# Pose Determination of a Mobile-Task Robot Using an Active Calibration of the Landmark 

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#### Abstract

A new method of estimating the pose of a mobile-task robot is developed based upon an active calibration scheme. The utility of a mobile-task robot is widely recognized, which is formed by the serial connection of a mobile robot and a task robot. For the control of the mobile robot, an absolute position sensor is necessary. This paper proposes an active calibration scheme to estimate the pose of a mobile robot that carries a task robot on the top. The active calibration scheme is to estimate a pose of the mobile robot using the relative position/orientation to a known object whose location, size, and shape are known a priori. Through the homogeneous transformation, the absolute position/ orientation of the camera is calculated and that is propagated to getting the pose of a mobile robot. With the experiments in the corridor, the proposed active calibration scheme is verified experimentally.


## 1. Introduction

Recently, in the factory automation, not only productivity but also flexibility is required to produce small quantity of several items in the same production line to overcome the versatile tastes of customers. A mobile robot carrying a task robot is a good alternative to conveyor lines for this purpose. Many of papers on mobile robots focused on how to avoid collisions against to obstacles, considering a mobile robot as a single carrier of an object. However, in the mobile-task robot, the mobile robot needs to be controlled very accurately since the end plate of the mobile robot is used as a base for the task robot [1].
In this paper, we proposed a method of measuring the position/orientation of the task robot base, that is, the pose of the mobile robot using the images of known objects
captured by a camera attached at the end of the task robot [12], [13]. A camera is represented as the pin-hole model, which approximates the camera system as linear such that camera parameters can be analyzed experimentally. Using the feature points on the objects whose locations are given a priori, we calculate the pose of the camera. After the calculation of the pose of the camera, we can obtain the homogeneous transformation matrix for the base of the task robot with respect to a world frame assuming that the kinematic parameters of the task robot are precisely given.

## 2. Active Calibration method of a Mobile/Task Robot

### 2.1 Perspective model of a camera

A perspective model of a camera represents the relationship in between the two dimensional object location on a image plane and the actual object location in a three dimensional space. Figure 1 represents a perspective model of a camera. Here, the coordinates, $\{\mathrm{W}\}$, is a world frame in the 3-D space, the coordinates, $\{\mathrm{C}\}$, is a camera frame whose origin is assigned at the center of the lens of the camera. Note that $z_{c}$ axis is coincident with the optical axis of the camera.


Fig. 1 Perspective model of camera.

A point in the space can be represented as a vector, $\mathrm{p}_{\mathrm{w}}=\left(x_{w}, y_{w}, z_{w}\right)$ w.r.t the reference frame, and it can be also represented as $\mathrm{p}_{\mathrm{c}}=\left(x_{c}, y_{c}, z_{c}\right)$ w.r.t the camera frame. The coordinates, $\{\mathrm{I}\}$, represents the image frame of the camera and $f$ is focal length of the camera.
The vector, $\mathrm{p}_{\mathrm{i}}=\left(x_{i}, y_{i}\right)$, on the image plane represents a feature point on the fixed object. $\mathrm{p}_{\mathrm{w}}$ w.r.t. the world frame corresponds to the vector, $\mathrm{p}_{\mathrm{c}}$, w.r.t. the camera frame. The coordinate, $\left(c_{x}, c_{y}\right)$ represents the center coordinates of the image in the image frame

### 2.2 Camera parameters

The camera parameters to be estimated can be classified into two categories: internal and external parameters. The specification related parameters of the camera and lens, for examples, the focal distance, $f$, and the image scale factor $S$, are internal parameters; the rotation matrix, $R$, and the translational vector, $T$ representing the pose of the moving camera are external parameters. By the coordinates transformation between the robot and camera frames, the positioning vector, $\mathrm{p}_{\mathrm{c}}$, represented in terms of the camera frame can be represented as $\mathrm{p}_{\mathrm{w}}$ in terms of the world frame.

$$
\begin{equation*}
\mathrm{p}_{\mathrm{w}}=R \mathrm{p}_{\mathrm{c}}+\mathrm{T} \tag{1}
\end{equation*}
$$

