Shape and location estimation using prior information obtained from the modified Newton-Raphson method

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Abstract: In most boundary estimation algorithms estimation in EIT (Electrical Impedance Tomography), anomaly boundaries can be expressed with Fourier series and the unknown coefficients are estimated with proper inverse algorithms. Furthermore, the number of anomalies is assumed to be available *a priori*. The prior knowledge on the number of anomalies may be unavailable in some cases, and we need to determine the number of anomalies with other methods. This paper presents an algorithm for the boundary estimation in EIT (Electrical Impedance Tomography) using the prior information from the conventional Newton-Raphson method. Although Newton-Raphson method generates so poor spatial resolution that the anomaly boundaries are hardly reconstructed, even after a few iterations it can give general feature of the object to be imaged such as the number of anomalies, their sizes and locations, as long as the anomalies are big enough. Some numerical experiments indicate that the Newton-Raphson method can be used as a good predictor of the unknown boundaries and the proposed boundary discrimination algorithm has a good performance.

Keywords: Electrical impedance tomography, boundary estimation, Prior information, Newton-Raphson method, Fourier series

1. INTRODUCTION

In electrical impedance tomography (EIT), the distribution of impedances inside an object ('image') is sought by using the relationship between the specified currents and the induced potentials on the object boundary. Usually, the boundary is partly covered by electrodes and the electrical signals are imposed and measured at the electrodes. Hence, the EIT problem that determines the impedance distribution using the over-determined data on the boundary is so-called inverse problem [1-2].

In general, the problem domain is discretized into small pixels, in each of which the impedance value is assumed to be constant. The unknowns to be determined by the inverse solver are the impedance values of pixels. Often, the EIT inverse problem is considered as an optimization problem to find the impedance distribution minimizing the difference between the measured voltages and the calculated ones with the assumed impedance distribution. As the search step to update the assumed impedance distribution, many methods have been proposed. For the stationary case where the impedance distribution does not change during the time taken to measure a full set of current-voltage data, the modified Newton-Raphson method (mNR) has been commonly used due to its good stability against the measurement error [1-2]. On the other hand, for the non-stationary cases where the impedance distribution is changed before acquiring a full set of current-voltage data, Kalman filter and its variations have been used and in this the inverse problem is thought of as a state estimation problem [3].

The approximation that the impedance is discretely distributed over the domain causes poor spatial resolution due to the diffusive characteristics of the problem. In some cases where the anomaly and the background have different but constant impedance values, the internal image can be reconstructed by the interfaces only; therefore some researchers have been interested in the direct estimation of the anomaly boundary [4-6]. Each of the boundaries is expressed as Fourier series independently and the inverse problem to determine the Fourier coefficients is formulated. So, the number of anomalies should be known *a priori*. In general, however, the number of anomalies may be unknown and we need to predict or determine it with other methods. In this paper, we propose a use of mNR for the determination of the number of anomalies and the initial guess for their sizes and locations. Even though the mNR has a critical drawback in the spatial resolution, the robustness to the measurement noise and the fast convergence to a certain extent are enough for the initial guess. Experiences show that even after 1~2 iterations it can give general feature of the object to be imaged such as the number of anomalies, their sizes and locations, as long as the anomalies are big enough.

2. MATHEMATICAL MODEL

2.1 Definition of region of object

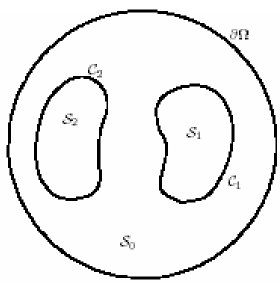


Fig.1. Example of a piecewise constant domain.

Let $\Omega \subset \Re^2$ denote a bounded domain and assume that Ω is divided into $m+1$ disjoint regions S_k which are bounded by smooth closed boundary curves and have constant resistivity values $\{\sigma_k, k = 0,1,\cdots,m\}$.

$$\Omega = \bigcup_{k=0}^{m} S_k, \qquad k = 0,\cdots,m \tag{1}$$

Let $\{C_\ell\} \subset \Omega$, $\ell = 1,2,\cdots,m$ denote the smooth outer boundary of region S_k. The outer boundary of the background region S_0 is $\partial\Omega$. Figure 1 shows the topology of the problem domain.

