

## Neuro-controller for a XY Positioning Table

Jun Oh Jang\*

\* Department of Computer Control Engineering, Uiduk University, Kyongju, 780-713, South Korea  
(Tel : +82-54-760-1624; E-mail: jojang@mail.uiduk.ac.kr)

**Abstract:** This paper presents control designs using neural networks (NN) for a XY positioning table. The proposed neuro-controller is composed of an outer PD tracking loop for stabilization of the fast flexible-mode dynamics and an NN inner loop used to compensate for the system nonlinearities. A tuning algorithm is given for the NN weights, so that the NN compensation scheme becomes adaptive, guaranteeing small tracking errors and bounded weight estimates. Formal nonlinear stability proofs are given to show that the tracking error is small. The proposed neuro-controller is implemented and tested on an IBM PC-based XY positioning table, and is applicable to many precision XY tables. The algorithm, simulation, and experimental results are described. The experimental results are shown to be superior to those of conventional control.

**Keywords:** Neural network, XY Positioning table, System nonlinearities, Tracking error bound

### 1. INTRODUCTION

Very accurate control is required in mechanical devices such as xy positioning tables, overhead crane mechanisms, robot manipulators, etc. For many of these devices, the performance is limited by deadzone, friction, and backlash [1-3]. Precise positioning, in particular, control of very small displacement is an especially difficult problem for micro positioning devices. Due to the nonanalytic nature of the system nonlinearities and the fact that their exact parameters are unknown such systems present a challenge for the control design engineer. A number of control strategies have been developed to overcome the problems caused by the nonlinearities effects. Lee and Tomizuka [4] proposed the robust motion controller design for high accuracy positioning systems. Cross-coupled control of biaxial feed drive servomechanisms is considered by Srinivansan and Kulkarni [5]. Adaptive high-precision control of positioning tables including theory and experiment is appeared in [6].

Recently, advances in the area of artificial neural networks have provided the potential for new approaches to the control of nonlinear systems through learning process. Artificial neural networks provide a distinctive computational paradigm by exploiting the massively parallel processing performed in their elementary processing elements called neurons. Relevant features of the NN in the control context include their ability to model arbitrary differential nonlinear functions, and their intrinsic on-line adaptation and learning capabilities. For example, Narendra and Parthasarathy [7] have shown by the simulations that the NN can be used to effectively for the identification and control of nonlinear dynamic processes. Lightbody and Irwin [8] proposed a direct model reference adaptive control structure using a linear controller and an NN in parallel in a chemical process and a missile control system. In robotics, Kawato et al. [9] used an hierarchical NN model as add-on component to the conventional linear controller in order to control the movement of a robot. Lewis et al. [10] proposed a multilayer neural-net robot controller with guaranteed tracking performance. Cui and Shin [11] proposed a direct control and coordination method using NN for a multiple robot system.

In this paper, we present the neuro-controller for a XY positioning table. The proposed neuro-controller is composed of a linear controller and an NN to compensate for system

nonlinearities. A rigorous design procedure with proofs is given that results in a PD tracking loop with an NN in the feedforward loop. We derive a practical bound on the tracking error from the analysis of the tracking error dynamics. The neuro-controller is implemented on a XY positioning table to show its efficacy in canceling the deleterious effects of system nonlinearities.

### 2. XY POSITIONING TABLE

The XY positioning table is constructed by mechanical connection of the servo system for each axis and these servo systems are controlled for each axis, independently [12]. The XY positioning table is depicted in Fig. 1. It is shown that the dynamics of a XY table can be written by

$$J_j \ddot{q}_j + B_j \dot{q}_j + T_{ff} + T_d = T_j, \quad j=1,2 \quad (1)$$

where  $q(t)$  is the position,  $J$  is the inertia,  $B$  is the damping,  $T_f$  is the nonlinear friction,  $T_d$  is the load disturbance, and  $T$  is system input. The nonlinear friction [13,14] is described by

$$T_{ff}(q_j) = \alpha_0 \operatorname{sgn}(\dot{q}_j) + \alpha_1 e^{-\alpha_2 |\dot{q}_j|} \operatorname{sgn}(\dot{q}_j), \quad j=1,2 \quad (2)$$

with constants  $\alpha_i > 0$ ,  $i=0,1,2$ . No assumption are made on the shape of the second nonlinear component  $\alpha_1 e^{-\alpha_2 |\dot{q}_j|} \operatorname{sgn}(\dot{q}_j)$  except that it vanishes beyond some critical relative velocity  $q_{jc}(t)$ . A model-free identifier such as an NN can be designed to capture such a characteristic, as described in the following.

