Fixed-Order $\mathcal{H}_\infty$ Controller Design for Descriptor Systems

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Abstract: For linear descriptor systems, we consider the $\mathcal{H}_\infty$ controller design problem via output feedback. Both static output feedback and dynamic one are discussed. First, in the case of static output feedback, we reduce our control problem to solving a bilinear matrix inequality (BMI) with respect to the controller coefficient matrix, a Lyapunov matrix and a matrix related to the descriptor matrix. Under a matching condition between the descriptor matrix and the measured output matrix (or the control input matrix), we propose setting the Lyapunov matrix in the BMI as being block diagonal appropriately so that the BMI is reduced to LMIs. For fixed-order dynamic $\mathcal{H}_\infty$ output feedback, we formulate the control problem equivalently as the one of static output feedback design, and thus the same approach can be applied.

Keywords: Descriptor system, $\mathcal{H}_\infty$ control, static output feedback, fixed-order dynamic output feedback, bilinear matrix inequality (BMI), linear matrix inequality (LMI)

1 Introduction

It is known that descriptor systems (also known as singular systems or implicit systems) have high abilities in representing dynamical systems. They can preserve physical parameters in the coefficient matrices, and describe the dynamic part, static part, and even improper part of the system in the same form. In this sense, descriptor systems are much superior to systems represented by state-space models.

There have been reported many works on descriptor systems, e.g., [1, 2, 3]. Among these works, Ref. [3] applied the linear matrix inequality (LMI) approach (e.g., [4, 5]) to $\mathcal{H}_\infty$ control problems for descriptor systems. However, the LMI-type conditions proposed there contain equality constraints, which may be little problem theoretically, but may cause big trouble in checking the conditions numerically. Because of quantization errors in digital computation, the equality constraints are fragile and in usual not satisfied exactly. To overcome this difficulty, Refs. [6, 7] derived strict LMI conditions for stability, robust stabilization and $\mathcal{H}_\infty$ control of linear descriptor systems. Since the strict LMIs are definite LMIs with no equality constraint, they are highly tractable and reliable when we use recent popular softwares for solving LMIs.

When focusing on $\mathcal{H}_\infty$ controller design for descriptor systems, we noticed that the controller proposed in Refs. [6, 7] is restricted to dynamic output feedback with the same descriptor form and the same order as that of the system, and thus the approach in Refs. [6, 7] can not be applied to fixed-order controller design or the design of controllers with non-descriptor form.

The above observations motivate us to consider fixed-order $\mathcal{H}_\infty$ controller (including the case of static output feedback) for descriptor systems. First, in the case of static output feedback, we use a lemma in Ref. [7] to express our $\mathcal{H}_\infty$ control problem as a bilinear matrix inequality (BMI) with respect to the controller coefficient matrix, a Lyapunov matrix and a matrix related to the descriptor matrix. Under a matching condition between the descriptor matrix and the measured output matrix (or the control input matrix), we propose setting the Lyapunov matrix in the BMI as being block diagonal appropriately so that the BMI is reduced to LMIs.

Next, we consider for the descriptor system a fixed-order dynamic output feedback $\mathcal{H}_\infty$ controller with non-descriptor form. By formulating the control problem as a new problem of static output feedback design, we show that we can adopt the same approach and reduce the controller design to solving LMIs, under the same matching condition as in the case of static output feedback. We emphasize that, although the obtained LMIs are sufficient but not necessary, they are strict LMIs and thus are easily solved by using many existing softwares (for example, LMI Control Toolbox [4, 8]). It is noted that the idea of setting the Lyapunov matrix in the BMI as being block diagonal is originated from Refs. [9, 10, 11]. A simple example is used to demonstrate the effectiveness of our results.

