# Cooperating Control of Multiple Nonholonomic Mobile Robots Carrying a Ladder with Obstacles

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**Abstract**: A cooperating control algorithm for two nonholonomic mobile robots is proposed. The task is composed of collision avoidance against obstacles and carrying a ladder. The front robot and the rear robot are called the leader and the follower, respectively. Each robot has a nonholonomic constraint so it cannot move in perpendicular directions. The environment is initially supposed to be unknown except target position. The torque that drives leader is determined by distance between the leader and the target position or the distance between it and the obstacles. The torque by target is attractive and the torque by obstacles is repulsive. The two mobile robots are supposed to be connected by link that can be expanded and contracted. The follower computes its torque using position and orientation information from the leader by communication. Simulation results show that the robots can drive to target position without colliding into the obstacles and maintain the distance in the allowable range.

**Keywords:** nonholonomic, mobile robot, cooperating control, collision avoidance

## 1. INTRODUCTION

Mobile robots have become very popular in various fields as industrial robots. Many researchers have been studying mobile robots are used in various environments. Robots should operate with other robots or systems in some environments such as factories. AGVs(Autonomous Guided Vehicles) are used in factory automation, to rescue human and to explore dangerous areas. Through cooperation with robots, robots can carry out these works faster and efficient. In factories, AGVs are used to carry objects to desired positions, conventional AGVs are constrained in size and power. To carry large objects, AGVs that are large enough and have enough power to carry it are required. To overcome the problems that costs and energy usage increase, many researchers studied the cooperating control of robots.

K. Kosuge *et al.* have been studying cooperating of mobile robots and manipulators [1], [2]. Compliance control and estimation for desired trajectory are proposed. Sensors for measuring the force caused by the other robots are needed. Also they proposed impedance control without force/torque sensors [3]. Z. Chen *et al.* proposed time optimal motion of two robots carrying a ladder [4]. Given the angle of two robots and the distance between the initial and final positions, the conditions that make move in optimal time using a lower bound of initial and final state and variational calculus are found. They assumed that robots move as same speed always and can move to any direction and there is no obstacle. Y. Asahiro *et al.* proposed a simple control algorithm for two omni-directional mobile robots that transport a long object, such as a ladder, through a 90 degree corner in a corridor [5]. The two robots used in the experiments do not have identical characteristics. J. Desai addressed the motion planning for multiple mobile manipulators [6]. Two mobile robots carry an object in an environment that has obstacles. Obstacles are considered as constraints and an optimal control algorithm is applied. The position of the obstacles must be known before the robots move. The computer generates and passes to the robots the optimal path considering collision avoidance with obstacles before departure. Therefore, the robots can not avoid the collision with the unknown obstacles.

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For cooperating of robots, it is needed to exchange the state information of each robot by sensing or communication. The former is defined as implicit communication and the latter is defined as explicit communication [8].

In this paper, a control algorithm for transporting an object to target position without collision with obstacles is proposed. Two mobile robots with nonholonomic constraint carry a ladder. Each robot has two wheels as shown in Fig. 1. It is assumed that the two mobile robots know both of their position and orientation.

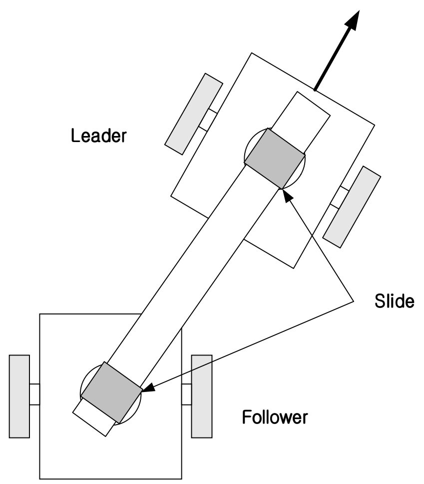



Fig. 1 Ladder transporting system by cooperating algorithm using two mobile robots

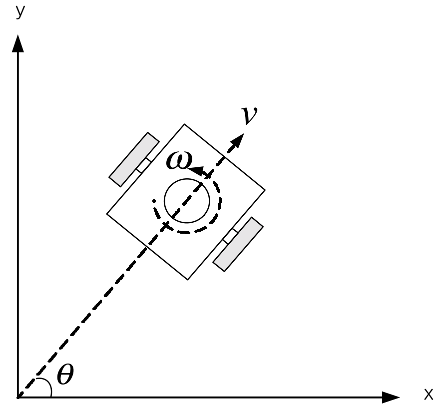


Fig.2 Nonholonomic mobile robot

On each robot there is a sliding pad that rotates freely as a compliant link that is supposed to be expanded and contracted. Therefore, each robot can move to any path without effect by each other. Along the progress direction, the front robot is defined as the leader and the rear one is the follower.

We analyze mobile robots that have a nonholonomic constraint and design control method to move the mobile robots to a target position in Sec.2. For collision avoidance against obstacles, a control algorithm using attractive and repulsive force is proposed. In Sec.3, a cooperating algorithm is proposed, which makes two robots move together so that they can maintain the distance and the error angle between them in allowable ranges. Simulation results in Sec.4 show that two robots carry an object to the target position while avoiding two obstacles. We conclude this paper in Sec. 5.

## 2. CONTROL METHOD FOR NONHOLONOMIC MOBILE ROBOT

### 2.1 Dynamics of Nonholonomic Systems

A two-wheeled mobile robot is described on the Cartesian frame in Fig. 2. The robot has two degree-of-freedom as the longitudinal and rotational movements but it has three coordinates as follows.

$$q = (x, y, \theta)^T \tag{1}$$

The dynamic equation of motion with $m$ constraints can be described by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = A^T(q)\lambda + S(q)\tau \tag{2}$$

where $M(q)$ is an $n\times n$ symmetric and positive definite inertia matrix, $C(q,\dot{q})$ is an $n\times n$ centripetal Coriolis matrix. $A(q)$ is an $m\times n$ matrix and $\lambda$ is an $m\times 1$ Lagrange multiplier vector. $S(q)$ is an $n\times m$ input transfer matrix and $\tau$ is an $n\times 1$ input torque vector.