A positioning vector for a feature point on the three dimensional object in terms of the camera frame, $\mathrm{p}_{\mathrm{c}}=\left(x_{c}, y_{c}, z_{c}\right)$ is mapped to a point $\mathrm{p}_{\mathrm{i}}=\left(x_{i}, y_{i}\right)$ on the two dimensional image frame using the camera perspective model [2], and it can be described as follows:

$$
\begin{align*}
& x_{i}=f \frac{x_{c}}{z_{c}}  \tag{2}\\
& y_{i}=f \frac{y_{c}}{z_{c}} \tag{3}
\end{align*}
$$

Since the scale values along $x$ axis and $y$ axis are different in the image frame, a point on the image frame, $\left(x_{i}, y_{i}\right)$ corresponds to a location in the image frame, $\left(x_{f}, y_{f}\right)$ according to the following relations:

$$
\begin{equation*}
x_{i}=S_{x}^{-1} \overline{x_{i}} \tag{4}
\end{equation*}
$$

$$
\begin{equation*}
x_{i}=S_{x}^{-1} \overline{x_{i}} \tag{5}
\end{equation*}
$$

where $\overline{x_{i}}=d_{x}\left(x_{f}-c_{x}\right), \overline{y_{i}}=d_{y}\left(y_{f}-c_{y}\right)$, and $S_{x}$ and $S_{y}$
represents the camera scale factor along the $x$ axis and $y$ axis, respectively.

## 3. Robot/Vision System

A task robot that has 5 links and a gripper, and a mobile robot that has 3 d.o.f are serially connected for this research. Figure 2 shows the coordinates assigned to this overall system.


Fig. 2 Link Coordinates of Mobile Robot Supporting a Task Robot.

The coordinates transformation relationship among the frames of the mobile robot supporting a task robot is shown in Figure 3.


Fig. 3 Coordinates Transformation of Robot/Vision System.

The calibration task can be decomposed into two steps. The first step is measuring the relative pose of the camera, ${ }^{W} H_{C}$, using the images of the fixed object. The second step is obtaining the homogeneous transformation of the mobile robot w.r.t. the world frame, ${ }^{W} H_{B}$, using ${ }^{W} H_{C},{ }^{B} H_{H}$ and ${ }^{H} H_{C}$. This process can be represented by the following equations.

$$
\begin{align*}
& { }^{W} H_{B}={ }^{W} H_{C} \cdot{ }^{B} H_{C}{ }^{T}  \tag{6}\\
& { }^{B} H_{C}={ }^{B} H_{H} \cdot{ }^{H} H_{C} \tag{7}
\end{align*}
$$

The structure of the task robot is generally assumed to be light and small. Therefore, ${ }^{B} H_{C}$, can be obtained accurately through the forward kinematics. In the following section, the process of obtaining ${ }^{W} H_{C}$ will be described in detail.
As it is shown in Figure 3, ${ }^{W} H_{C}$ is obtained as

$$
\begin{equation*}
{ }^{W} H_{C}={ }^{C} H_{O} \cdot{ }^{W} H_{O}{ }^{T} \tag{8}
\end{equation*}
$$

## 4. Parameter Estimation Using Linecorrespondence

Among the camera parameters, we obtain the internal parameters, $S$ and $f$, and the rotation matrix, $R$, in the first step. Later, the translational vector, $T$, is obtained using the correspondence of the feature points [3], [4].

Let us denote a straight line, J (refer to Figure 4) as

$$
\begin{equation*}
\mathrm{J}: \mathrm{P}_{\mathrm{j}}=\mathrm{n}_{\mathrm{w}} t+\mathrm{P}_{\mathrm{i}} \tag{9}
\end{equation*}
$$

where, $\mathrm{n}_{\mathrm{w}}$ represents the directional vector of the straight line, $t$ represents a constant, and $\mathrm{P}_{\mathrm{i}}$ and $\mathrm{P}_{\mathrm{j}}$ are positioning vectors for the point $U$ and $V$ w.r.t. the world frame, respectively.

A two dimensional line $L$ can be represented as follows:

$$
\begin{equation*}
\mathrm{L}: A x_{i}+B y_{i}+C=0 \tag{10}
\end{equation*}
$$

where, $A, B$ and $C$ can be determined by a constraint equation, $A^{2}+B^{2}+C^{2}=1$ and two equations corresponding to the two points. Substituting equation (2) and (3) into equation (10), we obtain the following plane equation:

$$
\begin{equation*}
\mathrm{M}: A x_{c}+B y_{c}+f^{-1} C z_{c}=0 \tag{11}
\end{equation*}
$$



Fig. 4. Projecting plane of a 3-D line and a 2-D line.