2 Problem Formulation and Preliminary Result

We consider the linear descriptor system

$$\begin{cases} E\dot{x} = Ax + B_1 w + B_2 u \\ z = C_1 x \\ y = C_2 x \end{cases} \tag{1}$$

where $x \in \mathcal{R}^n$ is the descriptor variable, $w \in \mathcal{R}^l$ is the disturbance input, $z \in \mathcal{R}^p$ is the controlled output, $u \in \mathcal{R}^m$ is the control input, and $y \in \mathcal{R}^q$ is the measured output. The matrices $E \in \mathcal{R}^{n\times n}$, $A \in \mathcal{R}^{n\times n}$, $B_1 \in \mathcal{R}^{n\times l}$, $B_2 \in \mathcal{R}^{n\times m}$, $C_1 \in \mathcal{R}^{p\times n}$, $C_2 \in \mathcal{R}^{q\times n}$ are constant, and the matrix E is singular, whose rank is denoted by $\operatorname{rank} E = r < n$. Without loss of generality, we assume that B_2 is of full column rank, and C_2 is of full row rank.

We first introduce some definitions [6] for the descriptor system (1) with $w \equiv 0, u \equiv 0$. The system has a unique solution for any initial condition, and is called *regular*, if $\det(sE - A) \not\equiv 0$. The finite eigenvalues of the matrix pair (E, A), i.e., the solutions of $\det(sE - A) = 0$, and the corresponding (generalized) eigenvectors define exponential modes of the system. If the finite eigenvalues are in the open left half-plane of s, the solution of the system decays exponentially. The infinite eigenvalues of (E, A) with the eigenvectors x satisfying $Ex = 0$ determine static modes. The infinite eigenvalues of (E, A) with generalized eigenvectors x_k satisfying the relations $Ex_1 = 0$ and $Ex_k = Ax_{k-1}$ $(k \geq 2)$ create impulsive modes. The system has no impulsive mode if and only if $\operatorname{rank} E = \deg\det(sE - A)$. The system is said to be *stable* if it is regular and has only decaying exponential modes and static modes,

For the system (1), we consider a static output feedback controller

$$u = Ky \tag{2}$$

where K is the gain matrix to be determined, and a dynamic output feedback controller

$$\begin{cases} \dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}y \\ u = \hat{C}\hat{x} + \hat{D}y \end{cases} \tag{3}$$

where $\hat{x} \in \mathcal{R}^{\hat{n}}$ is the state of the controller, $\hat{n} < n$ is a fixed-order, and $\hat{A}$, $\hat{B}$, $\hat{C}$, $\hat{D}$ are constant matrices to be determined. Then, the $\mathcal{H}_\infty$ control problem in this paper is formulated as follows.

Fixed-Order $\mathcal{H}_\infty$ Controller Design Problem: *Given a specified disturbance attenuation level $\gamma > 0$, design a static output feedback controller* (2) *and a dynamic output feedback controller* (3) *for the system* (1) *so that the resultant closed-loop system is stable and the $\mathcal{H}_\infty$ norm of the transfer function from w to z in the closed-loop system is less than γ.* If such a static output feedback controller (respectively, a dynamic output feedback controller) exists, we say the descriptor system (1) is stabilizable with $\mathcal{H}_\infty$ disturbance attenuation level γ via a static output feedback controller (2) (respectively, a dynamic output feedback controller (3)).

Next, we state a preliminary result, which plays an important role in the discussions later. We let matrices $V, U \in \mathcal{R}^{n\times(n-r)}$ be of full column rank and composed of bases of $\operatorname{Null} E$ and $\operatorname{Null} E^T$, respectively, and we decompose E as $E = E_L E_R^T$, where $E_L, E_R \in \mathcal{R}^{n\times r}$ are of full column rank.

