

# Aeroelastic Response of Wing-Flap system using Robust Control

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## 1. Introduction

The next generation of combat aircraft is likely to operate in more severe environmental conditions than in the past. This implies that such an aircraft, in addition to gust, will be exposed to blast, fuel explosions, sonic-booms, etc.

Under such conditions even if the flight speed of the aircraft is below the flutter speed, the wing structure will be subjected to large oscillations that can result in its failure by fatigue. Moreover, in some special events, occurring during the operational life of the aircraft such as escape maneuvers, significant decays of the flutter speed can occur, with dramatic consequences for the further evolution of the aircraft. All these facts fully underline the necessity of the implementation of an active control capability enabling one to fulfill two basic objectives: a) to enhance the subcritical aeroelastic response, in the sense of suppressing the wing oscillations in the shortest possible time, and b) to extend the flight envelope by suppressing flutter instability and so, contributing to a significant increase

of the allowable flight speed. With this in mind, in this paper the active aeroelastic control of a 2-D wing-flap system exposed to an incompressible flowfield will be investigated. In this context, a combined control strategy using LQG/LTR was implemented, and some of its performances put into evidence.

## 2. Configuration of the 2-D Wing-Flap Structural Model

Figure 1 shows the typical wing-flap that is considered in the present analysis. This model has been established for 2-D aeroelastic analysis, see e.g. [1,2]. The three degrees of freedom associated with the airfoil appear clearly from Fig.1. The pitching and plunging displacement are restrained by a pair of springs attached to the elastic axis(EA) with

spring constants  $K_a$  and  $K_h$ , respectively. The control flap is located at the trailing edge. A torsional flap spring is also attached at the hinge axis whose spring constant is  $K_b$ ;  $h$  denotes the plunge

displacement (positive downward),  $\alpha$  the pitch angle (measured from the horizontal at the elastic axis of the airfoil) and  $b$  is the flap deflection (measured from the axis created by the airfoil at the control flap hinge).

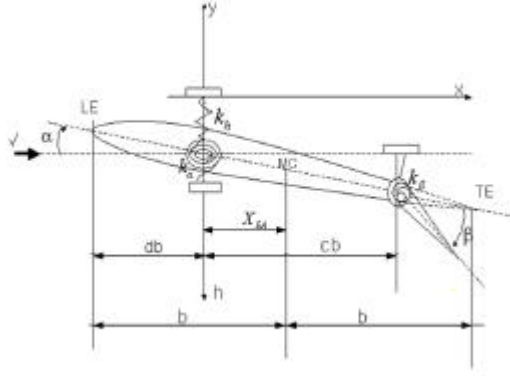


Fig1. Typical wing-flap section

### 3. Governing Equation of the Aeroelastic Model

The governing equations pertinent to the three degrees of freedom aeroelastic systems can be found in the classical aeroelasticity monographs. In matrix form the aeroelastic governing equations of the 2D wing-flap system can be written as [3,4]:

$$M\ddot{Y}(t) + KY(t) = - \begin{bmatrix} L_{new}(t) \\ M_{new}(t) \\ T_{new}(t) \end{bmatrix} + B u(t) + G w(t) \quad (1)$$

In the equation

$$Y(t) = \begin{bmatrix} \frac{h(t)}{b} & \alpha(t) & b(t) \end{bmatrix}^T \quad (2)$$

is the state-space vector.

$$M = \begin{bmatrix} bm & S_a & S_b \\ bS_a & I_a & I_b + bcS_b \\ bS_b & I_b + bcS_b & I_b \end{bmatrix} \quad (3)$$

$$K = \begin{bmatrix} bK_h & 0 & 0 \\ 0 & K_a & 0 \\ 0 & 0 & K_b \end{bmatrix} \quad (4)$$

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \quad (5)$$

denote the mass, stiffness and aerodynamic matrixes, respectively:  $u(t)$  is control input.  $w(t)$  is an external disturbance represented by a time-dependent external excitation, such as by a blast, sonic-boom or step pressure pulse;  $G$  is the disturbance-input matrix while  $B$  is the control input matrix.

The new aerodynamic load vector appearing in Eq. (1) is expressed in terms of its components as

$$L_{new}(t) = L(t) + L_G(t) \quad (6)$$

$$M_{new}(t) = M(t) + M_{yG}(t) \quad (7)$$

$$T_{new}(t) = T(t) + T_{yG}(t) \quad (8)$$

where  $L$ ,  $M$ , and  $T$  denote, respectively, the

aerodynamic lift (measurement positive in the upward direction), the pitching moment about the one-quarter chord of the airfoil (positive nose-down) and the flap torque applied to the flap hinge.