### 3. NEURO CONTROLLER OF A XY POSITIONING TABLE

We derive the neuro-controller for a XY positioning table so that the tracking error is guaranteed small and all internal state are bound. The each-axis dynamics of the XY positioning table may be rewritten from (1) by

$$J\ddot{q} + B\dot{q} + T_f + T_d = T \quad (3)$$

where  $q(t)$  is the position,  $J$  is the inertia,  $B$  is the damping,  $T_f$  is the nonlinear friction,  $T_d$  is the bounded

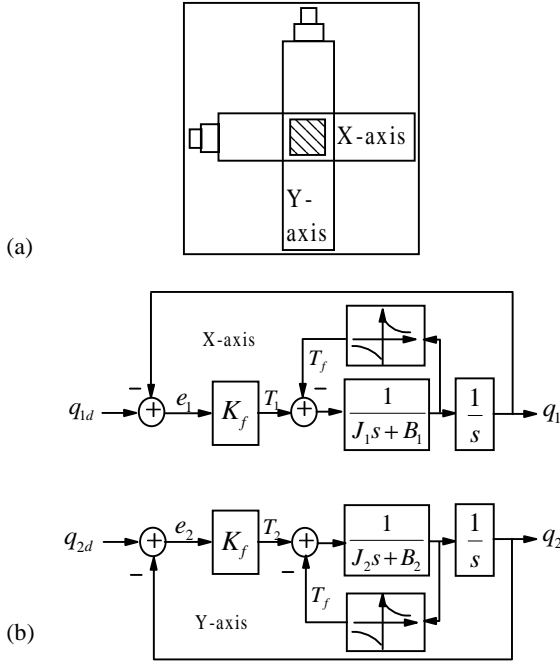


Fig. 1. XY positioning table (a) layout and (b) control structure.

unknown disturbance, and  $T$  is the system input. It is assumed that  $|T_d| < \tau_d$ , with  $\tau_d$ , a known positive constant. Given the desired trajectory  $q_d$ , the tracking error is expressed by  $e = q_d - q$  and the filtered tracking error by

$$r = \dot{e} + \Lambda e \quad (4)$$

with  $\Lambda$  a positive definite design parameter. Then (3) is a stable system so that  $e(t)$  is bounded as long as the controller guarantees that the filtered error  $r(t)$  is bounded.

Differentiating (4) and using (3), the dynamics of the system may be written in terms of the filtered tracking error as

$$J\dot{r} = -Br - T + f(q) + T_d \quad (5)$$

where the nonlinear plant function is defined as:

$$f(x) = J(\ddot{q}_d + \Lambda\dot{e}) + B(\dot{q}_d + \Lambda e) + T_f. \quad (6)$$

Vector  $x$  contains all the time signals needed to compute  $f(x)$ , and may be defined for instance as  $x = [e \ \dot{e} \ \dot{q}_d \ \ddot{q}_d]^T$ . It is noted that the function  $f(x)$  contains all the potentially unknown function, except for  $J$ ,  $B$  appearing in (5)- these latter terms cancel out in the stability proof.

Define a neuro-controller for each-axis as

$$T_j = T_{j1} + T_{j2} = K_{jf} r_j + T_{j2}, \quad j = 1, 2 \quad (7)$$

with the linear controller gain  $K_{jf} > 0$  and  $T_{j2}$ , an estimate of  $f$ , will be provided by some means not yet disclosed. The control structure implied by this scheme is shown in Fig. 2.

Substituting (7) into (5) yields the closed loop error dynamics

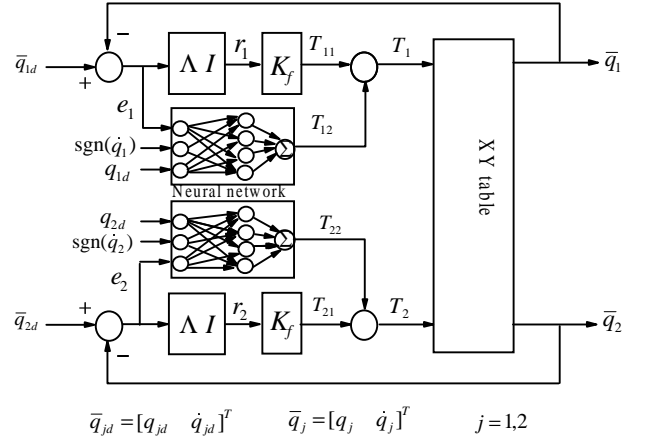


Fig. 2. Proposed neuro-control structure of XY positioning table.