Lemma 1. [7] The system (1) (with $u = 0$) is stable and $\|C_1(sE - A)^{-1}B_1\|_\infty < \gamma$ if and only if one of the following two conditions is satisfied:

(i) There exist a symmetric matrix $P \in \mathcal{R}^{n\times n}$ and a matrix $S \in \mathcal{R}^{(n-r)\times(n-r)}$ such that

$$\begin{bmatrix} \Phi_1 + \Phi_1^T + B_1 B_1^T & \Phi_{12} \\ \Phi_{12}^T & -\gamma^2 I \end{bmatrix} < 0\,,\ E_R^T P E_R > 0 \tag{4}$$

$$\Phi_1 = A(PE^T + VSU^T)$$
$$\Phi_{12} = (PE^T + VSU^T)^T C_1^T\,.$$

(ii) There exist a symmetric matrix $Q \in \mathcal{R}^{n\times n}$ and a matrix $R \in \mathcal{R}^{(n-r)\times(n-r)}$ such that

$$\begin{bmatrix} \Psi_1 + \Psi_1^T + C_1^T C_1 & \Psi_{12} \\ \Psi_{12}^T & -\gamma^2 I \end{bmatrix} < 0\,,\ E_L^T Q E_L > 0 \tag{5}$$

$$\Psi_1 = A^T(QE + URV^T)$$
$$\Psi_{12} = (QE + URV^T)^T B_1\,.$$

3 Static Output Feedback

In this section, we consider the design of a static output feedback controller (2) for the system (1) so that the resultant closed-loop system is stable with $\mathcal{H}_\infty$ disturbance attenuation level γ.

First, the closed-loop system obtained by applying the controller (2) to the system (1) is written as

$$\begin{cases} E\dot{x} = \bar{A}x + B_1 w\,, \quad \bar{A} = A + B_2 K C_2 \\ z = C_1 x \end{cases} \tag{6}$$

and the transfer function from w to z in (6) is

$$G_c(s) = C_1(sE - \bar{A})^{-1}B_1\,. \tag{7}$$

Then, according to Lemma 1, the closed-loop system (6) is stable and $\|G_c(s)\|_\infty < \gamma$ if and only if (4) (or (5)) is satisfied with A replaced by $\bar{A}$. To say it in other words, our control problem here is reduced to solving (4) (or (5)) with respect to P, S (or Q, R) and K. However, due to the products between $\bar{A}$ and (P, S) (or (Q, R)), (4) (or (5)) is a BMI with respect to the unknown matrices, and thus difficult to solve.

In this section, we assume that a matching condition holds between the descriptor matrix E and the measured output matrix C_2 (or the control input matrix B_2). By considering an equivalent block diagonal structure of the Lyapunov matrix P (or Q), we reduce the BMI to LMIs, for which there are many effective softwares (e.g., LMI Control Toolbox of MATLAB) [4, 8].

Assumption 1.(Matching Condition 1) There exists a constant matrix $\hat{C}_2$ such that $C_2 = \hat{C}_2 E$. ∎

To state and prove our first result, we need to find a nonsingular matrix T satisfying

$$C_2 T = \begin{bmatrix} I_q & 0 \end{bmatrix} . \tag{8}$$

This is always possible since C_2 is of full row rank. Using the matrix T, we make the following similarity transformation for the system matrices as

$$\begin{array}{l} \tilde{E} = T^{-1}ET , \quad \tilde{V} = T^{-1}V , \quad \tilde{U} = T^{-1}U \\ \tilde{A} = T^{-1}AT , \quad \tilde{B}_1 = T^{-1}B_1 \\ \tilde{B}_2 = T^{-1}B_2 , \quad \tilde{C}_1 = C_1 T . \end{array} \tag{9}$$

