Boundary estimation in electrical impedance tomography with multi-layer neural networks.

J. H. KIM[a], H. J. JEON[a], B. Y. CHOI[a], M.C.KIM[b], S. KIM[c] and K. Y. KIM[d]

[a] Department of Electronic Engineering, Kyungpook National University, Taegu, 702-701, Korea

(Tel: +82-53-940-8653; Fax: +82-53-9597336; Email: cjsrkd97@hanmail.net)

[b] Department of Chemical Engineering, Cheju National University, Cheju, 690-756, Korea

[c] Department of Nuclear & Energy Engineering, Cheju National University, Cheju, 690-756, Korea

[d] Department of Electrical and Electronic Engineering, Cheju National University, Cheju, 690-756, Korea

(Tel: +82-64-754-3664; Fax: +82-64-756-1745; Email: kyungyk@cheju.ac.kr)

Abstract: The boundary estimation problem is used to estimate the shape of organic depend on the phase of the cardiac cycle or interested in the detection of the location and size of anomalies with resistivity values different from the background tissues such as nuclear reactor. And we can use the method to solve the optimal solution such as modified Newton raphson, kalman filter, extended kalman filter, etc. But, this method consumes much time and is sensitive to the initial value and noise in the estimation of the unknown shape.

In the paper, we propose that multi-layer neural networks estimate the boundary of the unknown object using Fourier coefficient. This method can be used at the real time estimation and have strong characteristics at the noise and initial value. It uses voltage change; difference the homogeneous voltage to the non-homogeneous voltage, and change of Fourier coefficient change to train multi-layer neural network. After train, we can have real time estimation using this method.

Keywords: Tomography, Neural network, Boundary Estimation.

1. INTRODUCTION

In electrical impedance tomography(EIT) the distribution of impedances inside an object('image') is sought by applying specified currents at some electrodes, and performing measurements of the voltage at other electrodes. The impedance distribution is usually estimated in fixed element inside the object. The implicit assumption is most often that the impedance in each element is more dependent of the other element. In many cases, such as in the impedance imaging of the chest, the heart muscle and blood, this model might not be a feasible one. The poor spatial resolution of EIT that is due to diffusive characteristics of the problem does not usually allow for the estimation of internal boundaries in the interest. To improve spatial resolution, we have to increase fixed element. But the uncertain parameter has to increase too. So, the many researchers have the interest at the boundary estimation problem that wish to know the shape of the object more than the impedance distribution [3-6].

To use this method, we have an assumption that the impedance distributions are known a prior in the boundary estimation problem. And we can estimate the optimal solution for express the real object using many optimization methods[1-5]. The modified Newton Raphson method(mNR) that have the good performance is used usually in one of the many method[7][8]. But this method consumes much time and is sensitive to the noise. So, it is difficult to apply a field that fast response is required and system has noise.

In this paper, we suggest new algorithm using multi-layer neural network and the boundary of the object expresses using Fourier series.

This method can be used when the fast response is required, the system has noise.

It uses voltage change; difference the homogeneous voltage to the non-homogeneous voltage, and change of Fourier coefficient change to train multi-layer neural network. After train, we can have fast response using this method.

Finally, the results of simulation are given to demonstrate the validity of the proposed algorithm.

2. Method

2.1. Forward Problem

The forward problem is to compute the electrical potential when the injected current and the resistivity distribution are given. When electrical currents $I_l (l = 1,2,\cdots,L)$ are injected into the object $\Omega \in R^2$ through the electrodes $e_l (l = 1,2,\cdots,L)$ attached on the boundary $\partial\Omega$ and the resistivity distribution $\rho(x, y)$ is know for the Ω, the corresponding induced electrical potential $u(x, y)$ can be determined uniquely from following partial differential equation, which can be derived from the Maxwell equations:

$$\nabla \cdot \left(\frac{1}{\rho} \nabla u \right) = 0, \quad \text{in } \Omega \tag{1}$$