The second terms in the expressions (6)-(8) are due to the gust, in this respect, for the gust loading we have[5]

$$L_G(t) = \int_0^t I_{LG}(t-s) \frac{w_G}{V} ds \quad (9)$$

$$M_{yG}(t) = \int_0^t I_{MG}(t-s) \frac{w_G}{V} ds \quad (10)$$

$$T_{yG}(t) = \int_0^t I_{yG}(t-s) \frac{w_G}{V} ds \quad (11)$$

where  $w_G$  is the gust vertical velocity, while

$I_{LG}$ ,  $I_{MG}$  and  $I_{yG}$  are the related impulse functions.

For the present case of the incompressible flow, we have [5]:

$$I_{LG} = 4\mathbf{pj} \quad (12)$$

$$I_{MG} = I_{LG} (1/2 + x_{EA}/b) \quad (13)$$

$$I_{yG} = 0 \quad (14)$$

and where  $\mathbf{j}$  is the *Küssner's* function,

approximated by:

$$\mathbf{j}(t) = 1 - 0.5e^{-0.13t} - 0.5e^{-t} \quad (15)$$

In the time domain, the aerodynamic loads have the form

$$L(t) = \mathbf{pr}b^2[\ddot{h}(t) - ba\ddot{\alpha}(t) + \frac{b}{2p}\Phi_4\ddot{\mathbf{b}}(t) + V\dot{\mathbf{a}}(t) + \frac{V}{p}\Phi_3\dot{\mathbf{b}}(t)] + 2\mathbf{pr}VbD(t) \quad (16)$$

$$M(t) = \mathbf{pr}b^3[-a\ddot{h}(t) + b(\frac{1}{8} + a^2)\ddot{\alpha}(t) + \frac{b}{4p}\Phi_7\ddot{\mathbf{b}}(t) + (\frac{1}{2} - a)V\dot{\mathbf{a}}(t) + \frac{V}{2p}\Phi_6\dot{\mathbf{b}}(t) - \frac{V^2}{pb}\Phi_5\mathbf{b}(t) - 2\mathbf{pr}b^2(\frac{1}{2} + a)V D(t)] \quad (17)$$

$$T(t) = \mathbf{pr}b^2[(\frac{b}{2p}\Phi_4)\dot{h}(t) + (\frac{b^2}{4p}\Phi_7)\dot{\alpha}(t) + (\frac{b^2}{2p^2}\Phi_8)\dot{\mathbf{b}}(t) + (\frac{bV}{2p}\Phi_9)\dot{\mathbf{a}}(t) + (\frac{bV}{2p^2}\Phi_{11})\dot{\mathbf{b}}(t) + (\frac{V^2}{p^2}\Phi_{10})\mathbf{b}(t)] + \mathbf{pr}VbP(t) \quad (18)$$

The functions D(t) and P(t) are Duhamel

Integrals given by

$$D(t) = \int_0^t \Phi[\frac{(t-s)V}{b}]Q'_1(s)ds \quad (19)$$

$$P(t) = \int_0^t \Phi[\frac{(t-s)V}{b}]Q'_2(s)ds \quad (20)$$

where  $\Phi[\frac{(t-s)V}{b}]$  is the Wagner's function whose

argument is  $\frac{(t-s)V}{b}$ .

$$Q_1(t) = \frac{dQ_1(t)}{dt} = h''(t) + \mathbf{a}''(t)b + \frac{b}{2p}\Phi_2\mathbf{b}''(t) + V\mathbf{a}'(t) + \frac{V}{p}\Phi_1\mathbf{b}'(t) \quad (21)$$

$$Q_2(t) = \frac{dQ_2(t)}{dt} = \frac{b}{p}\Phi_8h''(t) +$$

$$\frac{b^2}{p}\Phi_8\mathbf{a}''(t) + \frac{b^2}{2p^2}\Phi_2\Phi_8\mathbf{b}''(t) + \frac{Vb}{p}\Phi_8\mathbf{a}'(t) + \frac{Vb}{p^2}\Phi_1\Phi_8\mathbf{b}'(t) \quad (22)$$

$$t = t \frac{V}{b} \quad (23)$$

While  $\Phi_i(\mathbf{f})$  are Theodorsen's Constants [1], where

$$\mathbf{f} = \arccos(-x_{flap}/b).$$

The standard two-term Jones exponential approximation of the Wagener's function is given by

$$\Phi(\mathbf{t}) = 1 - \mathbf{a}_1e^{-b_1t} - \mathbf{a}_2e^{-b_2t} \quad (24)$$