Kinematic constraints which are related with velocity can be described by

$$A(q)\dot{q} = 0\,. \tag{3}$$

Consider a matrix $G(q)$ whose columns are bases for the nullspace of $A(q)$ such that

$$A(q)G(q) = 0\,. \tag{4}$$

Since the constrained velocity is always in the null space of $A(q)$, from (3) and (4), we can find $(n-m)$ velocities $\overline{v}(t)$ such that

$$\dot{q} = G(q)\overline{v}(t) \tag{5}$$

where $G(q)$ transforms the velocity $\overline{v}(t)$ into the Cartesian coordinate $\dot{q}$.

The Lagrange multiplier vector can be eliminated by left-multiplying eq. (2) by $G^T(q)$, so the reduced dynamical model is obtained as follows.

$$G^T(q)(M(q)\ddot{q} + C(q,\dot{q})\dot{q}) = G^T(q)S(q)\tau \tag{6}$$

Substituting Eq.(5) into Eq. (6), this equation is simplified to follows

$$\overline{M}\dot{\overline{v}} + \overline{C}\overline{v} = \overline{\tau}\,. \tag{7}$$

where $\overline{M} = G^T MG$, $\overline{C} = G^T(M\dot{G} + CG)$, $\overline{\tau} = G^T S\tau$.

Dynamics of nonholonomic systems is derived in [9] and [10] in detail.

The nonholonomic constraint due to the non-sliding condition of two wheeled mobile robots is given by

$$[\sin\theta \;\; -\cos\theta \;\; 0]\dot{q} = 0 \tag{8}$$

O. J. Sø dalen showed that two-wheeled mobile robot with a nonholonomic constraint is completely controllable but not stabilizable by a smooth static state feedback control law as (5) [11].

Since $G(q)$ is transformation of velocity into Cartesian coordinates, it can be presented by

$$G(q) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \tag{9}$$

Actually, dynamic equation of the two wheeled robot can be described by

$$\overline{M}\dot{\overline{v}} + \overline{C}\overline{v} = \overline{\tau} \tag{10}$$

$$\dot{q} = G(q)\overline{v}(t)\,. \tag{11}$$

where

$$\overline{M} = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix},$$

$$\overline{C} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\overline{v} = \begin{bmatrix} v \\ \omega \end{bmatrix}$$

$m$ is the mass of the mobile robot, $I$ is the moment of inertia of the robot about the mass center, $v$ is the longitudinal velocity of the robot, and $\omega$ is the rotational velocity of the robot.

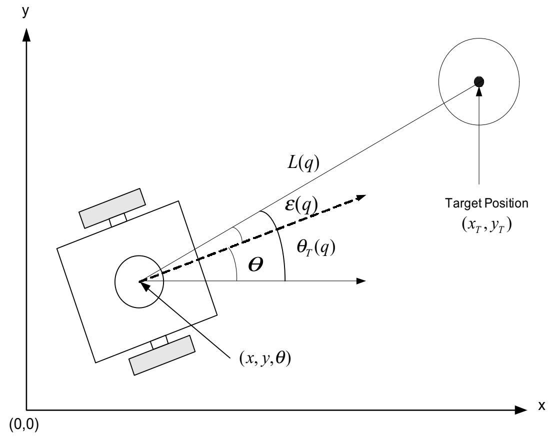



Fig.3 Control law to target position

The problem is to find a feedback control law $\overline{\tau}$, so that the closed loop system converges from any initial condition, $q(0)$, to a target position.

**2.2 Feedback control law for moving to target position**

A control law for two-wheeled mobile robot to move from any initial position to desired position is proposed. The robot is supposed to be able to start moving only forward. The longitudinal force of the robot is determined by a function of distance between current position of the mobile robot and target position and its derivative. The rotational torque is determined by an angle error between the target position and the current position of the mobile robot and its derivative. The force is proportional to the distance and its derivative. And the torque is proportional to the angle error and its derivative. The direction of the robot is determined to decrease distance and error angle between the robot and the target. Hence, target is shown as attracting the mobile robot. Fig. 3 shows the terms that are used in computing of longitudinal force and rotational torque of the robot. The two control inputs are composed of proportional and derivative term of the distance and angle error, respectively. The control inputs as follow.

$$\tau_T = \begin{bmatrix} K_{PL} L(q) + K_{DL} \dot{L}(q) \\ K_{PR} \varepsilon(q) + K_{DR} \dot{\varepsilon}(q) \end{bmatrix} \qquad (12)$$

where

$K_{PL}$ : Proportional gain of longitudinal force

$K_{DL}$ : Derivative gain of longitudinal force

$K_{PR}$ : Proportional gain of rotational torque

$K_{DR}$ : Derivative gain of rotational torque

$L(q) = \sqrt{(x - x_T)^2 + (y - y_T)^2}$

: Distance between current position of the robot and target position

$\dot{L}(q)$ : Derivative of the distance

$\varepsilon(q) = \theta_T - \theta$

: An angle error between target position and current position of the robot

$\dot{\varepsilon}(q)$ : Derivative of the angle error

**2.3 Feedback control law for obstacle avoidance**

While the mobile robot moves to the target position, there are able to be some obstacles. Therefore, an obstacle avoidance control algorithm is required. A simple obstacle avoidance algorithm that uses a distance measuring sensor and odometry from the robot is proposed. The mobile robot has only a distance measuring sensor at center of front of it. Using distance data between the mobile robot and each obstacle and derivative of it, the longitudinal force to avoid an obstacle is calculated. Magnitude of the rotational torque of the robot is computed by a similar method with getting the longitudinal force, but direction of the torque of it is determined by current rotating direction of the robot. In other words, if the robot is rotating in clockwise/counter clockwise direction, it is going to avoid the obstacles in direction of clockwise/counter clockwise. Fig. 4 shows that the robot avoids an obstacle. Before meeting an obstacle, the mobile robot moves with rotating along counter clockwise. Therefore, the mobile robot rotates in larger angle along counter clockwise.

