An Extended Robust H_∞ Filter for Nonlinear Constrained Uncertain System

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Abstract: In this paper, a robust filter is proposed to effectively estimate the system states in the case where system model uncertainties as well as disturbances are present. The proposed robust filter is constructed based on the linear approximation methods for a general nonlinear uncertain system with an integral quadratic constraint. We also derive the important characteristic of the proposed filter, a modified H_∞ performance index. Analysis results show that the proposed filter has robustness against disturbances, such as process and measurement noises, and against parameter uncertainties. Simulation results show that the proposed filter effectively improves the performance.

Keywords: extended robust H_∞ filter, nonlinear estimation, robust estimation, nonlinear uncertain system

1. INTRODUCTION

During the last four decades, the Kalman filter and the extended Kalman filter (EKF) have been widely used in the estimation problems. They require not only a precise system model, but also the statistical property of noise to achieve accurate performance. However, model uncertainty and incomplete statistical information often occur in real applications and make it difficult to precisely estimate the system states, potentially leading to very large estimation errors. These difficulties can be overcome by studying a robust filter [1,2,3].

Recently, a robust filter has received considerable attention. It has robustness against 1) statistical incompleteness of system noise, such as process noise and measurement noise, 2) system model uncertainty, and 3) sensitivity caused by parameter variation of the system model. They can be categorized as H_2 filters, H_∞ filters, and mixed H_2 / H_∞ filters [4,5,6]. The H_∞ filter minimizes the H_∞ norm of the transfer operator between the noise and the estimation error. Thus, the H_∞ filter is usually employed when the energy of the system noise is bounded and the statistical properties of the noise are unknown. This filter minimizes the highest energy gain of the estimation error for all initial conditions and noises. In particular, a robust H_∞ filter, a robust filter with a modified H_∞ performance, can be established for a system with model uncertainty as well as unknown statistical noise properties [3].

For a nonlinear system, a second order nonlinear filter and an extended Kalman filter have been widely utilized. Because the second order nonlinear filter considers higher order terms in the computation of its covariance, it is suitable for a highly nonlinear system. However, it has large computational complexity. Thus, the extended Kalman filter has been widely used for real system applications. Since the extended Kalman filter uses a linearized model of a nonlinear system with an abbreviation of higher order terms, excessive estimation errors occur when it is applied to a highly nonlinear system. In addition, the extended Kalman filter requires statistical information about noise, such as white Gaussian noise, which can hardly be obtained in real applications. Therefore, several studies have been conducted on the nonlinear robust filter. The H_∞ nonlinear filter with Hamilton-Jacobi inequality is the result of one such study, but its computation procedures for obtaining a filter are complicated and it is very difficult to use in real applications. To simplify complicated computation procedures, an approximation solution to the robust filtering problem has recently been developed based on a linearization method. The robust filter derived based on this approach is called the extended robust filter or extended H_∞ filter [3,7]. In [8], the nonlinear state estimation with similar characteristics is especially proposed for a nonlinear uncertain system with uncertainties described by an integral quadratic constraint.

In this paper, a new robust filter for nonlinear uncertain systems with an integral quadratic constraint is presented. The derivation is similar to that of [8], but the main contribution of this paper is a design of extended robust H_∞ filter and a derivation of the modified H_∞ performance index. The robust filter is constructed with local linearization of the system at the reference point. This approach extends the extended Kalman filter to a robust filter. By introducing a state estimation set that is the solution of Hamilton-Jacobi-Bellman partial differential equation and by solving locally the filtering problem, the robust filter is derived. Then the modified H_∞ performance index of the filter is derived and analyzed. The proposed filter is applied to an estimation problem by simulation

2. NONLINEAR ROBUST H_∞ FILTER

2.1 Problem formulation

Consider a nonlinear uncertain system described by

$$\dot{x}(t) = f(x(t)) + B_1(t)\Delta_1(t)N(x(t)) + B_2(t)w_0(t) \quad (1)$$

$$y(t) = h(x(t)) + \Delta_2(t)N(x(t)) + v_0(t) \quad (2)$$

where $B_1(t)\Delta_1(t)N(x(t))$ and $\Delta_2(t)N(x(t))$ represent the system uncertainties. $B_1(t)$ and $N(x(t))$ are known matrices. $\Delta_1(t)$ and $\Delta_2(t)$ are unknown matrices satisfying the condition

$$\left\| \begin{matrix} Q_1^{-1/2}\Delta_1(t) \\ R_1^{-1/2}\Delta_2(t) \end{matrix} \right\| \le 1$$

where Q_1 and R_1 are bounded positive definite matrices. $w_0(t)$ is the process noise and $v_0(t)$ is the measurement noise. They belong to the set of L_2 noises and the statistical properties are unknown.