The vector N is defined as a normal to the projecting
plane M,

$$
\mathrm{N}=\left[\begin{array}{lll}
A & B & f^{-1} C \tag{12}
\end{array}\right]^{T}
$$

Note that this normal vector N is always orthogonal to the three dimensional line J . The directional vector for the 3-D line J can be denoted as $\mathrm{n}_{\mathrm{c}}$ and represented in terms of the camera frame as

$$
\begin{equation*}
\mathrm{n}_{\mathrm{c}}=R^{T} \mathrm{n}_{\mathrm{w}} \tag{13}
\end{equation*}
$$

Since the 3-D line J is located on the projecting plane, M , and the directional vector of J is orthogonal to the normal vector of the projecting plane, we have

$$
\begin{align*}
& \mathrm{n}_{\mathrm{c}} \cdot \mathrm{~N}=0  \tag{14}\\
& \mathrm{n}_{\mathrm{w}}^{T} R \mathrm{~N}=0 \tag{15}
\end{align*}
$$

Let us consider two points in 3-D space, $U$ and $V$, and the corresponding two points in the image plane, $\mathrm{P}_{\mathrm{i}}\left(X_{i}, Y_{i}\right)$ and $\mathrm{P}_{\mathrm{j}}\left(X_{j}, Y_{j}\right)$. Plugging $\mathrm{P}_{\mathrm{i}}$ and $\mathrm{P}_{\mathrm{j}}$ coordinates into equation (10), and solving the two equations for the line coefficients, $A, B$ and $C$, we have

$$
\begin{equation*}
A=\left(Y_{j}-Y_{i}\right), B=\left(X_{j}-X_{i}\right), C=\left(X_{j} Y_{i}-X_{i} Y_{j}\right) \tag{16}
\end{equation*}
$$

Since the directional vector, $\mathrm{n}_{\mathrm{w}}$, of the 3-D line is parallel to the line passing through the two points, $U$ and $V$, it can be denoted as

$$
\begin{equation*}
\mathrm{n}_{\mathrm{w}}=\left(\mathrm{P}_{\mathrm{i}}-\mathrm{P}_{\mathrm{j}}\right) /\left\|\mathrm{P}_{\mathrm{i}}-\mathrm{P}_{\mathrm{j}}\right\| \tag{17}
\end{equation*}
$$

Utilizing $\overline{x_{i}}$ in (4) to separate $S$ from $x_{i}$, and substituting this into equation (16-a) through ( $16-\mathrm{c}$ ), the normal vector (12) is represented as

$$
\mathrm{N}=\left[\begin{array}{lll}
A & S^{-1} B & S^{-1} f^{-1} C \tag{18}
\end{array}\right]^{T}
$$

Now, let us describe the process of obtaining camera parameters using (15). Substituting (17) and (18) into (15), we have

$$
\left[\begin{array}{lll}
i & j & k
\end{array}\right]\left[\begin{array}{lll}
r_{1} & r_{2} & r_{3}  \tag{19}\\
r_{4} & r_{5} & r_{6} \\
r_{7} & r_{8} & r_{9}
\end{array}\right]\left[\begin{array}{c}
A \\
S^{-1} B \\
S^{-1} f^{-1} C
\end{array}\right]=0
$$

where the parameters $\mathrm{A}, \mathrm{B}$, and C are obtained by (16-a) through (16-c) and the directional vector $\mathrm{n}_{\mathrm{w}}=\left[\begin{array}{lll}i & j & k\end{array}\right]^{T}$ is obtained by (17)

Equation (19) can be changed to equation (20) by decomposing known variables and unknown variables as
follows:

$$
\left[\begin{array}{llllllll}
i A & i B & i C & j A & j C & k A & k B & k C
\end{array}\right]\left[\begin{array}{c}
S \cdot r_{1} \cdot r_{5}^{-1}  \tag{20}\\
r_{2} \cdot r_{5}^{-1} \\
f^{-1} \cdot r_{3} \cdot r_{5}^{-1} \\
S \cdot r_{4} \cdot r_{5}^{-1} \\
f^{-1} \cdot r_{6} \cdot r_{5}^{-1} \\
S \cdot r_{7} \cdot r_{5}^{-1} \\
r_{8} \cdot r_{5}^{-1} \\
f^{-1} \cdot r_{9} \cdot r_{5}^{-1}
\end{array}\right]=-j B
$$