The boundary conditions in the complete electrode model are given as:

$$\left. \left(u + z_l \frac{1}{\rho} \frac{\partial u}{\partial n} \right) \right|_{el} = U_l, \quad l = 1,2,\cdots,L \tag{2}$$

$$\int_{el} \frac{1}{\rho} \frac{\partial u}{\partial n} dS = I_l, \quad l = 1,2,\cdots,L \tag{3}$$

$$\left. \frac{1}{\rho} \frac{\partial u}{\partial n} \right|_{\partial\Omega \setminus \bigcup_{i=1}^{L} el} = 0 \tag{4}$$

where z_l is the effective contact impedance between lth electrode and electrolyte, U_l are the measured potential on the lth electrode, e_l is lth electrode, n is outward unit normal, and L is the number of electrodes. The boundary conditions (2)-(4) take into account the shunting effect (i.e. the voltage U_l is constant over the electrode e_l) and the additional voltage drop due to the contact impedance.

In addition, the follow two conditions for the injected currents and measured voltages are needed to ensure the existence and uniqueness of the solution [11]:

$$\sum_{l=1}^{L} I_l = 0 \,, \quad \sum_{l=1}^{L} U_l = 0 \tag{5}$$

In general, the forward problem cannot be solved analytically. So, we have to resort to the numerical solution. In this paper, we used the FEM to obtain the numerical solution. In the FEM, the object area is discretized into sufficiently small elements having a node at each corner and it is assumed that the resistivity distribution is constant within each element. The potential at each node and the "referenced" electrode voltages, defined bye the vector $v \in R^{(M+L-1)\times 1}$, are calculated by discretizing (1) into $Yv = c$, where $Y \in R^{(M+L-1)\times(M+L-1)}$ is so called stiffness matrix and M is the number of FEM nodes, Y and c are the functions of the resistivity distribution inside the object and the injected currents through the electrodes, respectively. For more details on the forward solution and the FEM approach, see[3]

2.2. Boundary expression and Measurement Voltage

In this paper, we express the boundary of the object using the Fourier series. And the representation equation is as follow[5].

$$C_\ell(s) = \begin{pmatrix} x \\ y \end{pmatrix} = \sum_{n=1}^{N_\theta} \begin{pmatrix} \gamma_n^{x_\ell} \theta_n^x(s) \\ \gamma_n^{y_\ell} \theta_n^y(s) \end{pmatrix} \tag{6}$$

where $C_\ell(s)$, $\ell = 1, \cdots, m$ is the boundary of the object, m is the number of the object, $\theta_n(s)$ are periodic and smooth basis function, we use basis function of the form:

$$\begin{aligned} &\theta_1^\beta(s) = 1 \\ &\theta_n^\beta(s) = \sin\left(2\pi \frac{n}{2} s\right), \qquad n = 2,4,6,\cdots, N_\theta - 1 \\ &\theta_n^\beta(s) = \cos\left(2\pi \frac{(n-1)}{2} s\right), \qquad n = 3,5,7,\cdots, N_\theta \end{aligned} \tag{7}$$

where $s \in [0\ 1]$, β denotes either x or y. And N_θ is the dimension of the Fourier series.

As we use equation (6), the boundaries of the object are identified with the vector γ of the shape coefficients.

$$\gamma = (\gamma_1^{x_1}, \cdots, \gamma_{N_\theta}^{x_1}, \gamma_1^{y_1}, \cdots, \gamma_{N_\theta}^{y_1}, \gamma_1^{x_m}, \cdots, \gamma_{N_\theta}^{x_m}, \gamma_1^{y_m}, \cdots, \gamma_{N_\theta}^{y_m})^T \tag{8}$$

where $\gamma \in R^{2mN_\theta}$.

The relation of the measured voltages on the electrodes and the Fourier coefficient γ above defined are very nonlinear. We have to make relation of both as follow.