Converting the uncertainties to the fictitious L_2 noises and introducing a freedom parameter, the uncertain system (1) and (2) can be transformed into an auxiliary system,

$$\dot{x}(t) = f(x(t)) + B(t)w(t) \quad (3)$$

$$y(t) = h(x(t)) + v(t) \quad (4)$$

where $B(t) = [\varepsilon B_1(t) \quad B_2(t)]$, $n(t) = \varepsilon^{-1}N(x(t))$, $w(t) = \begin{bmatrix} \Delta_1(t)n(x(t)) \\ w_0(t) \end{bmatrix}$, $v(t) = [I \quad I]\begin{bmatrix} \varepsilon\Delta_2(t)n(x(t)) \\ v_0(t) \end{bmatrix}$, and ε is a freedom parameter. With the combination of the state variables and $n(t)$, the filter output $z(x(t))$ is of the form

$$z(x(t)) = [(L(t)x(t))^T \quad (\gamma n(t))^T]^T \quad (5)$$

where γ is a given positive real value that indirectly indicates the level of noise attenuation in this robust filter design.

For the nonlinear uncertain system, suppose that the following constraint is satisfied.

$$\Phi(x(0)) + \int_0^T L_1(w,v)dt \le d + \int_0^T L_2(n,z)dt \quad (6)$$

where $0 \le t \le T$ and d is an assigned positive real number. To construct a robust filter, it is assumed that the system (3), (4), and (5) satisfy assumptions 1-6.

Assumption 1: Every function shown in (3)-(5) belongs to C^1 and the first derivative is bounded.
Assumption 2: The matrix $N(x(t))$ is bounded.
Assumption 3: The functions Φ, L_1, and L_2 belong to C^1 and are bounded nonnegative functions. They also satisfy

$$|\phi(x_2) - \phi(x_1)| \le \theta(1 + |x_2| + |x_1|)|x_2 - x_1| \quad (7)$$

where $\theta > 0$ and $\phi = \Phi$, L_1, or L_2.
Assumption 4: The function L_1 satisfies a coercivity condition,

$$L_1(w,v) \ge c|w|^2 \text{ where } c > 0.$$

Assumption 5: The matrix B is of full rank.
Assumption 6: The matrix $L(t)$ is bounded by

$$l_1 I \le L(t)^T L(t) \le l_2 I, \qquad \forall t$$

where l_1 and l_2 are positive real numbers.

2.2 Extended robust H_∞ filter

In this section, a robust filter with a modified H_2 filter structure and a modified H_∞ performance index is derived based on a local solution of the filtering problem. Similar to the development of the well known extended Kalman filter, we derive the filter by linearizing the system in the neighborhood of the estimated trajectory, $\hat{x}$.

Theorem 1[8]: Assume that the uncertain system (3), (4) with (6) satisfies assumption 1~6. Then, the corresponding set of possible states is given by

$$\chi_s = \{x \in R^n : V(x,s) \le d\}$$

where $V(x,t)$ is the unique viscosity solution of (8) in $C(R^n \times [0,s])$.

$$\frac{\partial}{\partial t}V + \max_w[\nabla_x V(f(x) + Bw) - L_1(w,v) + L_2(n(x), z(x))] = 0 \quad (8)$$

where $V(x,0) = \Phi(x(0))$. Assumptions 1-5 ensure that $V(x,t)$ is finite [9]. Q.E.D.