$$J_j \dot{r}_j = -(K_f + B_j)r_j + \tilde{f}_j + T_{jd}, \quad j = 1, 2 \quad (8)$$

where the nonlinear functional estimation error  $\tilde{f}_j$  is given

$$\text{by } \tilde{f}_j = f_j - T_{j2}.$$

Equation (8) is the error system wherein the tracking error is driven by the functional estimation error. In the remainder of the paper we shall use Eq. (8) to focus on selecting an NN training algorithm that the NN approximates the nonlinear plant function  $f_j$ .

A three layer NN in Fig. 3 has a network output given by

$$T_{j2} = \sum_{k=1}^4 [w_{mj}^i \cdot \sigma(\sum_{l=1}^3 v_{lm}^i \cdot x_l^i + v_{0m}^i) + w_{oj}^i], \quad j = 1, 2 \quad (9)$$

with notation  $\sigma(\cdot)$ , the activation function,  $v_{lm}$ , the inter-connection weights from first to second layer,  $w_{mj}$ , the inter-connection weights from second to third layer, The NN equation may be conveniently expressed in a vector format by defining  $\hat{W} = [w_{1,1}, w_{1,2}, \dots, w_{1,4}]^T$ ,

$\sigma(\cdot) = [\sigma_1(\cdot), \sigma_2(\cdot), \dots, \sigma_4(\cdot)]^T$ ,  $\sigma_i(\cdot) = \sigma(\cdot)$ , and a matrix format defining  $\hat{V}^T = [V_{lm}]$  Then,

$$T_{j2} = \hat{W}^T \sigma(\hat{V}^T x) \quad j = 1, 2. \quad (10)$$

A general function,  $f$ , can be modeled by an NN as:

$$f = W^T \sigma(V^T x) + \varepsilon \quad (11)$$

where  $W$  and  $V$  are constant ideal weight of the current weight  $\hat{W}$  and  $\hat{V}$  so that  $\varepsilon$  is bounded by a known constant  $\varepsilon_N$ , and  $\varepsilon$  is reconstruction error due to the NN structure [15, 16]. For practical situations, we assume that the ideal parameters are bounded by known positive values so that  $\|W\| < W_M$ ,  $\|V\| < V_M$  where  $\|\cdot\|$  is a norm. Define the parameter deviation or the parameter estimation error as:

$$\tilde{W} = W - \hat{W}, \quad \tilde{V} = V - \hat{V} \quad (12)$$

and the second layer output error for a given  $x$  as:

$$\tilde{\sigma} = \sigma - \hat{\sigma} = \sigma(V^T x) - \sigma(\hat{V}^T x). \quad (13)$$

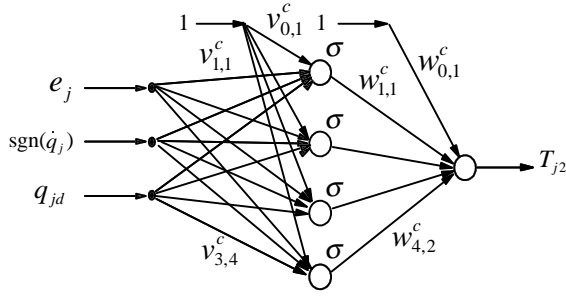


Fig. 3. Structure of NN.

The Taylor series expansion of the second layer output for a given  $x$  may be written as:

$$\sigma(V^T x) = \sigma(\hat{V}^T x) + \dot{\sigma}(\hat{V}^T x) \tilde{V}^T x + O(\tilde{V}^T x) \quad (14)$$

with  $\dot{\sigma}(\hat{z}) \equiv \frac{d\sigma(z)}{dz} \Big|_{z=\hat{z}}$ , and  $O(\cdot)$ , sum of higher order

terms. Denoting  $\dot{\sigma} = \dot{\sigma}(\hat{V}^T x)$ , we have

$$\tilde{\sigma} = \dot{\sigma}(\hat{V}^T x) \tilde{V}^T x + O(\tilde{V}^T x) = \dot{\sigma} \tilde{V}^T x + O(\tilde{V}^T x). \quad (15)$$

Now, define the NN functional estimate of (11) by:

$$T_{j2} = \hat{W}^T \sigma(\hat{V}^T x), \quad j=1,2 \quad (16)$$

with  $\hat{W}$ ,  $\hat{V}$  the current estimated values of the ideal weights  $W$ ,  $V$  as provided by the training algorithm subsequently to be discussed.