The longitudinal force is proportional to the distance between the robot and obstacles and its derivative. And the rotating torque is inverse proportional to the angle error and its derivative. The direction of the robot is determined to increase distance and error angle between the robot and the target. Hence, obstacles are shown as repulsing the mobile robot. The control inputs that are needed for the robot to avoid an obstacle are as follow. These control inputs are applied only when the distance measuring sensor of the mobile robot detects obstacles. Therefore, these inputs are zero if the sensor does not detect any obstacles.

$$\tau_O = \begin{bmatrix} K_{PLO} D(q) + K_{DLO} \dot{D}(q) \\ \mathrm{sgn}(\dot{\theta})(K_{PRO} D(q) + K_{DRO} \dot{D}(q)) \end{bmatrix} \qquad (13)$$

where

$K_{PLO}$ : Proportional gain of longitudinal force

$K_{DLO}$ : Derivative gain of longitudinal force

$K_{PRO}$ : Proportional gain of rotational torque

$K_{DRO}$ : Derivative gain of rotational torque

$D(q)$ : Distance between current position of the robot and an obstacle

$\dot{D}(q)$ : Derivative of the distance

$\mathrm{sgn}(\dot{\theta})$ : If $\dot{\theta}$ is positive, this is 1 and if negative, this is 0.

The control inputs of the robot are summation of the upper two inputs as follows.

$$\overline{\tau} = \tau_T + \tau_O \qquad (14)$$

These control inputs are limited by maximum torque of the actuators like below.

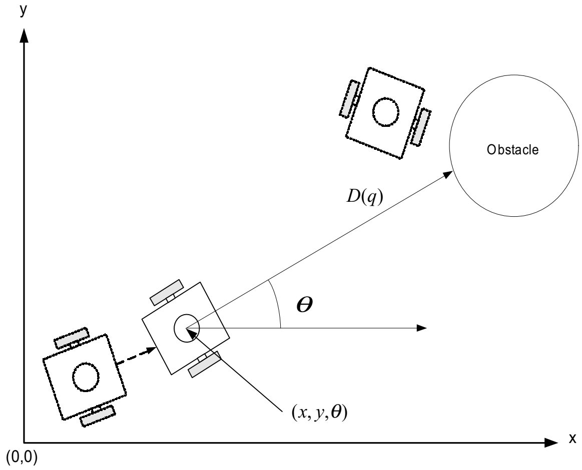



Fig.4. Obstacle avoidance

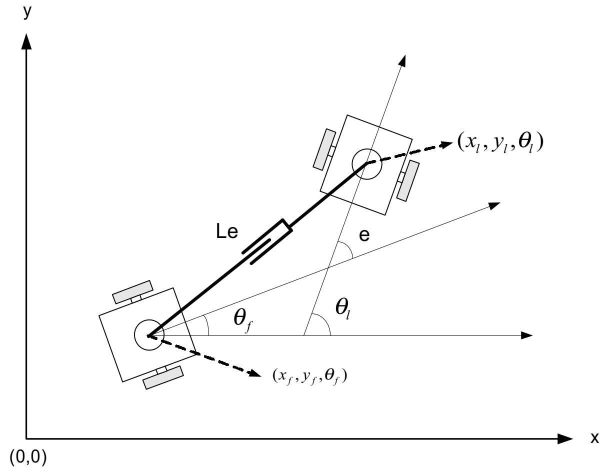



Fig.5. following motion of the follower

If $\overline{\tau} \geq \tau_{\max}$, $\overline{\tau} = \tau_{\max}$, and if $\overline{\tau} < \tau_{\min}$, $\overline{\tau} = \tau_{\min}$.

The role of moving to target position and avoid obstacles are for the front robot(Leader).

## 3. COOPERATING MOTION

### 3.1 Motion of follower

While the leader moves to the target position if the follower approaches too close to the leader, the ladder will shrink. And if the distance between two robots increases out of the range that they can hold the ladder, they will drop it. The follower tries to maintain the distance and angle between the leader and the follower in allowable range. It determines the control input force and torque by using the position and orientation information of the leader. If the distance between two mobile robots decreases, the follower makes its speed lower, and the follower increases its speed if the distance increases. Also, the follower tries to remain angle error between the orientations of the two mobile robots to zero. J. Borenstein [7] proposed Proportional-Integral controller to control for maintaining the distance between two mobile robots. In this paper, a Proportional-Derivative controller for same role that maintains the distance is proposed. Fig. 5 shows an operation of the follower that maintains the distance and zero angle error. Two robots are connected by compliant link that imitate a ladder on sliding pads of two robots. The control inputs of the follower are found like follow.