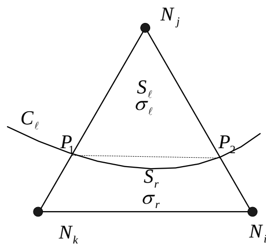


Fig. 1. A schematic representation of one FEM intercepted by the region boundaries $C_\ell(s)$.

When the one FEM mesh intercepted the boundaries of the object as Fig. 1., we have to solve the partial differential equation from FEM mesh separated. The FEM implementation of this relation is accomplished in 4 stages as follows [5]:

Step1: Classification of mesh nodes as nodes inside of outside a given boundaries $C_\ell(s)$. This is accomplished by counting the number of boundary crossings λ of a horizontal line drawn through the node $N_i = (x_i, y_i)^T$.

$$\begin{pmatrix} x_i \\ y_i \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix} = C_\ell \,. \tag{9}$$

Step2: Classification of the mesh element to sets of element inside the region and elements intercepted by the boundaries $C_\ell(s)$.

Step3: Determination of the position (x, y) in the intersection points $C_\ell(P_1)$ and $C_\ell(P_2)$ for each element, see Fig.1. The intersection of $C_\ell(s)$ with the edge from N_i to N_j is obtained from

$$\begin{pmatrix} x_i \\ y_i \end{pmatrix} + \varepsilon \begin{pmatrix} x_j - x_i \\ x_j - y_i \end{pmatrix} = C_\ell(s) \tag{10}$$

with condition $0 < \varepsilon \le 1$.

Step4: Computation of the partial differential equation in equation (1). In this process we approximate the boundary C_ℓ with a straight line from the intersection point $C_\ell(P_1)$ to the point $C_\ell(P_2)$ and then split the intercepted element into two parts. Therefore, we determine resistivity on the each FEM mesh as the rate of the triangle area [5].

$$\frac{\sigma_\ell S_\ell + \sigma_r S_r}{S_\ell + S_r} \int_{\Omega_m} \nabla\varphi_i \nabla\varphi_j dr \tag{11}$$

Where σ_ℓ is defined the resistivity of the object, σ_r is the resistivity of the background, S_ℓ is the area of the split triangle on the object, and S_r is the area of split triangle on the background. The integral part is the function of the FEM solver. This FEM solver is evaluated using the conventional global-to-local element mappings.

Finally, we can make the relation of the measurement voltage with the Fourier coefficient as above step.

2.3. Boundary estimation using neural network

In this paper, we use multi-layer neural network. The neural network trains weighting matrix W_1, W_2 using the relation of the change of the voltage on the electrode and the change of the furrier coefficient as Fig.2.

The following figure express multi-layer neural network.

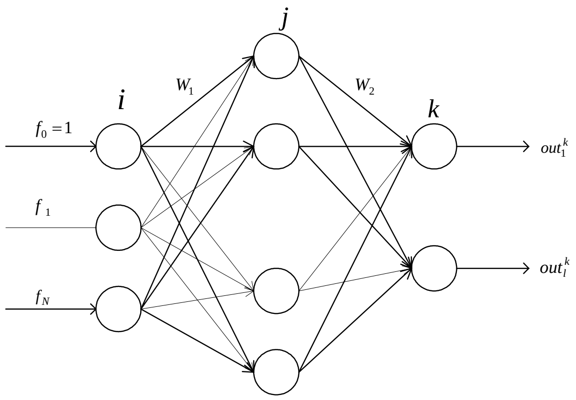


Fig. 2. Multi-layer neural network.

Where f_n is defined the change of the voltage on each frame, out_l^k is the output of the kth neuron.

To trains the weighting matrix W_1 , W_2 of the neural network, we use training pattern which is the change of the voltage (f_n) and the change of the Fourier coefficient ($\Delta\gamma_{known}$). The changes of the voltage are the difference of the voltage from reference shape to the changed shape. These voltages have to known beforehand to trains.