For one axis dynamics, select a control input torque using (7) and (16) as:

$$T = K_f r + \hat{W}^T \sigma(\hat{V}^T x). \quad (17)$$

Using (11) and (17), the closed loop error dynamics (8) become:

$$J\dot{r} = -(K_f + B)r + \hat{W}^T \sigma(\hat{V}^T x) - \hat{W}^T \sigma(\hat{V}^T x) + \varepsilon + T_d. \quad (18)$$

Adding and subtracting  $\hat{W}^T \dot{\sigma}$  yields:

$$J\dot{r} = -(K_f + B)r + \tilde{W}^T \dot{\sigma} - \hat{W}^T \dot{\sigma} + \varepsilon + T_d. \quad (19)$$

Adding and subtracting again  $\hat{W}^T \tilde{\sigma}$  yields:

$$J\dot{r} = -(K_f + B)r + \tilde{W}^T \dot{\sigma} + \hat{W}^T \tilde{\sigma} + \tilde{W}^T \tilde{\sigma} + \varepsilon + T_d. \quad (20)$$

Using the Taylor series approximation for  $\tilde{\sigma}$ , the closed loop error system becomes

$$J\dot{r} = -(K_f + B)r + \tilde{W}^T \dot{\sigma} + \hat{W}^T \dot{\sigma} \tilde{V}^T x + \delta + \varepsilon + T_d \quad (21)$$

where the disturbance  $\delta$  is

$$\delta = \tilde{W}^T \dot{\sigma} \tilde{V}^T x + \hat{W}^T O(\tilde{V}^T x). \quad (22)$$

The disturbance term,  $\delta$ , is bounded by a positive constant  $\delta_N$ , i.e.,  $|\delta| < \delta_N$ . It is important to note that the NN reconstruction error  $\varepsilon$ , the plant disturbance  $T_d$ , and the disturbance term  $\delta$  in the Taylor series expansion of  $f$  all have exactly the same influence as disturbance in the error system.

For the NN training algorithm to improve the tracking

performance of the closed loop system it is required to demonstrate that the tracking error,  $r$  is suitably small, a bound on the tracking error is derived by the following theorem.

*Theorem 1:* Let the reference signal be bounded. Take the control input for (3) as (17). Let an NN weights training rule be provided by

$$\dot{\hat{W}} = \hat{\sigma} r^T \quad (23)$$

$$\dot{\hat{V}} = x(\hat{\sigma} \hat{W} r)^T \quad (24)$$

Then, the tracking error  $r$  evolves within a practical bound,

$$|r| \leq \frac{\delta_N + \varepsilon_N + \tau_d}{K_f + B} \quad (25)$$

Proof: Define a Lyapunov function candidate for the error dynamics (8) as

$$L = \frac{1}{2} J r^2 + \frac{1}{2} \tilde{W}^T \tilde{W} + \frac{1}{2} \text{tr}(\tilde{V}^T \tilde{V}). \quad (26)$$

where  $\text{tr}(\cdot)$  is trace. Differentiating Eq. (26) yields

$$\dot{L} = J r \dot{r} + \frac{1}{2} J \dot{r}^2 + \tilde{W}^T \dot{\tilde{W}} + \tilde{V}^T \dot{\tilde{V}} \quad (27)$$

whence substitution of Eq. (21) yields

$$\begin{aligned} \dot{L} = & -(k_f + B)r^2 + \frac{1}{2} J \dot{r}^2 + \tilde{W}^T (\dot{\tilde{W}} + \dot{\sigma} r) \\ & + \text{tr}\{\tilde{V}^T (\dot{\tilde{V}} + x r \hat{W}^T \dot{\sigma})\} + r(\delta + \varepsilon + T_d) \end{aligned} \quad (28)$$