Theorem 1. Under Assumption 1, the system (1) is stabilizable with $\mathcal{H}_\infty$ disturbance attenuation level γ via a static output feedback controller (2) if there exist $\tilde{P} = \text{diag}\{\tilde{P}_1, \tilde{P}_2\} > 0$ $\left(\tilde{P}_1 \in \mathcal{R}^{q\times q},\ \tilde{P}_2 \in \mathcal{R}^{(n-q)\times(n-q)}\right)$, $S \in \mathcal{R}^{(n-r)\times(n-r)}$ and $W \in \mathcal{R}^{m\times q}$ such that the LMIs

$$\begin{bmatrix} \hat{\Phi}_1 + \hat{\Phi}_1^T + \tilde{B}_1\tilde{B}_1^T & \hat{\Phi}_{12} \\ \hat{\Phi}_{12}^T & -\gamma^2 I \end{bmatrix} < 0 \tag{10}$$

$$\hat{\Phi}_1 = \tilde{A}\tilde{P}\tilde{E}^T + \tilde{A}\tilde{V}S\tilde{U}^T + \tilde{B}_2 \begin{bmatrix} W & 0 \end{bmatrix} \tilde{E}^T$$

$$\hat{\Phi}_{12} = (\tilde{P}\tilde{E}^T + \tilde{V}S\tilde{U}^T)^T \tilde{C}_1^T$$

$$E_R^T T \tilde{P} T^T E_R > 0 \tag{11}$$

are satisfied.

When the above LMIs are feasible, one desired controller gain matrix is computed as

$$K = W\tilde{P}_1^{-1} . \tag{12}$$

Proof. Pre- and post-multiplying the LMI (10) respectively by $\begin{bmatrix} T & 0 \\ 0 & I \end{bmatrix}$ and its transpose, and then substituting all the matrices in (9) together with $P = T\tilde{P}T^T$, we obtain

$$\begin{bmatrix} \tilde{\Phi}_1 + \tilde{\Phi}_1^T + B_1 B_1^T & \Phi_{12} \\ \Phi_{12}^T & -\gamma^2 I \end{bmatrix} < 0 \tag{13}$$

$$\tilde{\Phi}_1 = APE^T + AVSU^T + B_2 \begin{bmatrix} W & 0 \end{bmatrix} T^T E^T$$

where Φ_{12} is the same as in Lemma 1.

It is easy to confirm $KC_2T\tilde{P} = \begin{bmatrix} W & 0 \end{bmatrix}$ from (8) and (12), and thus

$$B_2 \begin{bmatrix} W & 0 \end{bmatrix} T^T E^T = B_2 K C_2 P E^T . \tag{14}$$

Then, we use the fact $C_2 V = \hat{C}_2 E V = 0$ in (13) to obtain

$$\begin{bmatrix} \bar{\Phi}_1 + \bar{\Phi}_1^T + B_1 B_1^T & \Phi_{12} \\ \Phi_{12}^T & -\gamma^2 I \end{bmatrix} < 0 \tag{15}$$

$$\bar{\Phi}_1 = \bar{A}(PE^T + VSU^T) .$$

Since the LMI (11) is $E_R^T T\tilde{P}T^T E_R = E_R^T P E_R > 0$, we declare that the closed-loop system (6) with (12) is stable with $\mathcal{H}_\infty$ disturbance attenuation level γ, according to Lemma 1. ∎

Next, we consider the control problem under the following matching condition.

Assumption 2.(Matching Condition 2) There exists a constant matrix $\hat{B}_2$ such that $B_2 = E\hat{B}_2$. ∎

Similarly as before, we first find a nonsingular matrix X satisfying

$$XB_2 = \begin{bmatrix} I_m \\ 0 \end{bmatrix} . \tag{16}$$

This is always possible since B_2 is of full column rank. Using the matrix X, we make the following similarity transformation for the system matrices as

$$\begin{array}{l} \hat{E} = XEX^{-1} , \quad \hat{V} = (X^{-1})^T V , \quad \hat{U} = (X^{-1})^T U \\ \hat{A} = XAX^{-1} , \quad \hat{B}_1 = XB_1 \\ \hat{C}_1 = C_1 X^{-1} , \quad \hat{C}_2 = C_2 X^{-1} . \end{array} \tag{17}$$