To derive a robust filter, we consider a nonlinear uncertain system (3), (4) which satisfies an integral quadratic constraint given by

$$\begin{aligned} &(x(0) - x_0)^T M(x(0) - x_0) \\ &+ \frac{1}{2}\int_0^T [w(t)^T Q^{-1} w(t) + v(t)^T R^{-1} v(t)]dt \\ &\le d + \frac{1}{2}\int_0^T [n^T n + \gamma^{-2}(z_1 - z)^T(z_1 - z)]dt \end{aligned} \quad (9)$$

where $z_1 = z(\hat{x})$.

According to the theorem 1 and equation (9), the partial differential equation is obtained as

$$\begin{aligned} &\frac{\partial}{\partial t}V + \nabla_x V f(x) + \frac{1}{2}\nabla_x V B Q B^T \nabla_x V^T - \\ &\frac{1}{2}(y - h(x))^T R^{-1}(y - h(x)) + \frac{1}{2}n(x)^T n(x) \\ &+ \frac{1}{2}\gamma^{-2}(z_1 - z(x))^T(z_1 - z(x)) = 0 \end{aligned} \quad (10)$$

where $V(x,0) = (x - x_0)^T M(x - x_0)$.

The $\hat{x}(t)$, as an estimate value of the state variable $x(t)$ is defined to be

$$\hat{x}(t) = \arg\min_x V(x,t) \; . \tag{11}$$

Equation (11) satisfies two conditions:

$$\nabla_x V(\hat{x}(t),t) = 0 \tag{12}$$

$$\nabla_x^2 V(\hat{x}(t),t)\dot{\hat{x}}(t) + \frac{\partial}{\partial t}\nabla_x V(\hat{x}(t),t)^T = 0 \; . \tag{13}$$

The gradient of (10) with respect to x is given by

$$\begin{aligned}&\frac{\partial}{\partial t}\nabla_x V^T + \nabla_x f(x)^T \nabla_x V^T + \nabla_x^2 VBQB^T \nabla_x V^T \\ &+ \nabla_x^2 Vf(x) + \nabla_x h(x)^T R^{-1}(y - h(x)) \\ &+ \nabla_x n(x)^T n(x) - \gamma^{-2}\nabla_x z(x)^T (z_1 - z(x)) = 0 \; .\end{aligned} \tag{14}$$

Using (12) and (13) and evaluating at $x = \hat{x}$, (14) is simplified as

$$\begin{aligned}&\nabla_x^2 V(\hat{x},t)\dot{\hat{x}}(t) = \nabla_x^2 V(\hat{x},t) f(\hat{x}(t)) \\ &+ \nabla_x h(\hat{x}(t))^T R^{-1}(y - h(\hat{x}(t))) + \nabla_x n(\hat{x}(t))^T n(\hat{x}(t)) \\ &- \gamma^{-2}\nabla_x z(\hat{x}(t))^T (z_1 - z(\hat{x}(t))) \; .\end{aligned} \tag{15}$$

Furthermore, suppose that the matrix $\nabla_x^2 V(\hat{x},t)$ is nonsingular for all t, the dynamic equation of state estimate satisfying (11) can be written as

$$\begin{aligned}&\dot{\hat{x}}(t) = f(\hat{x}(t)) + (\nabla_x^2 V(\hat{x}(t),t))^{-1} \\ &[\nabla_x h(\hat{x}(t))^T R^{-1}(y - h(\hat{x}(t))) + \nabla_x n(\hat{x}(t))^T n(\hat{x}(t))]\end{aligned} \tag{16}$$

In addition, the gradient of (14) with respect to x is expressed as

$$\begin{aligned}&\frac{\partial}{\partial t}\nabla_x^2 V + \nabla_x f(x)^T \nabla_x^2 V^T + \nabla_x^2 f(x)^T \nabla_x V^T \\ &+ \nabla_x^2 V\nabla_x f(x) + \nabla_x^3 Vf(x) + \nabla_x^3 VBQB^T \nabla_x V^T \\ &+ \nabla_x^2 VBQB^T \nabla_x^2 V - \nabla_x h(x)^T R^{-1}\nabla_x h(x) \\ &+ \nabla_x^2 h(x)^T R^{-1}(y - h(x)) + \nabla_x^2 n(x)^T n(x) \\ &+ \nabla_x n(x)^T \nabla_x n(x) - \gamma^{-2}\nabla_x^2 z(x)^T (z_1 - z(x)) \\ &+ \gamma^{-2}\nabla_x z(x)^T \nabla_x z(x) = 0 \; .\end{aligned} \tag{17}$$