2.2 Forward Solver

The forward problem is to compute the electrical potential when the injected current and the resistivity distribution are given. When electrical currents $I_l (l = 1,2,\cdots,L)$ are injected into the object $\Omega \in \Re^2$ through the electrodes $e_l (l = 1,2,\cdots,L)$ attached on the boundary $\partial\Omega$ and the resistivity distribution $\rho(x,y)$ is known over the Ω, the potential distribution $u(x,y)$ can be determined uniquely from following partial differential equation, which can be derived from the Maxwell equations:

$$\nabla \cdot \left(\frac{1}{\rho} \nabla u \right) = 0, \qquad \text{in } \Omega \tag{2}$$

subject to the boundary conditions

$$\int_{e_l} \frac{1}{\rho} \frac{\partial u}{\partial n} dS = I_l, \qquad l = 1,2,\cdots,L \tag{3}$$

$$\left. \frac{1}{\rho} \frac{\partial u}{\partial n} \right|_{\partial\Omega \setminus \bigcup_{i=1}^{L} e_l} = 0 \tag{4}$$

where n is outward unit normal. From the complete electrode model taking into account the shunting effect (i.e. the voltage at the l-th electrode, U_l, is constant over the e_l) and the additional voltage drop due to the contact impedance between the l-th electrode and the object, z_l, we have

$$\left. \left(u + z_l \frac{1}{\rho} \frac{\partial u}{\partial n} \right) \right|_{e_l} = U_l, \quad l = 1,2,\cdots,L \tag{5}$$

In addition, the follow two conditions for the injected currents and measured voltages are needed to ensure the existence and uniqueness of the solution:

$$\sum_{l=1}^{L} I_l = 0, \quad \sum_{l=1}^{L} U_l = 0 \tag{6}$$

In general, since the forward problem cannot be solved analytically, we have to resort to the numerical solution. In this paper, we used the FEM to obtain the numerical solution. In the FEM, the object area is discretized into sufficiently small elements having a node at each corner and it is assumed that the resistivity distribution is constant within each element. The potential at each node and the "referenced" electrode voltages, defined by the vector $v \in \Re^{(M+L-1)\times 1}$, are calculated by discretizing Eq. (2) into $Yv = c$, where $Y \in \Re^{(M+L-1)\times(M+L-1)}$ is the admittance matrix and M is the number of FEM nodes. Y and c are the functions of the resistivity distribution inside the object and the injected currents through the electrodes, respectively. For more details on the forward solution and the FEM approach, see [7].

2.3 Prior information from mNR

In most previous researches for the boundary estimation in EIT, the boundary of each anomaly is expressed with Fourier series and the Fourier coefficients are determined with proper inverse solvers. Usually, the number of anomalies is assumed to be a known constant and the number of unknowns, namely the number of total Fourier coefficients to describe all the boundaries, is fixed during the solution. In some cases, the number of anomalies may be unknown and we have to determine it with other methods.

The conventional mNR shows poor spatial resolution in the reconstructed images and this drawback is why the direct boundary estimation has been introduced. However, the mNR is a robust algorithm as an inverse solver to EIT and after a few iterations, even 1 or 2, it can give valuable information on the internal impedance distribution like the number of anomalies and their approximate locations and sizes as long as they are big enough. Paying attention to such ability of the mNR, we consider a use of mNR as a predictor to generate prior information necessary for the boundary estimation.

In this section, we propose an algorithm to extract prior information on the number of anomalies and their approximate locations and sizes from the images by the mNR.