Theorem 2. Under Assumption 2, the system (1) is stabilizable with $\mathcal{H}_\infty$ disturbance attenuation level γ via a static output feedback controller (2) if there exist $\hat{Q} = \text{diag}\{\hat{Q}_1, \hat{Q}_2\} > 0$ $\left(\hat{Q}_1 \in \mathcal{R}^{m\times m},\ \hat{Q}_2 \in \mathcal{R}^{(n-m)\times(n-m)}\right)$, $R \in \mathcal{R}^{(n-r)\times(n-r)}$ and $W \in \mathcal{R}^{m\times q}$ such that the LMIs

$$\begin{bmatrix} \hat{\Psi}_1 + \hat{\Psi}_1^T + \hat{C}_1^T\hat{C}_1 & \hat{\Psi}_{12} \\ \hat{\Psi}_{12}^T & -\gamma^2 I \end{bmatrix} < 0 \tag{18}$$

$$\hat{\Psi}_1 = \hat{A}^T\hat{Q}\hat{E} + \hat{A}^T\hat{U}R\hat{V}^T + \hat{C}_2^T \begin{bmatrix} W^T & 0 \end{bmatrix} \hat{E}$$

$$\hat{\Psi}_{12} = (\hat{Q}\hat{E} + \hat{U}R\hat{V}^T)^T \hat{B}_1$$

$$E_L^T X^T \hat{Q} X E_L > 0 \tag{19}$$

are satisfied.

When the above LMIs are feasible, one desired controller gain matrix is computed as

$$K = \hat{Q}_1^{-1} W . \tag{20}$$

Proof. Pre- and post-multiplying the LMI (18) respectively by $\begin{bmatrix} X^T & 0 \\ 0 & I \end{bmatrix}$ and its transpose, and then substituting all the matrices in (17) together with $Q = X^T \hat{Q} X$, we obtain

$$\begin{bmatrix} \tilde{\Psi}_1 + \tilde{\Psi}_1^T + C_1^T C_1 & \Psi_{12} \\ \Psi_{12}^T & -\gamma^2 I \end{bmatrix} < 0 \tag{21}$$
$$\tilde{\Psi}_1 = A^T Q E + A^T U R V^T + C_2^T \begin{bmatrix} W^T & 0 \end{bmatrix} X E$$

where Ψ_{12} is the same as in Lemma 1.

It is easy to confirm $\hat{Q} X B_2 K = \begin{bmatrix} W^T & 0 \end{bmatrix}^T$ from (16) and (20), and thus

$$C_2^T \begin{bmatrix} W^T & 0 \end{bmatrix} X E = (B_2 K C_2)^T Q E\,. \tag{22}$$

Then, we use the fact $U^T B_2 = U^T E \hat{B}_2 = 0$ in (21) to obtain

$$\begin{bmatrix} \bar{\Psi}_1 + \bar{\Psi}_1^T + C_1^T C_1 & \Psi_{12} \\ \Psi_{12}^T & -\gamma^2 I \end{bmatrix} < 0 \tag{23}$$
$$\bar{\Psi}_1 = \bar{A}^T (Q E + U R V^T)\,.$$

Since the LMI (19) is $E_L^T X^T \hat{Q} X E_L = E_L^T Q E_L > 0$, we declare that the closed-loop system (6) with (20) is stable with $\mathcal{H}_\infty$ disturbance attenuation level γ, according to Lemma 1. ∎

Remark 1. Although Theorems 1 and 2 come up with dual forms, they are not equivalent and are supposed to deal with different matching conditions. Furthermore, the LMI conditions provided by the theorems are sufficient ones. Therefore, even in the case where both matching conditions (Assumptions 1 and 2) hold and thus both theorems can be applied, the LMI condition of one theorem would be satisfied while the other would not. ∎

Remark 2. When it is necessary, we can try to obtain a tight $\mathcal{H}_\infty$ disturbance attenuation level by considering the eigenvalue problem (EVP) [4]: "minimize γ, s.t. (10), (11) (or (18), (19)) with respect to $\tilde{P}$, S, W (or $\hat{Q}$, R, W)". ∎