$$\tau_f = \begin{bmatrix} K_{fPL} L_e(q) + K_{fDL} \dot{L}_e(q) \\ K_{fPR} e(q) + K_{fDR} \dot{e}(q) \end{bmatrix} \tag{15}$$

where

$K_{fPL}$ : Proportional gain of longitudinal force of the follower

$K_{fDL}$ : Derivative gain of longitudinal force of the follower

$K_{fPR}$ : Proportional gain of rotational torque of the follower

$K_{fDR}$ : Derivative gain of rotational torque of the follower

$$L_e(q) = \sqrt{(x_l - x_f)^2 + (y_l - y_f)^2}$$

: Distance between the two mobile robots

$\dot{L}_e(q)$ : Derivative of the distance

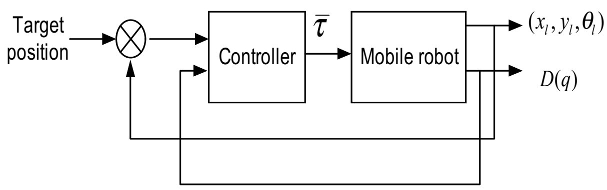
$$e(q) = \theta_l - \theta_f$$

: An angle error between the orientations of the two mobile robots

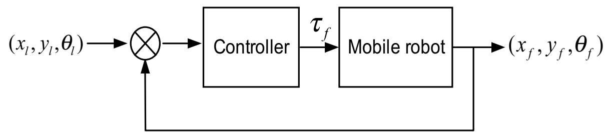
$\dot{e}(q)$ : Derivative of the angle error

### 3.1 Block diagram of the proposed controller

Fig. 6 shows the block diagrams of proposed controllers for two mobile robots. Fig.6 (a) is the diagram of the leader which leads to the desired target position and avoids with obstacles. Fig. 6 (b) is the diagram of the follower which maintaining the distance between two mobile robots and the error angle between orientations of the robots.




(a) Diagram of the leader




(b) Diagram of the follower

Fig. 6 Block diagram of the two robots

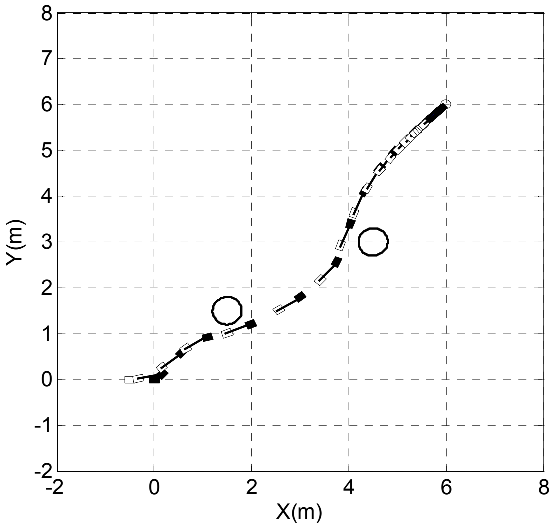


Fig. 7 Trajectories of the two robots

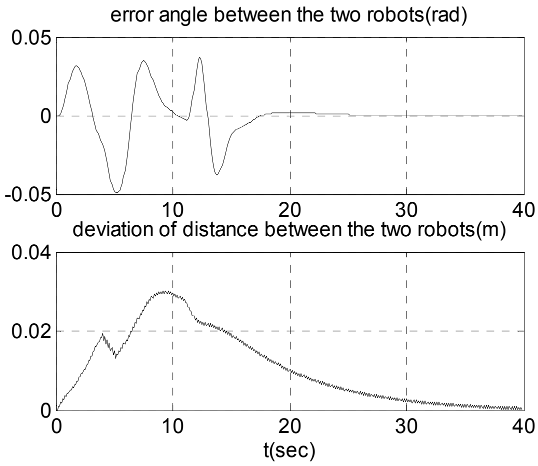


Fig. 8 Error between the two robots

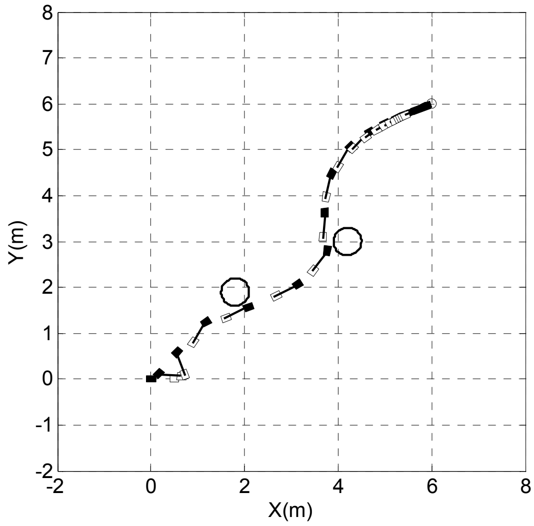


Fig. 9 Trajectories of the two robots

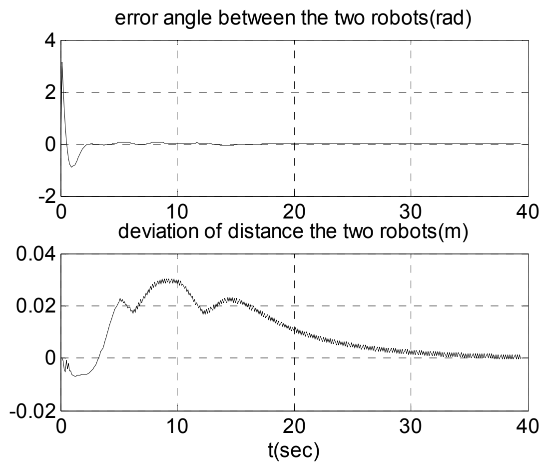


Fig. 10 Error between the two robots

## 4. SIMULATIONS

To verify the availability of the proposed controller, we have some simulations about two cases. One is the case that the leader is in front of the follower and the other is that the follower is in front of the leader. In real workspace, there can be time delay in communication for getting information of position and the orientation of the leader, but it is not considered in these simulations because it is very small time comparing with sampling time. The sampling time is 0.1second. The weight of the robot is 1kg and its inertia is $0.04\,kg \cdot m^2$. Size of the robots is $0.15 \times 0.2m$ and initial position of the leader is (0,0) in both cases. Length of the linkage that connects two mobile robots is 0.5m. It is assumed that the linkage which connects two mobile robots can be expanded and contracted as much as 0.1m. Two cases both have two obstacles which are circular cylinder with 30cm radius on Cartesian coordinates.