An image reconstruction in EIT is an inverse problem in which the impedance distribution of the interior is estimated based on the relationship between the applied currents and the measured voltages. In general, the inverse problem is formulated as an optimization problem to find the resistivity distribution minimizing the difference between the measured voltages and the calculated ones with the assumed resistivity distrubution. Due to the ill-posedness of the EIT inverse problem, a regularization should be introduced in the cost functional to be minimized:

$$\Phi(\rho_i) = \frac{1}{2} \left(\| U - V(\rho_i) \| + \alpha \| R_I \rho_i \| \right) \tag{7}$$

where the vectors V and $U \in \Re^{LP}$ are the computed and measured voltages, respectively and P is the number of injected current patterns. $R_I \in \Re^{N\times N}$ and α are regularization matrix and parameter, respectively. N denotes the number of elements in the FEM meshes. In this work, we use a deterministic approach called generalized Tikhonov regularization.

The most popular solver for the nonlinear minimization problem is the modified Newton Raphson method (mNR) and the search step is updated iteratively as follows:

$$\rho_{i+1} = \rho_i + \nabla\rho_i, \quad (8)$$

where

$$\nabla\rho_i = \{J(\rho_i)^T J(\rho_i) + \alpha R_I\}^{-1}\{J(\rho_i)^T (U - V(\rho_i)) + \alpha R_I \rho_i\}.$$

In the above, $J(\rho) \in \Re^{LP\times N}$ denotes the Jacobian of the mapping $V(\rho)|_{\rho=\rho_i}$ with respect to ρ and i is the iteration index.

As mentioned above, the reconstructed images by the mNR have very low spatial resolution, so that we should develop an algorithm to extract prior information from the blurred images. In this paper, we propose a simple classification criterion to discriminate anomalies from the background. Let's define a parameter of

$$\mu_j \equiv \frac{\rho_i(j)}{\rho_o}, \quad j = 1,2,\cdots,N \quad (9)$$

where ρ_o is an average resistivity over the object. With μ_j, all elements are classified into two groups: for resistive anomalies, if μ_j is larger than μ_{TH}, then the element belongs to TG (Target Group), otherwise the element belongs to BG (Background Group). For conductive anomalies, the classification will be reversed. The parameter, μ_{TH}, is a threshold to discriminate the anomalies.

From the number of the elements belonging to TG and their resistivity values, we can obtain the prior information as follows:

1. Determination of the number of the anomalies

- If two elements belonging to TG share their element boundary, they are classified into the same S_k.
- An element whose boundary is shared with the element belonging to S_k is a member of S_k.

2. Determination of anomaly locations and sizes

- The center location of *k*-th anomaly is approximated as the arithmetic average of the node points belonging to S_k.
- The shape of anomaly is initially approximated as a circle and the size is expressed in terms of its radius. The radius of *k*-th anomaly is approximated from the total area of the elements belonging to S_k.

2.4 Boundary estimation problem

From the prior information obtained from the mNR, now, we can proceed with the boundary estimation. The anomaly boundary is expressed in terms of Fourier series [5]:

$$C_\ell(s) = \begin{pmatrix} x_\ell \\ y_\ell \end{pmatrix} = \sum_{n=1}^{N_\theta} \begin{pmatrix} \gamma_n^{x_\ell}\theta_n^x(s) \\ \gamma_n^{y_\ell}\theta_n^y(s) \end{pmatrix}, \quad \ell = 1,2,\cdots,m, \quad (1)$$

where $C_\ell(s)$, $\ell = 1,\cdots,m$ is the boundary of the ℓ-th anomaly, m is the number of anomalies identified from mNR, and $\theta_n(s)$ are periodic and smooth basis functions:

$$\theta_1^\beta(s) = 1$$
$$\theta_n^\beta(s) = \sin\left(2\pi\frac{n}{2}s\right), \quad n = 2,4,6,\cdots,N_\theta - 1 \quad (11)$$
$$\theta_n^\beta(s) = \cos\left(2\pi\frac{(n-1)}{2}s\right), \quad n = 3,5,7,\cdots,N_\theta$$

where $s \in [0\ 1]$, β denotes either x or y. And N_θ is the order of Fourier series.