We can define the change of the voltage on the electrodes as following equation [9].

$$f_n = \frac{v_n^i - v_n^{ref}}{v_n^i + v_n^{ref}}, \quad \begin{matrix} i = 1, 2, \cdots, s_j \\ n = 1, 2, \cdots, N \end{matrix} \tag{12}$$

Where v_n is defined the nth voltage from measurement voltage (U_l) on each frame, s_j is the number of the target sampling data. And v^{ref} is the voltage of the reference voltage.

We can define the change of the Fourier coefficient about the change of the voltage in a next equation (9).

$$\Delta\gamma_{known} = \frac{\gamma^i - \gamma^{ref}}{2}, \qquad i = 1, 2, \cdots, 2mN_\theta \tag{13}$$

Where $\Delta\gamma_{known}$ is the difference of the ith coefficient from the reference coefficient γ^{ref} and this variable is variable to know beforehand to train.

Using the beforehand knowing parameter ($f_n, \Delta\gamma_{known}$), we can update the weighting matrix by learning algorithm as follow step:

Step 1: Initially, set all weighting, W_1 and W_2 , random numbers. This weight controlled suitable interval. Select the parameters η and α .

Step 2: Randomly take one unmarked pair($f_n, \Delta\gamma_{known}$) of the training set for the further steps and mark it as used.

Step 3: The forward calculation.

- Input neuron.

$$out^i = f_0, f_1, \cdots, f_n \tag{14}$$

- The input and the output of the hidden layer.

$$net^j = \sum_{i=1}^{n} (W_1 \cdot Out^i), \qquad Out^j = g(net^j) \tag{15}$$

- The input and the output of the output layer.

$$net^k = \sum_{j=1}^{J} (W_2 \cdot Out^j), \qquad Out^k = g(net^k) \tag{16}$$

where Out^i, Out^j, Out^k are output of the neuron in the input layer, the hidden layer and the output layer. net^i, net^j, net^k are input of the neuron in the each layer.

The used activation function and its derivative are given by:

$$g(x) = \frac{1}{1 + e^{-bx}} \tag{17}$$

where b is the threshold of the activation function.

Step 4: The backward calculation.

$$\begin{aligned} \delta^k &= (\Delta\gamma_{known} - Out^k) \cdot f'(net^k) \\ \delta^j &= f'(net^j) \cdot \sum_k (\delta^k \cdot W_2) \end{aligned} \tag{18}$$

Step 5: Calculate the weight changes and update the weights.

$$\begin{aligned} \Delta W_1 &= \eta \cdot \delta^j \cdot Out^i \\ \Delta W_2 &= \eta \cdot \delta^k \cdot Out^j \end{aligned} \tag{19}$$

$$W_1(k+1) = W_1(k) + \Delta W_1 + \alpha \cdot (W_1(k) - W_1(k-1))$$
$$W_2(k+1) = W_2(k) + \Delta W_2 + \alpha \cdot (W_2(k) - W_2(k-1))$$

Step 6: Repeat from step 2 while there are unused pair in the training.

Step 7: Set all pairs in the training set to unused.

Step 8: Repeat from step 2 until the stop condition is true.

Training continues until the overall error in one training cycle is sufficiently small, this stop condition is given by:

$$E < \varepsilon \tag{20}$$

This acceptable error ε has to be selected carefully, if ε is too large the network is under-trained and lacks in performance, if ε is selected too small the network will be biased towards the training set.
The used performance Index error E calculated by:

$$E = \sum (\Delta\gamma_{known} - Out^k)^2 \, . \tag{21}$$

We can estimate the change of the fourier coefficient using the change of the voltage in the actual phantom and the updated weights W_1, W_2.

$$\Delta\hat{\gamma} = g\left(W_2 \cdot g(W_1 \cdot \tilde{f}_n + b)\right) + b) \tag{22}$$

where $\Delta\hat{\gamma}$ is the estimated coefficient, and $\tilde{f}_n$ is the change of the voltage in the actual phantom.