$\mathbf{a}_1 = 0.165; \mathbf{a}_2 = 0.335; b_1 = 0.041; b_2 = 0.32$

By replacing Eq. (24) in Eqs. (16) and (18), one obtain for D(t) and P(t) the expressions:

$$D(t) = Q_1(t) - \mathbf{a}_1B_1(t) - \mathbf{a}_2B_2(t) \quad (25)$$

$$P(t) = Q_2(t) - \mathbf{a}_1A_1(t) - \mathbf{a}_2A_2(t) \quad (26)$$

where

$$Q_1(t) = \dot{h}(t) + \dot{\mathbf{a}}(t)b + \frac{b}{2p}\Phi_2\dot{\mathbf{b}}(t) + V\mathbf{a}(t) + \frac{V}{p}\Phi_1\mathbf{b}(t) \quad (27)$$

$$Q_2(t) = \frac{b}{p}\Phi_8\dot{h}(t) + \frac{b^2}{p}\Phi_8\dot{\alpha}(t) + \frac{b^2}{2p^2}\Phi_2\Phi_8\dot{\mathbf{b}}(t) + \frac{Vb}{p}\Phi_8\mathbf{a}(t) + \frac{Vb}{p^2}\Phi_1\Phi_8\mathbf{b}(t) \quad (28)$$

#### 4. Design of Control Law

In this paper, we designed LQG/LTR control law which related the Kalman filter[7]:

$$\dot{\hat{\mathbf{x}}} = (A + \bar{L}C)\hat{\mathbf{x}}(t) + Bu_c^0(\hat{\mathbf{x}}(t)) - \bar{L}y(t) \quad (29)$$

Here  $u_c^0$  is represented according to the law:

$$u_c^0(\hat{\mathbf{x}}(t)) = \bar{K}\hat{\mathbf{x}} \quad (30)$$

with

$$\bar{K} = -RB^T\bar{P} \text{ and } \bar{L} = -\bar{\Pi}C^TW^{-1} \quad (31)$$

The matrices  $\bar{P}$  and  $\bar{\Pi}$  are the unique symmetric,

positive definite solutions of the algebraic Riccati equations.

The time domain equations of a full order observer based controller are well known and are given by [8]

$$\dot{\hat{x}} = A\hat{x} + Bu + K_f(y - C\hat{x} - Du) \quad (32)$$

$$u = \hat{u} - F\hat{x} \quad (33)$$

The time domain dynamic equations of a Luenberger observer based controller are given by

$$\dot{v} = Lv + Gu + Hy \quad (34)$$

$$-\hat{u} = Pv + Vy \quad (35)$$

$$L = A - K_f C, \quad G = B - K_f D, \quad H = K_f, \quad P = F,$$

$$V = 0 \quad T = I \quad (36)$$

## 5. Numerical Simulations

The considered geometrical and physical characteristics of the 2D wing-flap system are identical to the ones in the work by Edwards[2]

$$b = 0.9143m \quad c = 1.0 \quad x_{EA} = -0.4$$

$$I_a = 26.9 \frac{kg \cdot m^2}{m} \quad I_b = 0.6726 \frac{kg \cdot m^2}{m}$$

$$K_a = 100^2 I_a; \quad K_b = 500^2 I_b; \quad K_n = 50^2 m$$

$$m = 128.7236 \frac{kg}{m} \quad r = 0.002378$$

$$S_a = 23.5398 kg \quad S_b = 1.471 kg$$

Corresponding to these data, the flutter speed is

$$V_f = 271.25 \frac{m}{sec}$$

In Fig. 2 through 3, the open/closed response time-histories of the quantities  $(\tilde{h}(\equiv h/b), \mathbf{a}, \mathbf{b})$  of the aeroelastic system operating in the close proximity of the flutter boundary ( $V_f = 889 ft/sec$ ) and subjected to a blast load, represented in the absence of the control, LQG control, LQG/LTR control, respectively.