The unknowns to be determined are the Fourier coefficients:

$$\gamma = (\gamma_1^{x_1},\cdots,\gamma_{N_\theta}^{x_1},\gamma_1^{y_1},\cdots,\gamma_{N_\theta}^{y_1},\gamma_1^{x_m},\cdots,\gamma_{N_\theta}^{x_m},\gamma_1^{y_m},\cdots,\gamma_{N_\theta}^{y_m})^T \quad (12)$$

Thus, $\gamma \in \Re^{2mN_\theta}$.

The mapping $F: \gamma \mapsto V(\gamma)$ to map Fourier coefficients to the voltages measured at the electrodes is nonlinear, so for the solution of γ we linearize about a reference point (ρ_*, γ_*):

$$V = V_* + J_F(\gamma - \gamma_*), \quad (13)$$

where V_* are the calculated potentials that correspond to (ρ_*, γ_*) and J_F is the Jacobian matrix, $J_{F,ij} = \partial V_i / \partial\gamma_j$, which is calculated by the perturbation method. For a given γ_*, there may be finite element meshes crossing anomaly boundaries and for such elements the finite element integration should be performed separately for the anomaly and for the background. The columns $J_F^{r_m}$ of J_F are then obtained by perturbing each of the coefficients γ^{r_m} by predetermined $\delta\gamma^{r_m}$ and calculating the resulting voltages on the boundary.

$$J(\gamma)^{r_m} = (\delta\gamma^{r_m})^{-1}\delta V, \quad (14)$$

where $\delta V = V - V_*$ and $r_m = 1,2,\cdots,2mN_\theta$. The cost function to determine γ can be constructed much like Eq. (7):

$$\Phi(\gamma_k) = \frac{1}{2}\left(\| U - V(\gamma_k) \| + \alpha \| R_I \gamma_k \|\right) \quad (15)$$

In this, we introduce the regularization again to stabilize the solution against ill-posedness, and we can find the Fourier coefficients by minimizing the cost function.

Of course, we start with the prior information obtained from the mNR as an initial guess. The above nonlinear optimization problem is solved using the iterative Newton-Raphson method as follows:

$$\gamma_{k+1} = \gamma_k + \Delta\gamma_k, \quad (16)$$

where,

$$\Delta\gamma_k = \{J(\gamma)^T J(\gamma) + \alpha R_I\}^{-1}\{J(\gamma)^T (U - V(\gamma)) + \alpha R_I \gamma_k\}.$$

After updating the Fourier coefficients, for the next iteration, we should select meshes crossing the updated boundary, and then divide each mesh into two regions to conduct the finite element integration separately.

3. NUMERICAL SIMULATIONS

To show the effectiveness of our approach, we carried out numerical experiments with synthetic data. We consider a 16 channel EIT system. The resistivity contrast of the anomaly to the background is set to 2. The domain is discretized into 1968 triangular meshes with 1049 nodes. The threshold to discriminate anomalies from the background is set to $\mu_{TH} = 1.4$. Fourier modes larger than 3 are truncated, namely $N_\theta = 3$. As for the regularization, we use

$$R_I = I \in R^{2mN_\theta \times 2mN_\theta}, \quad \alpha = 0.0005 \qquad (17)$$

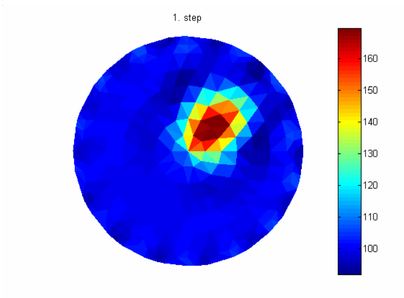
and we define the RMS (root-mean-squared) error as following to evaluate the quality of the solution.

$$e_{rms} = \sqrt{\left(\frac{(U - V(\gamma))^T (U - V(\gamma))}{U^T U} \right)} \qquad (18)$$

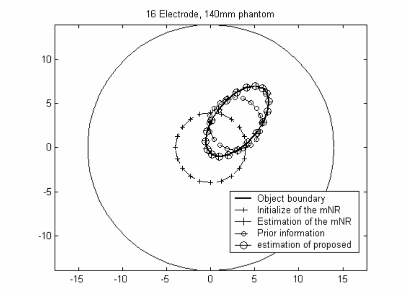
We have prior information on the locations and sizes anomalies as well as the number of anomalies. For the investigation of the effect of the size information on the convergence, we conducted two kinds of calculations for the same configuration with and without size information.