Since  $\dot{\tilde{W}} = -\dot{\hat{W}}$  with  $W$  constant (and similarly for  $\dot{\tilde{V}}$ ), the tuning rules (23)-(24) and the assumption  $|\dot{J}| = 0$  give

$$\dot{L} = -(K_f + B)r^2 + r(\delta + \varepsilon + T_d) \quad (29)$$

and

$$\begin{aligned} \dot{L} \leq & -(k_f + B)r^2 + |r|(\delta_N + \varepsilon_N + \tau_d) \\ = & -|r|[(K_f + B)|r| - (\delta_N + \varepsilon_N + \tau_d)] \end{aligned} \quad (30)$$

Thus  $\dot{L}$  is negative as long as the term in the brace is positive, which implies

$$|r| > \frac{\delta_N + \varepsilon_N + \tau_d}{K_f + B}. \quad (31)$$

According to the Lyapunov theorem, the tracking error decrease as long long as the tracking error is bigger than the right hand side of Eq. (31). This implies Eq. (32) gives a practical bound on the tracking error

$$|r| \leq \frac{\delta_N + \varepsilon_N + \tau_d}{K_f + B}. \quad (32)$$

The NN reconstruction error  $\varepsilon$ , the bounded disturbance  $T_d$ , and the higher order Taylor series terms  $\delta$  increase the bound on  $|r|$ . However, a small tracking error bound may be achieved by reducing  $\delta_N$  by tuning the NN and by decreasing the reconstruction error  $\varepsilon$  by properly selecting the structure of the NN. Note that since the linear controller gain  $K_f$  is determined according to the design of linear controller,  $K_f$ , cannot be increased arbitrarily. However, large  $K_f$  may decrease the tracking error bound as long as the linear controller maintains the stability of control system.

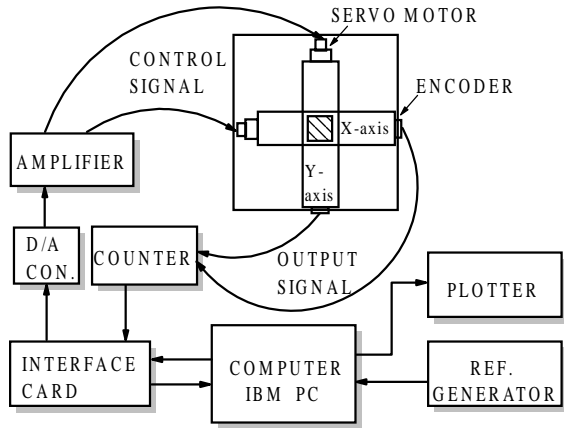


Fig. 4. Experimental setup.

### 4. SIMULATION AND EXPERIMENTAL RESULTS

In this section, we illustrate the effectiveness of a proposed neuro-controller by computer simulations and experiments on a XY positioning table[17]. The experimental setup is shown in Fig. 4. The Y-axis of the XY table was placed on the X-axis. The actuators of the XY table were two dc servo motors. Each motor was controlled independently by the same servo controller. Ball screws were connected to the motors and let the table move, and an IBM PC was connected to the XY table through the A/D and D/A converters. The main control algorithm is implemented at a 100 Hz sampling rate via an IBM PC with an 486DX-66 micro processor. The proposed algorithm is written in C language.

The parameters of the XY table are estimated as

$$J_1 = 0.0143 [Kg \cdot m^2] \text{ and } B_1 = 0.945 [N \cdot m],$$

$$J_2 = 0.0135 [Kg \cdot m^2] \text{ and } B_2 = 0.927 [N \cdot m] \quad (33)$$

for the X- and Y-axis, respectively. The allowable moving area of the XY table is 25[cm].

We simulate the XY positioning table with nonlinear friction nonlinearity using PD controller. The desired trajectory is selected by

$$q_{1d}(t) = \begin{cases} 2 & 0 < t < 1 \\ 2 \cos[\frac{\pi(t-1)}{2}] & 1 < t < 5 \\ 2 & 5 < t < 6 \end{cases}$$

$$q_{2d}(t) = \begin{cases} 2 & 0 < t < 1 \\ 2 \sin[\frac{\pi(t-1)}{2}] & 1 < t < 5 \\ 2 & 5 < t < 6 \end{cases} \quad (34)$$

The friction set at  $\alpha_0 = 0.06$ ,  $\alpha_1 = 0.01$ , and  $\alpha_2 = 3.0$  for X-axis, and  $\alpha_0 = 0.062$ ,  $\alpha_1 = 0.011$ , and  $\alpha_2 = 3.0$  for Y-axis. The gains of PD controllers are chosen as  $K_f = 3.0$  and  $\Lambda = 0.4$ . As shown in Fig. 5(a), the performance is degraded by friction nonlinearities(locus i). Therefore the neuro-controller was trained as described in