Using (12) and (13), evaluating at $x = \hat{x}$, and neglecting high order gradient terms, then (17) is reduced to the approximated equation (18)

$$\begin{aligned}&\dot{\Pi} + \nabla_x f(\hat{x})^T \Pi + \Pi\nabla_x f(\hat{x}) + \Pi BQB^T \Pi \\ &- \nabla_x h(\hat{x})^T R^{-1}\nabla_x h(\hat{x}) + \nabla_x n(\hat{x})^T \nabla_x n(\hat{x}) \\ &+ \gamma^{-2}\nabla_x z(\hat{x})^T \nabla_x z(\hat{x}) = 0\end{aligned} \tag{18}$$

where $\Pi = \nabla_x^2 V$ and $\Pi(0) = M$ [8]. The corresponding differential equation for $P(t) = \Pi(t)^{-1}$ from (18) is

$$\begin{aligned}&\dot{P}(t) = P(t)\nabla_x f(\hat{x})^T + \nabla_x f(\hat{x})P(t) + BQB^T \\ &- P(t)[\nabla_x h(\hat{x})^T R^{-1}\nabla_x h(\hat{x}) \\ &- \nabla_x n(\hat{x})^T \nabla_x n(\hat{x}) - \gamma^{-2}\nabla_x z(\hat{x})^T \nabla_x z(\hat{x})]P(t) \; .\end{aligned} \tag{19}$$

From these results, a robust filter can be summarized as

$$\begin{aligned}&\dot{\hat{x}}(t) = f(\hat{x}(t)) + P(t)\nabla_x h(\hat{x}(t))^T R^{-1}(y - h(\hat{x}(t))) \\ &+ \varepsilon^{-2}P(t)\nabla_x N(\hat{x}(t))^T N(\hat{x}(t))\end{aligned} \tag{20}$$

$$z(\hat{x}(t)) = \begin{bmatrix}(L(t)\hat{x}(t))^T & (\frac{\gamma}{\varepsilon}N(\hat{x}(t)))^T\end{bmatrix}^T \tag{21}$$

$$\begin{aligned}&\dot{P}(t) = P(t)\nabla_x f(\hat{x})^T + \nabla_x f(\hat{x})P(t) + \varepsilon^2 B_1 Q_1 B_1^T \\ &+ B_2 Q_2 B_2^T - P(t)[\nabla_x h(\hat{x})^T R^{-1}\nabla_x h(\hat{x}) \\ &- \varepsilon^{-2}\nabla_x N(\hat{x})^T \nabla_x N(\hat{x}) - \gamma^{-2}\nabla_x z(\hat{x})^T \nabla_x z(\hat{x})]P(t)\end{aligned} \tag{22}$$

where $P(0) = M(0)^{-1}$, $\hat{x}(0) = x_0$, and M is a matrix which reflects the initial errors of the estimate.

The proposed filter has the structure of an H_2 filter but with (21) and $\gamma^{-2}P(t)\nabla_x z(\hat{x})^T \nabla_x z(\hat{x})P(t)$ in (22). However, by virtue of (21) and $\gamma^{-2}P(t)\nabla_x z(\hat{x})^T \nabla_x z(\hat{x})P(t)$, this filter can have modified H_∞ performance index, as shown in the next section.

2.3 Analysis of extended robust H_∞ filter

In this section, the analytical performances of the filter proposed in section 2.1 are investigated. We will derive a modified H_∞ performance index, which is the energy ratio between the disturbances and the estimation error, as an important characteristic of the filter.