At first, we consider a single anomaly whose boundary is $\gamma = [3\ 3\ 2\ \ 3\ 0\ 4]$. After only one iteration, as can be seen in Fig. 2(a), the impedance image obtained by the mNR shows there is one anomaly and informs us of its approximate location and size. From the resistivity distribution by the mNR, we estimate a good initial guess for the boundary estimation as shown in Fig. 2(b) and the results agree with the target very well. Regardless of the use of prior information on size, both results quite satisfactory as illustrated in Fig. 2(b). However, Fig. 2(c) indicates that the prior information on size will accelerate the convergence.

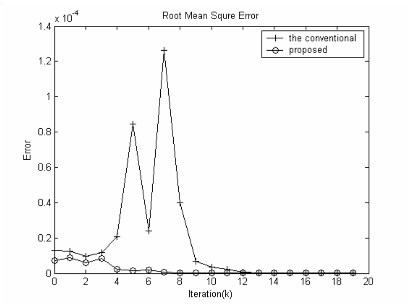
Secondly, the boundary estimation for two anomalies is attempted. The boundaries are $\gamma = [-2\,4\,1\ -2\,0\,3\ \ 6\,3\,0\ 6\,0\,2]$. The results are given in Fig. 3, from which we can observe the same trend as the case of single anomaly.



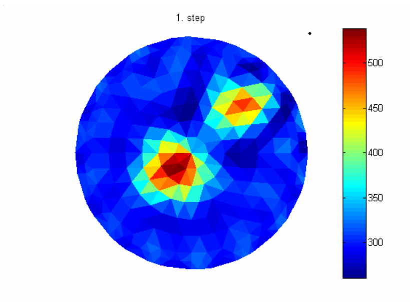
(a) Prior information from mNR



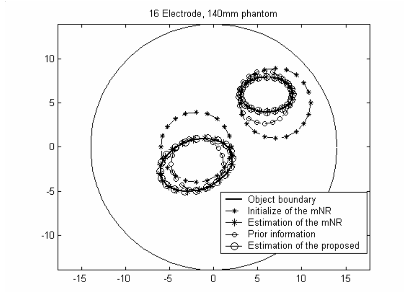
(b) Boundary estimation.



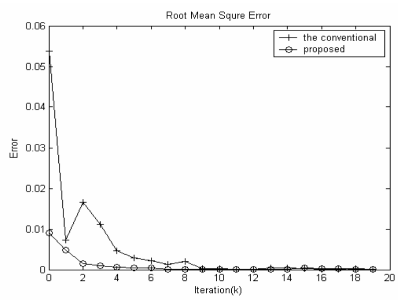
(c) RMS error
Fig. 2. Single anomaly.



(a) Prior information from mNR



(b) Boundary estimation



(c) RMS error
Fig.3. the result of the two object

4. CONCLUSIONS

The conventional boundary estimation problem in EIT assumes that we know the number of anomalies. But, the assumption may not be available. As a predictor for the prior information for the boundary estimation, we propose a use of the mNR since it can give some valuable information on the number of anomalies and their approximate locations and size even after a few iterations, although it suffers from poor spatial resolution that is why the mNR cannot be used for the boundary estimation. With a tentative result by the mNR, we developed an algorithm to discriminate anomalies from the background and to estimate their locations and sizes. Numerical experiments were conducted to verify the performance of the proposed algorithm and the results show its effectiveness especially in the convergence rate. Since for the prior information from the mNR only 1 or 2 iterations are enough, the extra cost engaged in generating the prior information is small and it can be compensated by fast convergence.

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