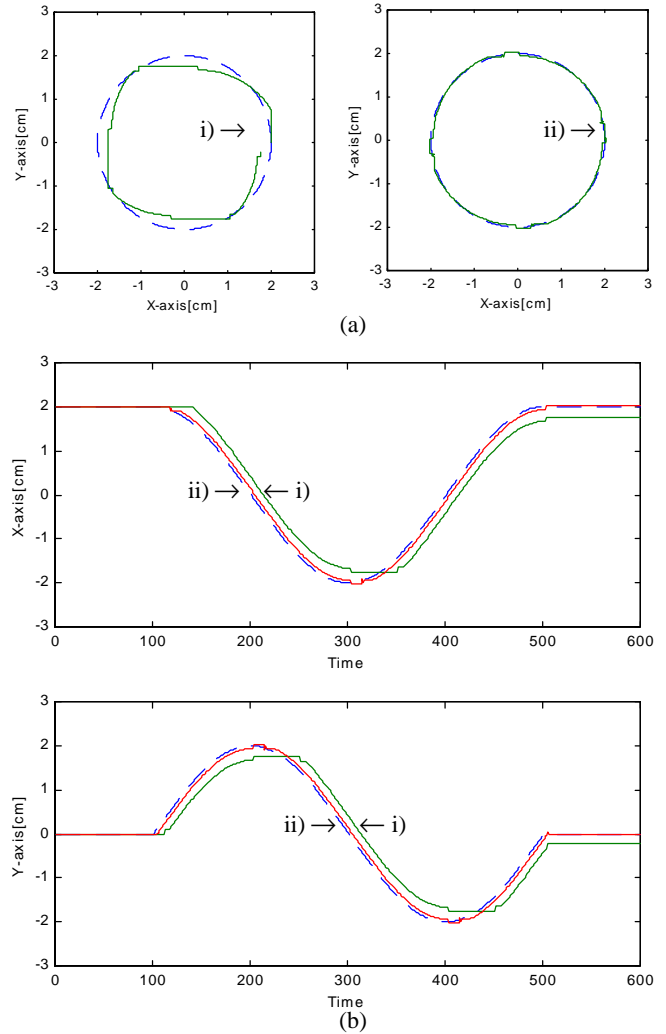


Fig. 5. Simulation results for the circle (a) locus and (b) trajectory. (dotted line: desired, i) without compensation, ii) with compensation)

Section 3 in order to compensate for the effects of the nonlinearities. The input vector  $x$  can be taken as  $x_j = [e_j(k), \text{sgn}(\dot{q}_j(k)), q_{jd}(k)]^T$ . The sgnum function  $\text{sgn}(\cdot)$  is needed for Coulomb friction terms. The simulation result of the XY table with the neuro-controller is also shown in Figure(locus ii). The trajectories of each axis are included in Fig. 5(b). Experimental results are shown in Fig. 6, which show similar phenomena to those found in simulation. The proposed method exhibits an improvement in its locus and trajectory response compared with the PD controller.

We also applied it to modified circle and described as

$$q_{1d}(t) = \begin{cases} 2.4 & 0 < t < 1 \\ 2 \cos[\frac{\pi(t-1)}{2}] + \frac{2}{5} \cos[\frac{5\pi(t-1)}{2}] & 1 < t < 5 \\ 2.4 & 5 < t < 6 \end{cases}$$