The estimate errors can be defined to be

$$\zeta(t) = x(t) - \hat{x}(t) \tag{23}$$

and the dynamic equation of the estimation errors $\zeta(t)$ is expressed as

$$\begin{aligned}&\dot{\zeta}(t) = (A(t) - K(t)C(t))\zeta(t) + B(t)w(t) \\ &- P(t)\nabla_x n(\hat{x}(t))^T n(\hat{x}(t)) + \varphi(x(t),\hat{x}(t)) \\ &- K(t)\chi(x(t),\hat{x}(t)) - K(t)v(t)\end{aligned} \tag{24}$$

where $A(t) = \frac{\partial f}{\partial x}(\hat{x}(t))$, $C(t) = \frac{\partial h}{\partial x}(\hat{x}(t))$, and $K(t) = P(t)C(t)^T R^{-1}$. Nonlinear functions $\varphi(x(t),\hat{x}(t))$ and $\chi(x(t),\hat{x}(t))$ are defined as

$$\begin{aligned}&f(x(t)) - f(\hat{x}(t)) = A(t)(x(t) - \hat{x}(t)) + \varphi(x(t),\hat{x}(t)) \\ &h(x(t)) - h(\hat{x}(t)) = C(t)(x(t) - \hat{x}(t)) + \chi(x(t),\hat{x}(t)) \; .\end{aligned}$$

And $\varphi(x(t),\hat{x}(t))$, $\chi(x(t),\hat{x}(t))$ are higher order terms in the Taylor expansion. We make assumption 7 and assumption 8.

Assumption 7: $n(t) = \varepsilon^{-1}N(x(t)) = \varepsilon^{-1}N(t)x(t)$.

Assumption 8: There exist positive real numbers, $\varepsilon_\varphi, \varepsilon_\chi, k_\varphi$, and k_χ, to bound the nonlinear terms $\varphi(x(t),\hat{x}(t))$ and $\chi(x(t),\hat{x}(t))$ as follows:

$$\begin{aligned}&\|\varphi(x(t),\hat{x}(t))\| \le k_\varphi \|x(t) - \hat{x}(t)\|^2 , \quad \|\zeta\| \le \varepsilon_\varphi \\ &\|\chi(x(t),\hat{x}(t))\| \le k_\chi \|x(t) - \hat{x}(t)\|^2 , \quad \|\zeta\| \le \varepsilon_\chi\end{aligned}$$

Lemma 1[1]: Suppose that A1 and A8 are satisfied. For estimation errors $\|\zeta\| \le \varepsilon_1$, there exist real numbers k such that

$$(x(t)-\hat{x}(t))^T P(t)^{-1}[\varphi(x(t),\hat{x}(t))-K\chi(x(t),\hat{x}(t))] \le k\|x(t)-\hat{x}(t)\|^3 \quad (25)$$

where $\varepsilon_1 = \min(\varepsilon_\varphi, \varepsilon_\chi)$, $k = \frac{k_\varphi}{p_1} + \frac{c_2 k_\chi}{r}$, $\|C(t)\| \le c_2$, and p_1 is the lower bound of $P(t)$ and $rI \le R$.

Suppose that a function is chosen as

$$\bar{V}(\zeta(t)) = \zeta(t)^T P(t)^{-1}\zeta(t) \quad (26)$$

where $P(t)$ is the solution of (22). Differentiating $\bar{V}(\zeta(t))$ over time yields

$$\dot{\bar{V}}(\zeta(t)) = \dot{\zeta}(t)^T P(t)^{-1}\zeta(t) + \zeta(t)^T \dot{P}(t)^{-1}\zeta(t) + \zeta(t)^T P(t)^{-1}\dot{\zeta}(t) . \quad (27)$$

Substituting (22) and (24) in (27), it is easy to show that (27) becomes

$$\begin{aligned}\dot{\bar{V}}(\zeta(t)) &= \zeta(t)^T[-\gamma^{-2}L(t)^T L(t)]\zeta(t) + w^T Q^{-1} w - s^T s \\ &+ v^T R^{-1} v - \eta^T R^{-1}\eta + 2\varphi^T P(t)^{-1}\zeta(t) \\ &- 2(K(t)\chi)^T P(t)^{-1}\zeta(t) \\ &+ \varepsilon^{-2}\{\zeta(t)^T[-2\nabla_x N(\hat{x}(t))^T \nabla_x N(\hat{x}(t))]\zeta(t) \\ &- \zeta(t)^T \nabla_x N(\hat{x}(t))^T N(\hat{x}(t)) \\ &- (\nabla_x N(\hat{x}(t))^T N(\hat{x}(t)))^T \zeta(t)\}\end{aligned} \quad (28)$$

where $s = Q^{-\frac{1}{2}}w - (B(t)Q^{\frac{1}{2}})^T P(t)^{-1}\zeta(t)$, $Q = Q^{\frac{1}{2}}Q^{\frac{1}{2}T}$, $\eta = v + C(t)\zeta(t)$, and $R = R^{\frac{1}{2}}R^{\frac{1}{2}T}$.