Note that even though there are nine variables $r_{1} \cdots r_{9}$ in $R$, only three of them are independent. By the superposition, a matrix which equation can be represented as

$$
\begin{equation*}
M_{8 \times 8} \mathrm{X}_{8 \times 1}=\mathrm{B}_{8 \times 1} \tag{21}
\end{equation*}
$$

where $X_{8 \times 1}$ is the second vector in (20). Now, the unknown camera parameters can be obtained by multiplying the inverse of $M$ at both sides of (21). Therefore, instead of multiplying inverse of $M_{8 \times 8}$ directly, the matrix is decomposed by the singular value decomposition (SVD) as $M_{8 \times 8}=U D V^{T}$, and the matrix $\mathrm{X}_{8 \times 1}$ is obtained as follows:

$$
\begin{equation*}
\mathrm{X}=V \cdot D^{-1} \cdot U^{T} \cdot \mathrm{~B} \tag{22}
\end{equation*}
$$

where $D \in R^{8 \times 8}$ is a diagonal matrix with positive element $D \in R^{8 \times 8}$ and $V \in R^{8 \times 8}$ are eigen-vector matrices.
Now, the correspondence of the feature points are used to calculate the translational vector $T$ of the camera frame. By changing (1) as (23), and substituting this equation into (2) and (3), we have the coordinates of the feature point on the image frame as (24) and (25):

$$
\begin{align*}
& \mathrm{p}_{\mathrm{c}}=R^{-1} \mathrm{p}_{\mathrm{r}}-R^{-1} T=\mathrm{p}_{\mathrm{w}}^{\prime}-T^{\prime}  \tag{23}\\
& x_{i}=f \frac{x_{w}^{\prime}-T_{x}^{\prime}}{z_{w}^{\prime}-T_{z}^{\prime}}  \tag{24}\\
& y_{i}=f \frac{y_{w}^{\prime}-T_{x}^{\prime}}{z_{w}^{\prime}-T_{z}^{\prime}} \tag{25}
\end{align*}
$$

where $\mathrm{p}_{\mathrm{w}}^{\prime}=\left(x_{w}^{\prime}, y_{w}^{\prime}, z_{w}^{\prime}\right)^{T}$ and $T^{\prime}$ represent a positioning vector and a translation vector in terms of the camera frame, respectively. To obtain the translation vector $T^{\prime}$ from (24) and (25), the following matrix equation is derived:

$$
\left[\begin{array}{ccc}
f & 0 & -x_{i}  \tag{26}\\
0 & f & -y_{i}
\end{array}\right]\left[\begin{array}{l}
T_{x}^{\prime} \\
T_{y}^{\prime} \\
T_{z}^{\prime}
\end{array}\right]=\left[\begin{array}{l}
f \cdot x_{r}^{\prime}-x_{i} \cdot z_{r}^{\prime} \\
f \cdot y_{r}^{\prime}-y_{i} \cdot z_{r}^{\prime}
\end{array}\right]
$$

Therefore, if we have two feature points, four linear equations are obtained for the three unknowns, $T_{x}^{\prime}, T_{y}^{\prime}$, and, $T_{z}^{\prime}$.

## 5. Active Calibration using Coniccorrespondence

## 5.1 conic parameter estimation

We apply the least square method for the curve fitting algorithm to obtain the conic parameter matrix.
Let's consider the second order equation (27) defining the conics in a plane. In an image plane, a second-order conic equation is represented as

$$
\begin{equation*}
a X^{2}+b X Y+c Y^{2}+d X+e Y+f=0 \tag{27}
\end{equation*}
$$

where $a, b, c, d, e$, and $f$ are conic parameters that determine the shape of the second order equation.

Equation (27) is re-formulated with a general vector $\mathrm{u}=\left[\begin{array}{lll}X & Y & 1\end{array}\right]^{T}$ and a coefficient matrix, $A$ as follow:

$$
\begin{equation*}
\mathbf{u}^{T} A \mathbf{u}=0 \tag{28}
\end{equation*}
$$

where the $A$ matrix is defined as a symmetric conic coefficient matrix, that is,

$$
A=\left[\begin{array}{ccc}
a & b / 2 & d / 2  \tag{29}\\
b / 2 & c & e / 2 \\
d / 2 & e / 2 & f
\end{array}\right]
$$

In order to estimate the conic coefficient matrix A , we need to get the image coordinates, $\left(x_{i}, y_{i}\right)$ 's, on the conic.