Remark 3. In Theorems 1 and 2, we required $\tilde{P} > 0$ and $\hat{Q} > 0$, respectively. It can be easily understood from the proofs and Lemma 1 that we can relax the conditions by requiring instead $\tilde{P}_1 > 0$ with symmetric $\tilde{P}_2$ and $\hat{Q}_1 > 0$ with symmetric $\hat{Q}_2$, respectively. ∎

In the end of this section, we present a simple example. The system (1) under consideration is defined by the coefficient matrices

$$\begin{aligned} A &= \begin{bmatrix} 0.42 & 1.33 & 1.76 & 1.45 \\ 1.51 & 1.26 & 0.14 & 0.40 \\ 0.00 & 1.70 & 1.12 & 1.09 \\ 0.66 & 1.37 & 1.32 & 0.46 \end{bmatrix} \\ E &= \begin{bmatrix} 0.33 & 0.83 & 0.34 & 1.11 \\ 0.48 & 0.77 & 0.73 & 1.11 \\ 0.21 & 0.41 & 0.28 & 0.57 \\ 0.27 & 0.49 & 0.38 & 0.69 \end{bmatrix} \\ B_1 &= \begin{bmatrix} 0.46 & 0.62 \\ 0.43 & 1.87 \\ 1.77 & 0.43 \\ 1.31 & 0.63 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.72 & 0.66 \\ 0.58 & 1.19 \\ 1.13 & 1.00 \\ 0.97 & 0.87 \end{bmatrix} \\ C_1 &= \begin{bmatrix} 0.21 & 0.00 & 0.67 & 0.85 \\ 0.76 & 0.33 & 0.63 & 0.69 \end{bmatrix} \\ C_2 &= \begin{bmatrix} 0.84 & 1.77 & 1.05 & 2.43 \end{bmatrix}. \end{aligned} \tag{24}$$

For this system, we set $\gamma = 1.2$ as the desired $\mathcal{H}_\infty$ disturbance attenuation level and aim to design a static $\mathcal{H}_\infty$ output feedback controller.

It is easy to see that Assumption 1 holds between E and C_2, and thus Theorem 1 is available. Solving the two LMIs in Theorem 1, we obtain the controller gain matrix

$$K = \begin{bmatrix} 2.14 \\ -3.06 \end{bmatrix}. \tag{25}$$

Using the existing result [6], we can confirm that the closed-loop system composed of (24) and (25) is stable with $\mathcal{H}_\infty$ disturbance attenuation level less than γ.

4 Dynamic Output Feedback

In this section, we consider the design of a dynamic output feedback controller (3) for the system (1) so that the resultant closed-loop system is stable with $\mathcal{H}_\infty$ disturbance attenuation level γ.

We first write the controller coefficient matrices $\hat{A}$, $\hat{B}$, $\hat{C}$ and $\hat{D}$ in a single matrix

$$K = \begin{bmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{bmatrix} \in \mathcal{R}^{(\hat{n}+m)\times(\hat{n}+q)} \tag{26}$$

and introduce the notations

$$\begin{aligned} [\breve{A} \;\; \breve{B}_1 \;\; \breve{B}_2] &= \left[\begin{array}{cc|c|cc} A & 0_{n\times\hat{n}} & B_1 & 0_{n\times\hat{n}} & B_2 \\ 0_{\hat{n}\times n} & 0_{\hat{n}\times\hat{n}} & 0_{\hat{n}\times l} & I_{\hat{n}} & 0_{\hat{n}\times m} \end{array}\right] \\ \begin{bmatrix} \breve{C}_1 \\ \breve{C}_2 \end{bmatrix} &= \left[\begin{array}{cc} C_1 & 0_{p\times\hat{n}} \\ \hline 0_{\hat{n}\times n} & I_{\hat{n}} \\ C_2 & 0_{q\times\hat{n}} \end{array}\right], \quad \breve{E} = \begin{bmatrix} E & 0 \\ 0 & I_{\hat{n}} \end{bmatrix}. \end{aligned} \tag{27}$$