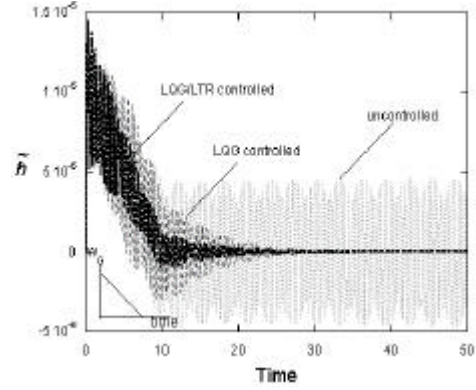


Fig.2 Open/closed loop plunging time history of the aeroelastic system under blast

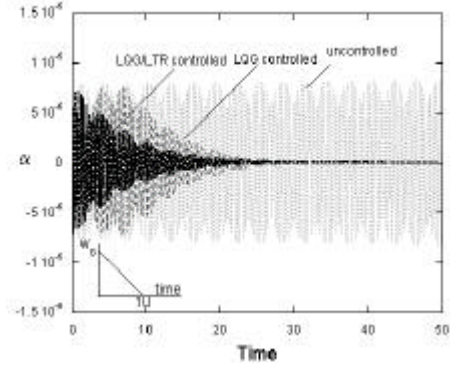


Fig.3 The counterpart of Fig 2. for the pitching displacement

Fig.4 and Fig.5 the open/closed-loop dimensionless plunging time-history of the aeroelastic system operating in the proximity of the flutter instability and exposed to a sonic-boom pressure pulse presented. In the previous case the combined control reveals its efficiency to suppress the flutter instability and simultaneously, the oscillations of the wing.

Fig.6 is the singular value decomposition in frequency domain. In this figure, we saw that the frequency response of LQR, LQG, LQG/LTR controlled response. As the loop transfer gain increase, the performance of LQG/LTR controller is similar to the LQR controller.

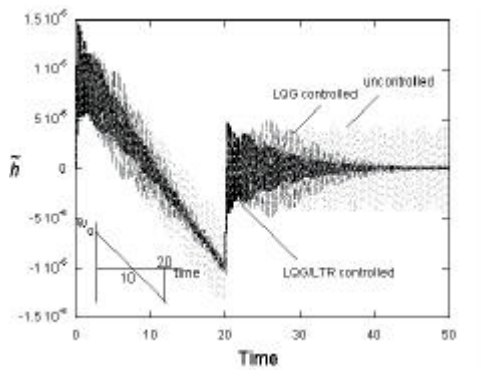


Fig.4 Open/close loop plunging time-history of the aeroelastic system under a sonic-boom pulse

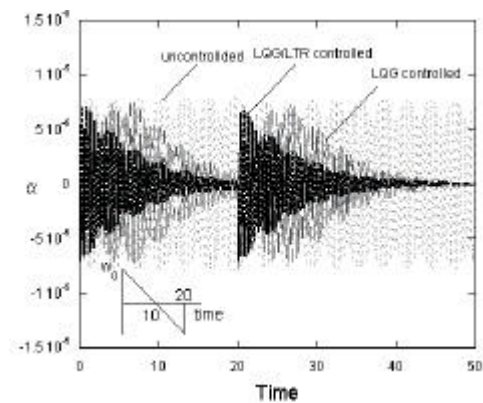


Fig.5 The counterpart of Fig.4 for the pitching displacement

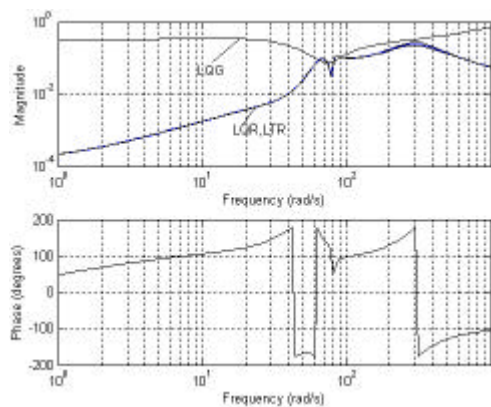


Fig.6. The Singular Value Decomposition in frequency domain.

**Closure**

The goal is to implement a robust control capability

that utilizes the deflected flap as to suppress the flutter instability or enhance the subcritical aeroelastic response to gust or blast loads. To this end, the high efficiency of the implemented LQG/LTR control strategy was presented.

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