As shown in Fig. 7 obstacles are located at $(1.5, 1.5)$, $(4.5, 3)$ in coordinates, respectively. Since the leader is in front of the follower they move to the target directly. The simulation elapsed for 40 seconds. Maximum length between two robots is 0.5301m and minimum length is 0.5000m. Maximum deviation of angle error is 0.0368 rad.

Second case is shown in Fig. 9. Obstacles are located at $(1.8, 1.9)$, $(4.2, 3)$ in coordinates, respectively. Since the leader is in behind of the follower the follower turns around near initial position. The simulation elapsed for 39.5 seconds. Maximum length between two robots is 0.5304m and minimum is 0.4930m. Maximum deviation of angle error is 3.1416rad which is initial angle. After turning around of the follower error angle has about 0.07rad as maximum.

From two simulation results, we can show that the distance between two robots increases at instant after robot just started and that come across the obstacles. At near the target position distance and error angle between two robots converge to about zero.

## 5. CONCLUSIONS

A control algorithm for multiple two-wheeled mobile robots with obstacles was proposed. The two mobile robots are connected by a compliant linkage. The front robot defined as leader moves to the target position given by operator and avoids with unknown obstacles using distance measuring sensor. The follower in back of the leader maintains the constant distance and same angle with leader. Some simulations conducted to verify the availability of the proposed control algorithm. Two simulation results showed the two mobile robots can move to the desired position without collision with obstacles.

It is assumed that there is no time delay during communicating between the two mobile robots. To implement more reasonable system, time delay should be considered. There are not factors causing local minimum in environment where the mobile robots move. Some algorithms that make the mobile robots escape local minimum cases will be studied. And the method that changes leadership to drive the multiple mobile robots for more efficient traveling will be applied.

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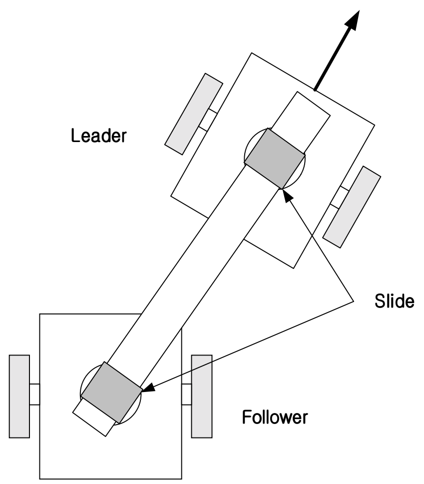


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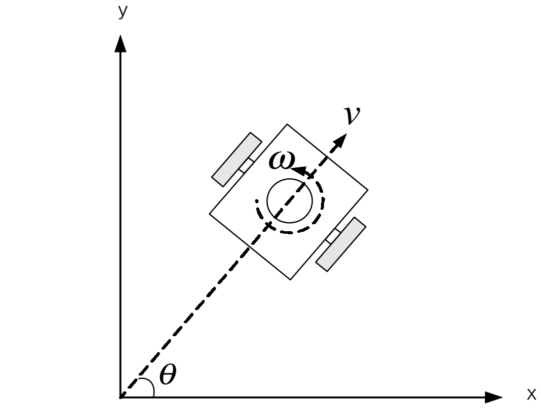

Fig.2 Nonholonomic mobile robot

On each robot there is a sliding pad that rotates freely as a compliant link that is supposed to be expanded and contracted. Therefore, each robot can move to any path without effect by each other. Along the progress direction, the front robot is defined as the leader and the rear one is the follower.

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The dynamic equation of motion with $m$ constraints can be described by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = A^T(q)\lambda + S(q)\tau \tag{2}$$

where $M(q)$ is an $n \times n$ symmetric and positive definite inertia matrix, $C(q,\dot{q})$ is an $n \times n$ centripetal Coriolis matrix. $A(q)$ is an $m \times n$ matrix and $\lambda$ is an $m \times 1$ Lagrange multiplier vector. $S(q)$ is an $n \times m$ input transfer matrix and $\tau$ is an $n \times 1$ input torque vector.

Kinematic constraints which are related with velocity can be described by

$$A(q)\dot{q} = 0\ . \tag{3}$$

Consider a matrix $G(q)$ whose columns are bases for the nullspace of $A(q)$ such that

$$A(q)G(q) = 0\ . \tag{4}$$

Since the constrained velocity is always in the null space of $A(q)$, from (3) and (4), we can find $(n-m)$ velocities $\overline{v}(t)$ such that

$$\dot{q} = G(q)\overline{v}(t) \tag{5}$$

where $G(q)$ transforms the velocity $\overline{v}(t)$ into the Cartesian coordinate $\dot{q}$.

The Lagrange multiplier vector can be eliminated by left-multiplying eq. (2) by $G^T(q)$, so the reduced dynamical model is obtained as follows.

$$G^T(q)(M(q)\ddot{q} + C(q,\dot{q})\dot{q}) = G^T(q)S(q)\tau \tag{6}$$

Substituting Eq.(5) into Eq. (6), this equation is simplified to follows

$$\overline{M}\dot{\overline{v}} + \overline{C}\overline{v} = \overline{\tau}\ . \tag{7}$$

where $\overline{M} = G^T M G$, $\overline{C} = G^T(M\dot{G} + CG)$, $\overline{\tau} = G^T S\tau$.

Dynamics of nonholonomic systems is derived in [9] and [10] in detail.

The nonholonomic constraint due to the non-sliding condition of two wheeled mobile robots is given by

$$[\sin\theta \ \ -\cos\theta \ \ 0]\dot{q} = 0 \tag{8}$$

O. J. Sø dalen showed that two-wheeled mobile robot with a nonholonomic constraint is completely controllable but not stabilizable by a smooth static state feedback control law as (5) [11].