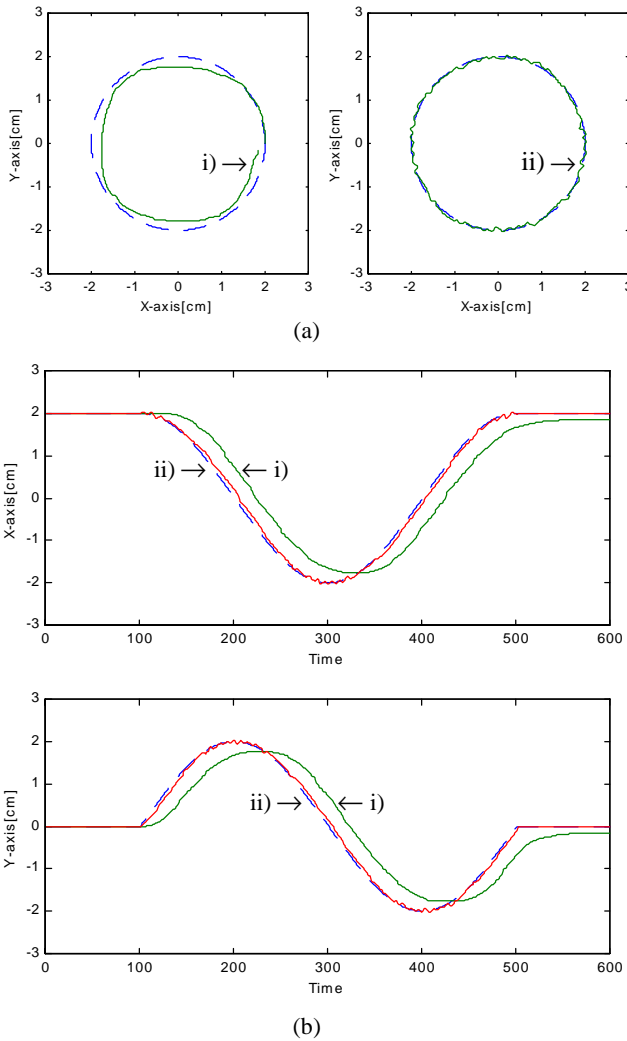


Fig. 6. Experimental results for the circle (a) locus and (b) trajectory. (dotted line: desired, i) without compensation, ii) with compensation)

$$q_{2d}(t) = \begin{cases} 0 & 0 < t < 1 \\ 2 \sin\left[\frac{\pi(t-1)}{2}\right] + \frac{2}{5} \sin\left[\frac{5\pi(t-1)}{2}\right] & 1 < t < 5 \\ 0 & 5 < t < 6 \end{cases} \quad (35)$$

Experimental results are given in Fig. 7. The locus of the PD controller without compensation is different from the desired locus because of the friction effects. However, the locus of the proposed method is almost same as the desired locus, which means the proposed method compensates for friction effects. The result of the proposed method is better than that of the PD controller without compensation from the point of view of closing the desired locus.

### 5. CONCLUSIONS

The neuro-control scheme has been proposed for a XY positioning table. The neuro-controller includes a PD tracking

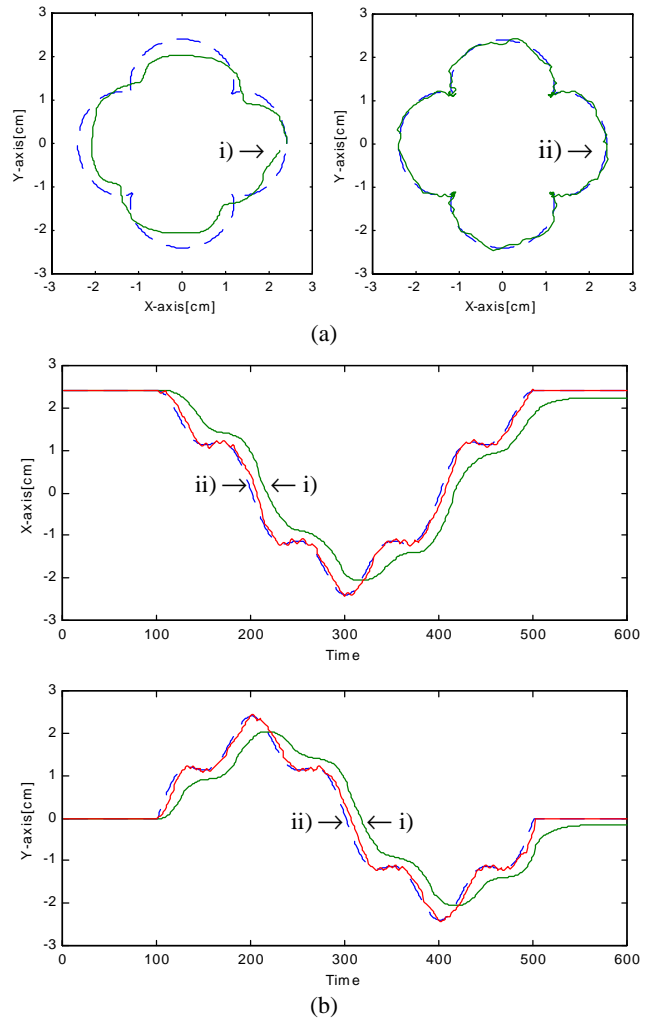


Fig. 7. Experimental results for the modified circle (a) locus and (b) trajectory. (dotted line: desired, i) without compensation, ii) with compensation)

loop for stabilization of fast dynamics and an NN loop used to compensate for the effects of the system nonlinearities. Using nonlinear stability techniques, the bound on tracking error is derived from the tracking error dynamics. Implementation on an actual XY positioning table shows the efficacy of the proposed technique.