Utilizing the assumption 7 and the triangle inequality property, (28) can be expressed as

$$\begin{aligned}\dot{\bar{V}}(\zeta(t)) &\le \zeta(t)^T[-\gamma^{-2}L(t)^T L(t)]\zeta(t) + w^T Q^{-1} w + v^T R^{-1} v \\ &+ 2\varphi^T P(t)^{-1}\zeta(t) - 2(K(t)\chi)^T P(t)^{-1}\zeta(t) \\ &+ \varepsilon^{-2} N(\hat{x}(t))^T N(\hat{x}(t)) .\end{aligned} \quad (29)$$

Applying Lemma 1 to (29), we obtain the following inequality,

$$\begin{aligned}\dot{\bar{V}}(\zeta(t)) &\le \zeta(t)^T[-\gamma^{-2}L(t)^T L(t)]\zeta(t) + 2k\|\zeta(t)\|^3 \\ &+ w^T Q^{-1} w + v^T R^{-1} v + \varepsilon^{-2} N(\hat{x}(t))^T N(\hat{x}(t)) .\end{aligned} \quad (30)$$

Provided that the estimate errors satisfy $\|\zeta(t)\| \le \varepsilon_2$, (30) can be modified to

$$\begin{aligned}\dot{\bar{V}}(\zeta(t)) &\le \zeta(t)^T[-\gamma^{-2}L(t)^T L(t)]\zeta(t) + 2k\|\zeta(t)\|^3 \\ &\quad + w^T Q^{-1} w + v^T R^{-1} v + \varepsilon^{-2} N(\hat{x}(t))^T N(\hat{x}(t)) \\ &\le -\frac{\gamma^{-2} l_1}{2}\|\zeta(t)\|^2 + w^T Q^{-1} w + v^T R^{-1} v \\ &\quad + \varepsilon^{-2} N(\hat{x}(t))^T N(\hat{x}(t)) \\ &\le -\frac{\gamma^{-2} l_1}{2 l_2}\zeta(t)^T[L(t)^T L(t)]\zeta(t) + w^T Q^{-1} w + v^T R^{-1} v \\ &\quad + \varepsilon^{-2} N(\hat{x}(t))^T N(\hat{x}(t))\end{aligned} \quad (31)$$

where $\varepsilon_2 = \min(\varepsilon_1, \frac{\gamma^{-2} l_1}{4k})$.

Finally, the performance index of the derived filter is obtained as follows. By integrating both sides of (31), the modified H∞ performance index J is expressed as

$$J = \frac{\|L\zeta\|_2^2}{\|w\|_{2Q^{-1}}^2 + \|v\|_{2R^{-1}}^2 + \|N(x)\|_{2\varepsilon^{-2}}^2 + \zeta(0)^T P(0)^{-1}\zeta(0)} < \gamma^2\mu^{-1} = \gamma_t^2 \quad (32)$$

where $\mu = \frac{l_1}{2 l_2}$. The performance index J of the robust filter is less than γ_t^2 . As μ is less than 1, the new value γ_t is always greater than γ . γ_t is not only an index of disturbance attenuation level, but also an important parameter describing filter's estimation ability in the worst case. Decreasing γ_t means that robustness of the filter increases. Equation (32) shows that the proposed filter guarantees robustness against the noises, including process noise and measurement noise and the system model uncertainty. On the contrary, when the extended Kalman filter or the H_2 filter is applied to the nonlinear system, the performance index (32) cannot be defined since the value of γ is ∞. Therefore they cannot guarantee robustness against noise and uncertainty and cannot have the effect of disturbance attenuation.

3. EXAMPLE

To verify the performance of the proposed filter, an FM demodulation problem [10 p. 200,8] is considered. For the FM demodulation problem, the extended Kalman filter is commonly applied.