Since $G(q)$ is transformation of velocity into Cartesian coordinates, it can be presented by

$$G(q) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \tag{9}$$

Actually, dynamic equation of the two wheeled robot can be described by

$$\overline{M}\dot{\overline{v}} + \overline{C}\overline{v} = \overline{\tau} \tag{10}$$

$$\dot{q} = G(q)\overline{v}(t)\ . \tag{11}$$

where

$$\overline{M} = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix},$$

$$\overline{C} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\overline{v} = \begin{bmatrix} v \\ \omega \end{bmatrix}$$

$m$ is the mass of the mobile robot, $I$ is the moment of inertia of the robot about the mass center, $v$ is the longitudinal velocity of the robot, and $\omega$ is the rotational velocity of the robot.

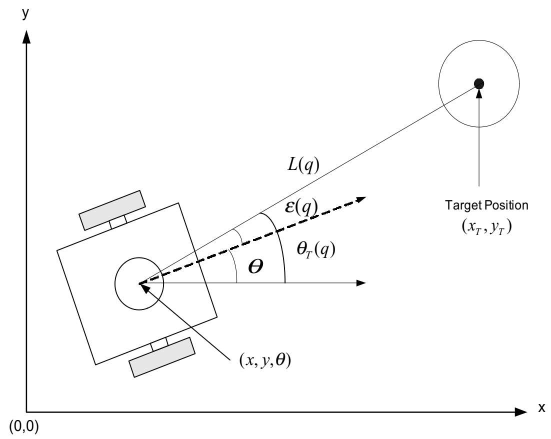


Fig.3 Control law to target position

The problem is to find a feedback control law $\overline{\tau}$, so that the closed loop system converges from any initial condition, $q(0)$, to a target position.

**2.2 Feedback control law for moving to target position**

A control law for two-wheeled mobile robot to move from any initial position to desired position is proposed. The robot is supposed to be able to start moving only forward. The longitudinal force of the robot is determined by a function of distance between current position of the mobile robot and target position and its derivative. The rotational torque is determined by an angle error between the target position and the current position of the mobile robot and its derivative. The force is proportional to the distance and its derivative. And the torque is proportional to the angle error and its derivative. The direction of the robot is determined to decrease distance and error angle between the robot and the target. Hence, target is shown as attracting the mobile robot. Fig. 3 shows the terms that are used in computing of longitudinal force and rotational torque of the robot. The two control inputs are composed of proportional and derivative term of the distance and angle error, respectively. The control inputs as follow.

$$\tau_T = \begin{bmatrix} K_{PL}L(q) + K_{DL}\dot{L}(q) \\ K_{PR}\varepsilon(q) + K_{DR}\dot{\varepsilon}(q) \end{bmatrix} \quad (12)$$

where

$K_{PL}$ : Proportional gain of longitudinal force

$K_{DL}$ : Derivative gain of longitudinal force

$K_{PR}$ : Proportional gain of rotational torque

$K_{DR}$ : Derivative gain of rotational torque

$L(q) = \sqrt{(x - x_T)^2 + (y - y_T)^2}$

: Distance between current position of the robot and target position

$\dot{L}(q)$ : Derivative of the distance

$\varepsilon(q) = \theta_T - \theta$

: An angle error between target position and current position of the robot

$\dot{\varepsilon}(q)$ : Derivative of the angle error

**2.3 Feedback control law for obstacle avoidance**

While the mobile robot moves to the target position, there are able to be some obstacles. Therefore, an obstacle avoidance control algorithm is required. A simple obstacle avoidance algorithm that uses a distance measuring sensor and odometry from the robot is proposed. The mobile robot has only a distance measuring sensor at center of front of it. Using distance data between the mobile robot and each obstacle and derivative of it, the longitudinal force to avoid an obstacle is calculated. Magnitude of the rotational torque of the robot is computed by a similar method with getting the longitudinal force, but direction of the torque of it is determined by current rotating direction of the robot. In other words, if the robot is rotating in clockwise/counter clockwise direction, it is going to avoid the obstacles in direction of clockwise/counter clockwise. Fig. 4 shows that the robot avoids an obstacle. Before meeting an obstacle, the mobile robot moves with rotating along counter clockwise. Therefore, the mobile robot rotates in larger angle along counter clockwise.

The longitudinal force is proportional to the distance between the robot and obstacles and its derivative. And the rotating torque is inverse proportional to the angle error and its derivative. The direction of the robot is determined to increase distance and error angle between the robot and the target. Hence, obstacles are shown as repulsing the mobile robot. The control inputs that are needed for the robot to avoid an obstacle are as follow. These control inputs are applied only when the distance measuring sensor of the mobile robot detects obstacles. Therefore, these inputs are zero if the sensor does not detect any obstacles.

$$\tau_O = \begin{bmatrix} K_{PLO}D(q) + K_{DLO}\dot{D}(q) \\ \mathrm{sgn}(\dot{\theta})(K_{PRO}D(q) + K_{DRO}\dot{D}(q)) \end{bmatrix} \quad (13)$$

where

$K_{PLO}$ : Proportional gain of longitudinal force

$K_{DLO}$ : Derivative gain of longitudinal force

$K_{PRO}$ : Proportional gain of rotational torque

$K_{DRO}$ : Derivative gain of rotational torque

$D(q)$ : Distance between current position of the robot and an obstacle

$\dot{D}(q)$ : Derivative of the distance

$\mathrm{sgn}(\dot{\theta})$ : If $\dot{\theta}$ is positive, this is 1 and if negative, this is 0.