### ACKNOWLEDGEMENTS

This paper was supported by grant No. R05-2003-000-11047 -0 from Korea Science & Engineering Foundation.

### REFERENCES

- [1] J. O. Jang and P. G. Lee, "Neuro-fuzzy control for DC motor friction compensation," in *Proc. IEEE Conf. Decision Contr.*, Sydney, Australia, Dec. 2000, pp. 3550-3555.
- [2] J. O. Jang, P. G. Lee, S. B. Park, and I. S. Ahn, "Backlash compensation of systems using fuzzy logic," in *Proc. 2001 Amer. Contr. Conf.*, Arlington, VA, 24-27

- June, 2001, pp.4788-4789.
- [3] J. O. Jang, "A deadzone compensator of a DC motor system using fuzzy logic control," *IEEE Trans. Syst., Man, Cybernet. C*, vol. 31, no. 1, pp. 42-48, 2001.
- [4] H. S. Lee and M. Tomizuka, "Robust motion controller design for high-accuracy positioning systems," *IEEE Trans. Ind. Electron.*, vol. 43, no. 1, pp. 48-55, 1996.
- [5] K. Srinivasan and P. K. Kulkarni, "Cross coupled control of biaxial feed drive servomechanisms," *ASME J. Dynamic Syst. Measur. Contr.*, vol. 112, pp. 225-232, 1990.
- [6] W. Li and X. Cheng, "Adaptive high-precision control of positioning tables-Theory and experiment," *IEEE Trans. Contr. Syst. Technol.*, vol. 2, pp. 265-270, 1994.
- [7] K. S. Narendra, and K. Parthasarathy, "Identification and control of dynamical systems using neural networks," *IEEE Trans. Neural Networks*, vol. 1, no. 1, pp. 4-27, 1990.
- [8] G. Lightbody and G. W. Irwin, "Direct neural model reference adaptive control," *IEE Proc.-Control Theory and Applications*, vol. 142, pp. 31-43, 1995.
- [9] M. Kawato, Y. Uno, M. Isobe, and R. Suzuki, "A hierarchical model for voluntary movement and its application to robotics," *IEEE Contr. Syst. Mag.*, vol. 8, no. 4, pp. 8-15, 1988.
- [10] F. L. Lewis, A. Yesildirek, and K. Liu, "Multilayer neural net robot controller with guaranteed tracking performance," *IEEE Trans. Neural networks*, vol. 7, no. 2, pp. 388-399, March 1996.
- [11] X. Cui and K. G. Shin, "Direct control and coordination using neural networks," *IEEE Trans. Systems, Man, and Cybernetics*, vol. 23, pp. 686-697, 1993.
- [12] S. Goto, M. Nakamura, and N. Kyura, "Accurate contour control of mechatronic servo systems using Gaussian networks," *IEEE Trans. Ind. Electron.*, vol. 43, no. 4, pp. 469-476, 1996.
- [13] C. Canudas, P. Noel, A. Aubin, and B. Brogliato, "Adaptive friction compensation in robot manipulators: Low velocities," *Int. J. Robot Res.* vol. 10, no. 3, pp. 189-199, 1991.
- [14] B. Friedland and Y. Park, "On adaptive friction compensation," *IEEE Trans. Automat. Contr.*, vol. 37, no. 10, pp. 1069-1612, 1992.
- [15] J. O. Jang and G. J. Jeon, "A parallel neuro-controller for DC motors containing nonlinear friction," *Neuro-computing*, vol. 30, no. 1-4, pp. 233-248, 2000.
- [16] D. E. Rumelhart, G. E. Hinton, and G. E. Williams, "Learning internal representations by error propagation", In Rumelhart D. E, & McClelland J.L., *Parallel Distributed Processing*, vol. 1 pp.318-362. Cambridge, MA, USA: MIT Press, 1986.
- [17] J. O. Jang "Deadzone compensation of a XY positioning table using fuzzy logic ," Submitted to IEEE Trans. IE, 2003