3. Simulation

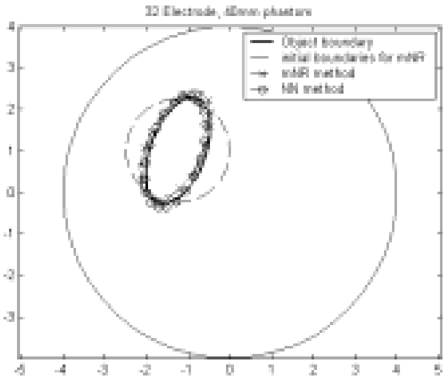
To show the effectiveness of our approach, we assume that all the 32 channels are measured for a given current pattern. In the computation of the synthetic data by the FEM-model, the body Ω was discretized into 3104 triangular element, the number of nodes being 1681. We used the phantom to have a radius $4\,cm$, and the resistivity of the object is $600\Omega cm$, the resistivity of the background is $300\Omega cm$.

For the current injection we use the opposite current patterns, the input of the neural network are the change of the voltage to have 512 and the change of the coefficient to have the target sampling data 2608(s_j) . The number of units in the hidden layer has been selected 48.

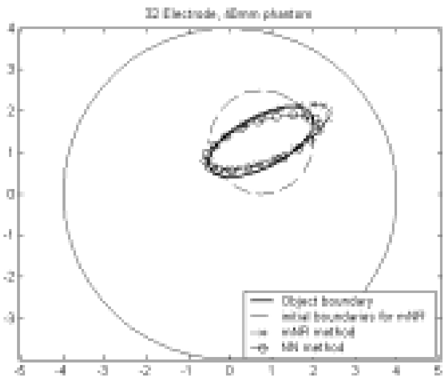
We assume that the shape of the object has sufficiently to an oval form as like bubble. Consequently, the dimension of the Fourier series assumes $N_\theta = 3$.

We do computer simulation at one object to demonstrate usefulness of proposed scheme.

The results of the simulation with 1% random noise are shown in Fig. 3. It can be seen that the results with the proposed approach are not better than with the modified Newton Raphson.



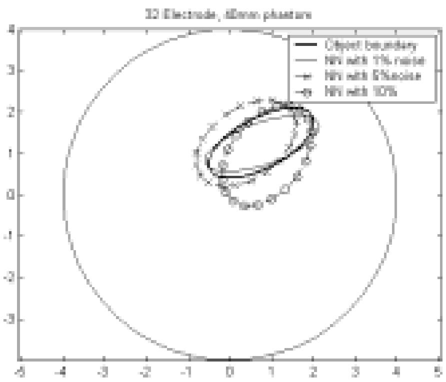
(a) $\gamma = [\,0.75\ \ 1.25\ \ 0.20\ \ 1.25\ \ 0.40\ \ 0.75]$



(b) $\gamma = [\,-1.25\ \ 0.75\ \ 0.20\ \ 1.00\ \ 0.25\ \ 1.25]$

Fig. 3. Fourier coefficient estimated by multi-layered neural network with 1% noise.

We compared modified Newton Raphson to proposed scheme in the Fig. 4., when noise of 1%, 5% and 10% were added to the simulated voltage. It can be seen that the results with modified Newton Raphson can not guarantee that it can converge with 5% noise. But, the proposed scheme guarantees that it can converge with 5% and 10% noise.