### 5.2 Active calibration using conic object

If we use a camera for the active calibration, multiple images need to be captured at different postures of the camera. Figure 5 displays the geometric structure of the camera capturing two images at the different postures. The relationship between two-camera coordinate system is represented as

$$
\begin{equation*}
\mathrm{p}_{\mathrm{c} 1}={ }^{1} R_{2} \mathrm{p}_{\mathrm{c} 2}+{ }^{1} t_{2} \tag{30}
\end{equation*}
$$

where ${ }^{1} R_{2}$ represents a rotation matrix from $\left\{C_{1}\right\}$ to $\left\{C_{2}\right\}$ and ${ }^{1} t_{2}$ is a translation vector from $\left\{C_{1}\right\}$ to $\left\{C_{2}\right\}$.


Fig. 5. Geometric structure for capturing conic images.

## 6. Experiments

### 6.1 Robot Localization used Landmark

To begin with, the 2-D landmark used by IRL-2001 is shown in Figure 6. The primary pattern of landmark is a 10 cm black square block on white background and a 5 cm square block. The image corners are then automatically extracted by camera parameters, and displayed on Figure 6 and the blue squares around the corner points show the limits of the corner finder window. The corners are extracted to an accuracy of about 0.1 pixel.


Fig. 6. The landmark pattern and size used by IRL-2001.

The extrinsic parameters, relative positions of the landmark with respect to the camera, are then shown in a form of a 3D plot as Figure 8. And on Figure 9, every camera position and orientation are represented by red pyramid, therefore we can see the location and the orientation of a mobile robot in the indoor environment.


Fig. 7. A landmark locations detected by camera.


Fig. 8. Relative position of the landmark w.r.t the camera


Fig. 9. Mobile robot positionand orientation.
To measure the relative distance of the landmark from the mobile robot, we first measure the distance of image from the fixed position in $I R L$-lab corridor. The predefined values of the landmark defined in this section are given as follows the origin of coordinates is equal to the origin of mobile robot, a Y-axis is fit to the front of mobile robot and an X -axis is perpendicular with Y -axis.

Table 1 lists the data measured in IRL-lab corridor. The Left direction marks negative. From table 1, we find the maximum and the minimum error on distance is 0.32 m and 0.13 m , respectively.
It shows that the distance error becomes less and less by frames, which composes the environment map. And so, we can use it to measure the relative distance of the mobile robot.

Table 1. The result of relative distance (Dim.:m).

| Frame <br> Number | World <br> Coordinate <br> Distance | Image <br> Coordinate <br> Distance | Error |
| :---: | :---: | :---: | :---: |
| 1 | 7.81 | 8.13 | 0.32 |
| 2 | 7.02 | 7.30 | 0.28 |
| 3 | 6.28 | 6.53 | 0.25 |
| 4 | 5.06 | 4.89 | 0.17 |
| 5 | 5.52 | 5.39 | 0.13 |
| 6 | 6.32 | 6.46 | 0.14 |

## 7. Conclusions

A new active calibration scheme is developed to estimate the position/orientation of a mobile robot working in the various environments. In this research, the position of mobile robot is obtained by the active calibration scheme using the images captured by a camera at the hand of the task robot. In detail, the correspondence of the image coordinates to the real object coordinates is the basis for this scheme. This scheme is applied to the work-pieces of both the polygonal and the cylindrical.
For a polygonal object composed of points or lines, while the task robot is approaching to the object, the position of the camera is estimated using the line-correspondence between the lines on the image captured by the camera and the lines of the real object. For the cylindrical or ball-shaped objects, there are not enough line segments for the calibration based on the line-correspondence. For these objects, the coniccorrespondence scheme is developed. That is, two conic parameter matrices that can be obtained from the two consecutive elliptic images and a homogeneous transformation matrix are used for obtaining the position and orientation of the camera. Note that the homogeneous transformation matrix which defines the relationship between the two frames where the images are captured can be calculated by using the joint angles of the robot. Our future research topics are reducing the estimation errors and capturing two successive frames effectively adapting the environmental variations.

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