The control inputs of the robot are summation of the upper two inputs as follows.

$$\overline{\tau} = \tau_T + \tau_O \quad (14)$$

These control inputs are limited by maximum torque of the actuators like below.

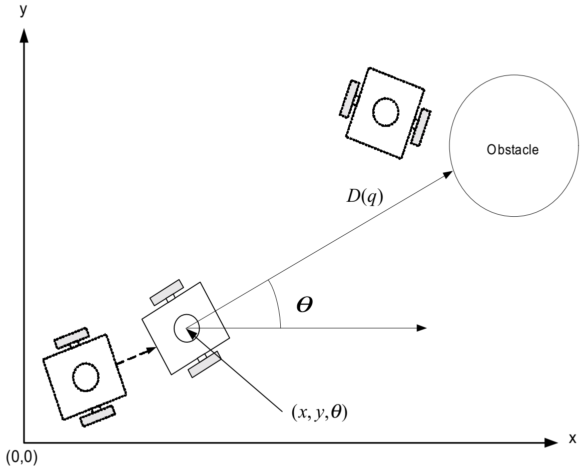



Fig.4. Obstacle avoidance

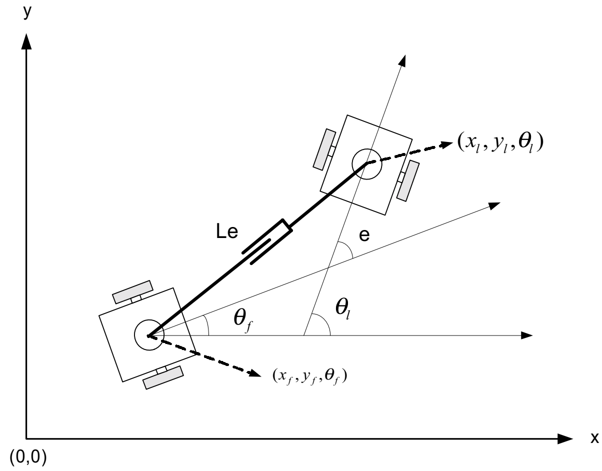



Fig.5. following motion of the follower

If $\overline{\tau} \geq \tau_{max}$, $\overline{\tau} = \tau_{max}$, and if $\overline{\tau} < \tau_{min}$, $\overline{\tau} = \tau_{min}$.

The role of moving to target position and avoid obstacles are for the front robot(Leader).

## 3. COOPERATING MOTION

### 3.1 Motion of follower

While the leader moves to the target position if the follower approaches too close to the leader, the ladder will shrink. And if the distance between two robots increases out of the range that they can hold the ladder, they will drop it. The follower tries to maintain the distance and angle between the leader and the follower in allowable range. It determines the control input force and torque by using the position and orientation information of the leader. If the distance between two mobile robots decreases, the follower makes its speed lower, and the follower increases its speed if the distance increases. Also, the follower tries to remain angle error between the orientations of the two mobile robots to zero. J. Borenstein [7] proposed Proportional-Integral controller to control for maintaining the distance between two mobile robots. In this paper, a Proportional-Derivative controller for same role that maintains the distance is proposed. Fig. 5 shows an operation of the follower that maintains the distance and zero angle error. Two robots are connected by compliant link that imitate a ladder on sliding pads of two robots. The control inputs of the follower are found like follow.

$$\tau_f = \begin{bmatrix} K_{fPL} L_e(q) + K_{fDL} \dot{L}_e(q) \\ K_{fPR} e(q) + K_{fDR} \dot{e}(q) \end{bmatrix} \qquad (15)$$

where

$K_{fPL}$ : Proportional gain of longitudinal force of the follower

$K_{fDL}$ : Derivative gain of longitudinal force of the follower

$K_{fPR}$ : Proportional gain of rotational torque of the follower

$K_{fDR}$ : Derivative gain of rotational torque of the follower

$$L_e(q) = \sqrt{(x_l - x_f)^2 + (y_l - y_f)^2}$$

: Distance between the two mobile robots

$\dot{L}_e(q)$ : Derivative of the distance

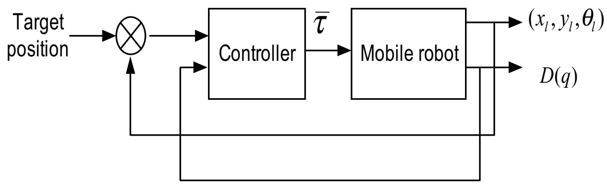
$$e(q) = \theta_l - \theta_f$$

: An angle error between the orientations of the two mobile robots

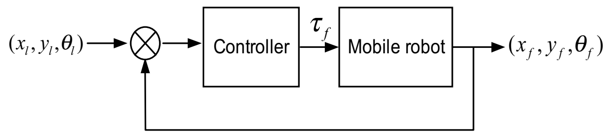
$\dot{e}(q)$ : Derivative of the angle error

### 3.1 Block diagram of the proposed controller

Fig. 6 shows the block diagrams of proposed controllers for two mobile robots. Fig.6 (a) is the diagram of the leader which leads to the desired target position and avoids with obstacles. Fig. 6 (b) is the diagram of the follower which maintaining the distance between two mobile robots and the error angle between orientations of the robots.