Then, the closed-loop system obtained by applying the controller (3) to the system (1) is written in a compact form as

$$\begin{cases} \breve{E}\dot{\breve{x}} = (\breve{A} + \breve{B}_2 K \breve{C}_2)\breve{x} + \breve{B}_1 w \\ z = \breve{C}_1 \breve{x} \end{cases} \tag{28}$$

where $\breve{x} = [x^T \ \hat{x}^T]^T \in \mathcal{R}^{n+\hat{n}}$. Therefore, our control problem is equivalent to the problem of designing a static output feedback $u = Ky$ for the linear descriptor system

$$\begin{cases} \breve{E}\dot{\breve{x}} = \breve{A}\breve{x} + \breve{B}_1 w + \breve{B}_2 u \\ z = \breve{C}_1 \breve{x} \\ y = \breve{C}_2 \breve{x}\,. \end{cases} \tag{29}$$

It is easy to see that $\operatorname{rank} \breve{E} = r + \hat{n} < n + \hat{n}$,

$$\begin{aligned} \breve{E} &= \begin{bmatrix} E_L & 0 \\ 0 & I_{\hat{n}} \end{bmatrix} \begin{bmatrix} E_R & 0 \\ 0 & I_{\hat{n}} \end{bmatrix}^T = \breve{E}_L \breve{E}_R^T \\ \breve{E}\breve{V} &= \breve{E} \begin{bmatrix} V \\ 0 \end{bmatrix} = 0\,, \quad \breve{E}^T \breve{U} = \breve{E}^T \begin{bmatrix} U \\ 0 \end{bmatrix} = 0\,, \end{aligned} \tag{30}$$

and

$$\begin{aligned} \breve{C}_2 \breve{T} &= \breve{C}_2 \begin{bmatrix} 0 & T \\ I_{\hat{n}} & 0 \end{bmatrix} = \begin{bmatrix} I_{q+\hat{n}} & 0 \end{bmatrix} \\ \breve{X}\breve{B}_2 &= \begin{bmatrix} 0 & I_{\hat{n}} \\ X & 0 \end{bmatrix} \breve{B}_2 = \begin{bmatrix} I_{m+\hat{n}} \\ 0 \end{bmatrix}. \end{aligned} \tag{31}$$

Furthermore, when Assumption 1 is true, a matching condition between $\breve{E}$ and $\breve{C}_2$ is also true since

$$\breve{C}_2 = \begin{bmatrix} 0 & I \\ C_2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & I \\ \hat{C}_2 & 0 \end{bmatrix} \breve{E}\,, \tag{32}$$

and when Assumption 2 is true, a matching condition between $\breve{E}$ and $\breve{B}_2$ is also true since

$$\breve{B}_2 = \begin{bmatrix} 0 & B_2 \\ I & 0 \end{bmatrix} = \breve{E} \begin{bmatrix} 0 & \hat{B}_2 \\ I & 0 \end{bmatrix}. \tag{33}$$

Therefore, under Assumptions 1 and 2, respectively, Theorems 1 and 2 are available for the design of fixed-order $\mathcal{H}_\infty$ dynamic output feedback controller design with some notation modifications in the LMIs (10), (11), (18) and (19). For brevity, we omit the precise descriptions here.

5 Conclusion

In this paper, we have considered fixed-order $\mathcal{H}_\infty$ controller design problems via output feedback for linear descriptor systems. In both cases of static output feedback and dynamic one, we have expressed the control problem as a bilinear matrix inequality (BMI) with respect to the controller coefficient matrix, a Lyapunov matrix and a matrix related to the descriptor matrix. Under a matching condition between the descriptor matrix and the measured output matrix (or the control input matrix), we have proposed setting the Lyapunov matrix in the BMI as being block diagonal appropriately so that the BMI is reduced to LMIs. We suggest that the approach in this paper should also be useful for other synthesis problems involving constraints on controller structure.

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