the performance of combined control method is compared with the exiting control methods such as the uncompensated controller that assumes $\tau_k = 0$ and the controller of [2] that uses the mean value of τ to calculate the controller gain of [2]. Then the system model with double integrators is employed and the total time delay τ is between 0 and sampling period T.
System equations are written as follows.

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u,$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x.$$

where initial control input and state transition probability matrix are denoted as

$u(0) = 5$, $T = 0.15$

$$Q = \begin{bmatrix} 0.85 & 0.1 & 0.05 \\ 0.1 & 0.8 & 0.1 \\ 0.05 & 0.1 & 0.85 \end{bmatrix}.$$

The control effort is shown in Fig. 6 where $J(t_N)$ is the overall control effort. In the figure, the double dashed line represents the uncompensated controller, the dashed line represent the controller of [2], and the solid line represents the combined controller. In Fig. 6, cost of each controller is almost same for small delay cases. The cost of the uncompensated controller is increased for medium delay and the combined controller is be shown to have the smallest cost for large delay.

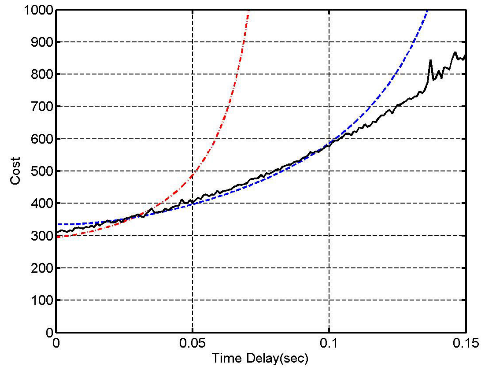

Fig. 6 Cost functions of the three controllers

Figs. 8~13 illustrate the plant output and the control input of all the three controllers for the random delay history shown in Fig. 7. In Figs. 8~9, plant output and control input of the uncompensated controller are shown to be slow and fluctuating to reach the steady state, while Figs. 10~11 show the performance of the controller of [2] while Figs. 12~13 show the performance of the combined controller. In the figures, it is obvious that the combined controller has superior performance over the controller of [2] in both transient response and stability.

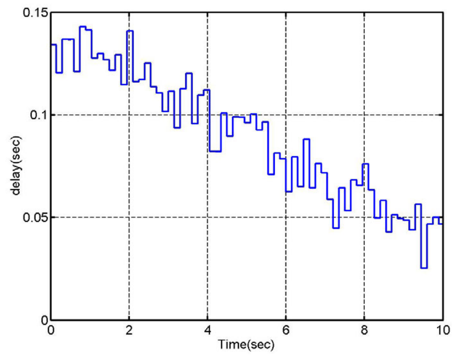

Fig. 7 Random delay history

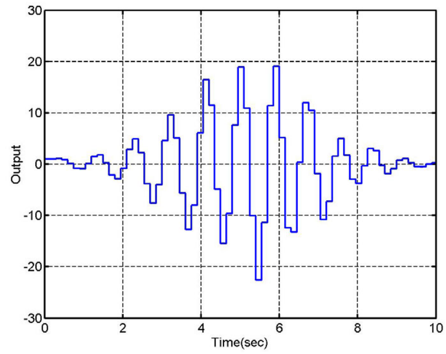

Fig. 8 Plant output using the uncompensated controller.

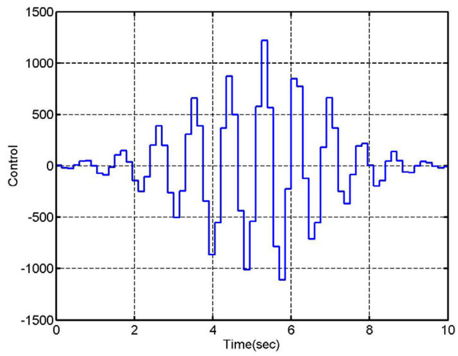

Fig. 9 Control input using the uncompensated controller

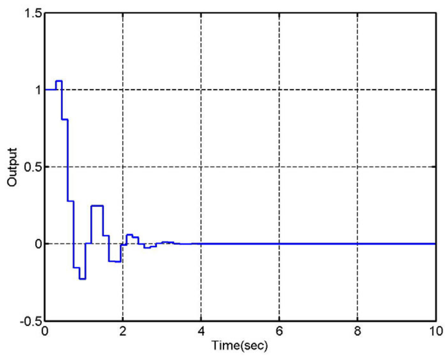

Fig. 10 Plant output using the controller of [2]

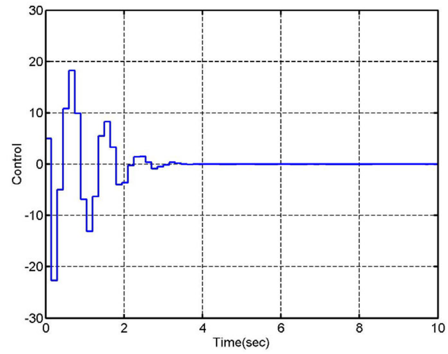

Fig. 11 Control input using the controller of [2]

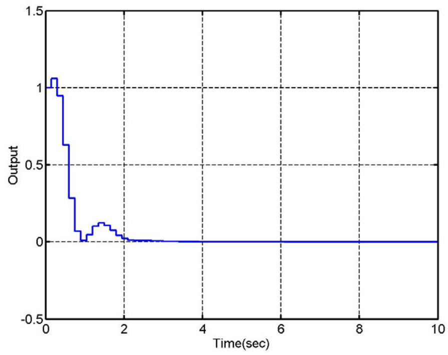

Fig. 12 Plant output using the combined controller.

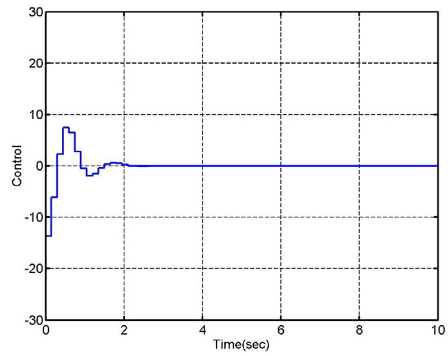

Fig. 13 Control input using the combined controller.

7. CONCLUSION

In this paper we propose a combined control scheme based on the IMM structure to compensate the random delay induced in a network system. The combined controller is shown to have superior performance over the existing controllers.

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