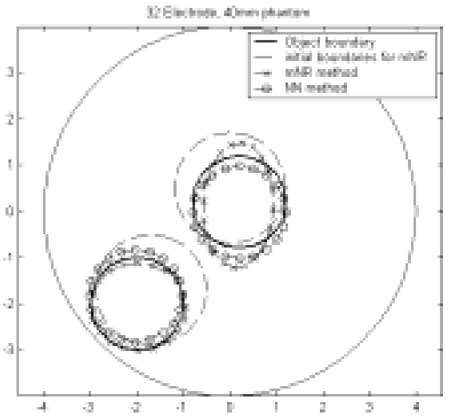


(a) $\gamma = [\,-1.25\ \ 0.75\ \ 0.20\ \ 1.00\ \ 0.25\ \ 1.25]$

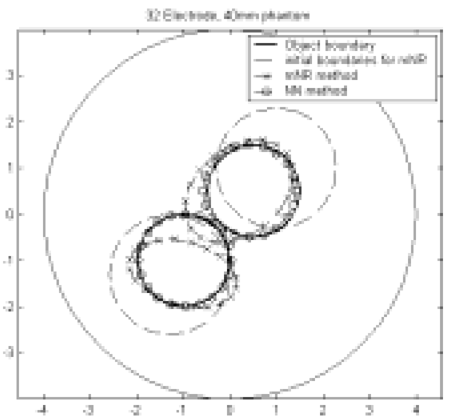
Fig. 4. Fourier coefficient estimated by multi-layered neural network with 1%, 5%, and 10% noise.

In the next simulation, we consider the multi-object, when there exist noise 0.1%, 1% and 3%, see Fig. 5, Fig.6. The reference voltages in this simulation are voltages on the phantom to have homogeneous resistivity. The used object in the simulation not to be trained, we can estimate the various object using this algorithm.

We can see the performance of the proposed scheme better than modified Newton Raphson method with 0.1% noise, see Fig. 5.



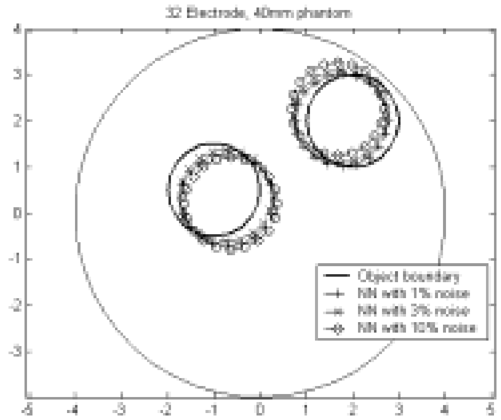
(a) $\gamma = [\ 0.2\ \ 1\ \ \ 0\ \ \ 0.2\ \ 0\ 1\text{-}2\ \ \ 1\ \ \ 0\ \ \text{-}2\ \ 0\ 1\ \ ]$



(b) $\gamma = [\ 0.5\ \ 1\ \ \ 0\ \ \ 0.5\ \ 0\ 1\ \text{-}1\ \ \ 1\ \ \ 0\ \ \ \ \text{-}1\ \ \ 0\ 1\ \ ]$

Fig. 5. Fourier coefficient estimated by multi-layered neural network with 0.1% noise.

If a measured voltage, calculation using FEM, has over 1% of noise, the conventional mNR method not to be obtained to the object. But the proposed scheme to have robust characteristic against the noise can estimate the coefficient of the object, see Fig.6.



(a) $\gamma_3 = [1\ \ 1\ \ 0\ \ \ 1\ \ 0\ \ 1\ \ \text{-}2\ \ 1\ \ 0\ \ 1.5\ \ 0\ \ 1]$

Fig. 6. Fourier coefficient estimated by multi-layered neural network with 1%, 3%, and 10% noise.

4. Conclusions

We proposed multi-layer neural network to estimate the shape of the object promptly using the relation the changes of the measurement voltage and the change of the Fourier coefficient. And we obtained the characteristic of the robustness against noise.

In the numerical simulation, we did simulation about multi-object in the phantom. The proposed scheme had a good performance more than the mNR with noise. And we obtained the fast processing time more than the mNR.

In the further study, we need to modify the parameter, the number of the hidden neuron, the training pattern data, to improve the performance.

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