We consider the following nonlinear uncertain system

$$\begin{bmatrix}\dot{\lambda}(t)\\ \dot{\theta}(t)\end{bmatrix} = \begin{bmatrix}-1+\Delta(t) & 0\\ 1 & 0\end{bmatrix}\begin{bmatrix}\lambda(t)\\ \theta(t)\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix} w_0(t) \quad (33)$$

$$y(t) = \sqrt{2}\sin(t+\theta(t)) + v(t) \quad (34)$$

with an integral quadratic constraint

$$\int_0^{100}[50 w_0(t)^2 + 100 v(t)^2]dt \le 10 . \quad (35)$$

The initial conditions of the system (33) are assumed to be known as $\lambda(0)=0$ and $\theta(0)=0$. The disturbances, which satisfy the above constraint, also satisfy the following integral quadratic constraint.

$$\int_0^{100}[50 w_0(t)^2 + 100 v(t)^2]dt \le 10 + \frac{1}{2}\int_0^{100}[\gamma^{-2}\delta\lambda(t)^2 + \varepsilon^{-2}\delta\lambda(t)^2 + \gamma^{-2}\delta\theta(t)^2]dt .$$

For the extended robust H∞ filer, the filter output is designed to be

$$z(t)=\begin{bmatrix}\lambda(t)\\ \theta(t)\\ \gamma\varepsilon^{-1}\lambda(t)\end{bmatrix}.$$

In this system model, we want to estimate the variable $\lambda(t)$, $\theta(t)$ and $y(t)$ is the measured FM signal. The system is simulated with unknown disturbance signals $w_0(t)$ and $v(t)$ those satisfy (35) and are not white Gaussian noises.

Our extended robust H_∞ filer, (20) and (22), processes the measured signal $y(t)$. The result of simulation is illustrated in figure 1 for $\lambda(t)$ with true trajectory. The result of the proposed filter is somewhat noisy, but it is tracking the true trajectory. In this simulation, the modified H_∞ performance index is 1.5. For the purpose of comparison, we simulated the extended Kalman filter for the same nonlinear uncertain system. This result is illustrated in figure 2. The estimation result of the extended Kalman filter has large errors and can be considered to be divergent.

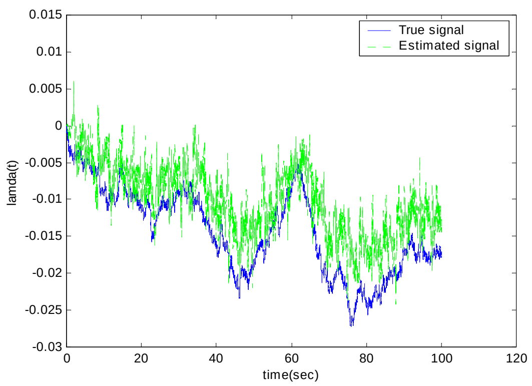

Figure 1. Simulation results of the extended robust H_∞ filer

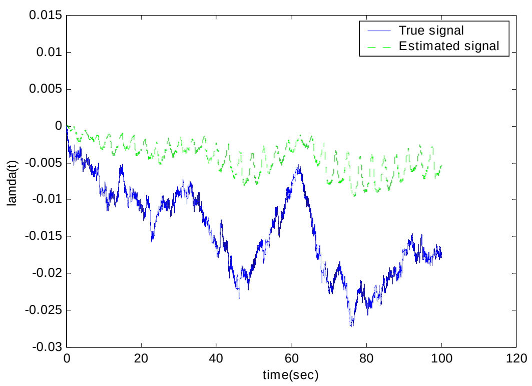

Figure 2. Simulation results of the extended Kalman filer

4. CONCLUSION

The extended robust H_∞ filer has been proposed. It has been derived by considering a nonlinear uncertain system with an integral quadratic constraint and by introducing the notion of a local solution to the filtering problem. The proposed filter possesses the modified H_∞ performance index. Thus, we can know the energy gain from disturbances to estimation errors of the proposed filter, and contrarily for the prespecified level of energy gain, we can design a robust filter, if it exists. The simulation results for an FM demodulation have shown that the proposed filter is robust to the uncertainty and can yield more accurate results than the extended Kalman filter.

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