(a) Diagram of the leader




(b) Diagram of the follower

Fig. 6 Block diagram of the two robots

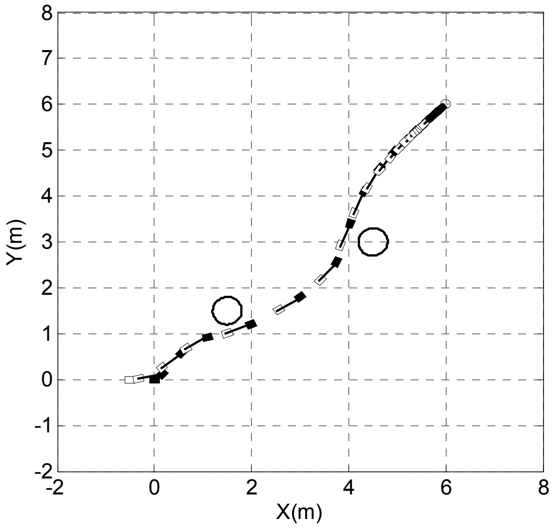



Fig. 7 Trajectories of the two robots

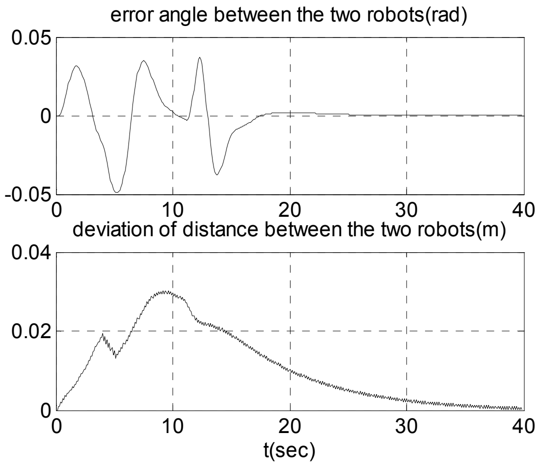



Fig. 8 Error between the two robots

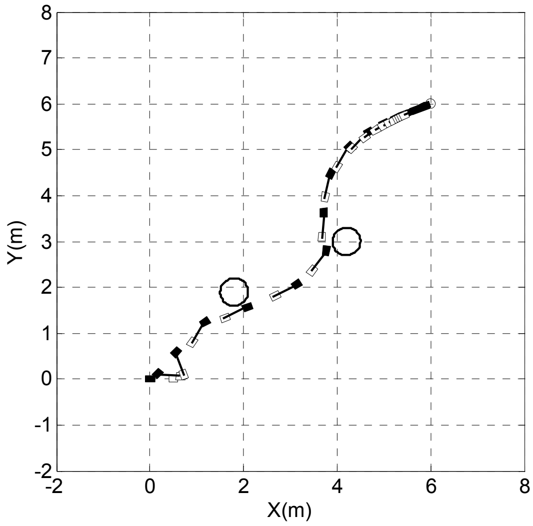



Fig. 9 Trajectories of the two robots

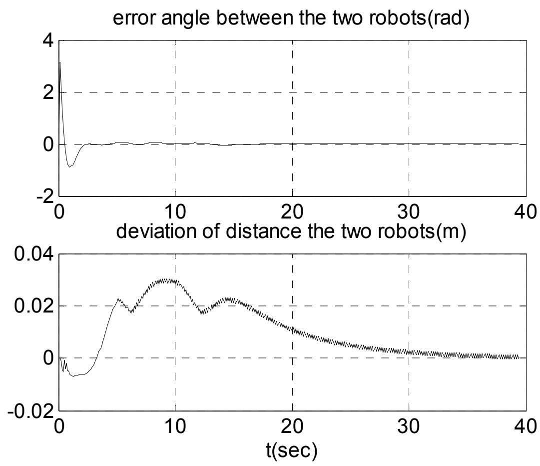



Fig. 10 Error between the two robots

## 4. SIMULATIONS

To verify the availability of the proposed controller, we have some simulations about two cases. One is the case that the leader is in front of the follower and the other is that the follower is in front of the leader. In real workspace, there can be time delay in communication for getting information of position and the orientation of the leader, but it is not considered in these simulations because it is very small time comparing with sampling time. The sampling time is 0.1second. The weight of the robot is 1kg and its inertia is $0.04\,kg \cdot m^2$. Size of the robots is $0.15 \times 0.2m$ and initial position of the leader is (0,0) in both cases. Length of the linkage that connects two mobile robots is 0.5m. It is assumed that the linkage which connects two mobile robots can be expanded and contracted as much as 0.1m. Two cases both have two obstacles which are circular cylinder with 30cm radius on Cartesian coordinates.
As shown in Fig. 7 obstacles are located at $(1.5, 1.5)$, $(4.5, 3)$ in coordinates, respectively. Since the leader is in front of the follower they move to the target directly. The simulation elapsed for 40 seconds. Maximum length between two robots is 0.5301m and minimum length is 0.5000m. Maximum deviation of angle error is 0.0368 rad.

Second case is shown in Fig. 9. Obstacles are located at $(1.8, 1.9)$, $(4.2, 3)$ in coordinates, respectively. Since the leader is in behind of the follower the follower turns around near initial position. The simulation elapsed for 39.5 seconds. Maximum length between two robots is 0.5304m and minimum is 0.4930m. Maximum deviation of angle error is 3.1416rad which is initial angle. After turning around of the follower error angle has about 0.07rad as maximum.

From two simulation results, we can show that the distance between two robots increases at instant after robot just started and that come across the obstacles. At near the target position distance and error angle between two robots converge to about zero.

## 5. CONCLUSIONS

A control algorithm for multiple two-wheeled mobile robots with obstacles was proposed. The two mobile robots are connected by a compliant linkage. The front robot defined as leader moves to the target position given by operator and avoids with unknown obstacles using distance measuring sensor. The follower in back of the leader maintains the constant distance and same angle with leader. Some simulations conducted to verify the availability of the proposed control algorithm. Two simulation results showed the two mobile robots can move to the desired position without collision with obstacles.

It is assumed that there is no time delay during communicating between the two mobile robots. To implement more reasonable system, time delay should be considered. There are not factors causing local minimum in environment where the mobile robots move. Some algorithms that make the mobile robots escape local minimum cases will be studied. And the method that changes leadership to drive the multiple mobile robots for more efficient traveling will be applied.

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