Autopilot Design for Agile Missile with Aerodynamic Fin and Thrust Vecotring Control

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Abstract: This paper is concerned with a control allocation strategy using the dynamic inversion which generates the nominal control input trajectories, and autopilot design using the time-varying control technique which is time-varying version of pole placement of linear time-invariant system for an agile missile with aerodynamic fin and thrust vectoring control. Dynamic inversion can decide the amount of the deflection of each control effector, aerodynamic fin and thrust vectoring control, to extract the maximum performance by combining the action of them. Time-varying control technique for autopilot design enhance the robustness of the tracking performance for a reference command. Nonlinear simulations demonstrates the dynamic inversion provides the effective nominal control input trajectories to achieve the angle of attack command, and time-varying control technique exhibits good robustness for a wide range of angle of attack.

Keywords: dynamic inversion, time-varying control, control allocation, autopilot, agile missile

1. Introduction

The modern control system of an agile missile has the many challenges due to the stringent required performance such as fast time response, high angle of attack, and high maneuver. Usually, to achieve the required performance, the agile missiles combine the new control effectors (thrust vectoring, side thrusters) with the conventional control surface(aerodynamic fin) because thrust vectoring control and side thrusters can provide additional moments and forces to achieve the reference command [1],[2]. However, managing each of a group of control devices with the independent control logic sometimes can result in reduced missile controllability and efficiency. For example, at the launch phase of the missile, the aerodynamic fin has low control authority due to low speed. Hence the missile must be controlled by simultaneously using the aerodynamic fin and the additional control effectors. Therefore, for the super-maneuverability of the agile missile, control allocation for control effector family is needed.

On the other hand, the dynamics of an agile missile is inherently nonlinear and may vary rapidly with time. Furthermore, this dynamics is highly uncertain since exact aerodynamic data for vehicles operating under such conditions are difficult to obtain and may in fact be a poor approximation to the actual dynamics. These and other concerns have prompted researchers to look beyond the classical methods. Most nonlinear control techniques are based on linearizing the equations of motion at each equilibrium point or by the application of nonlinear feedback as known variously as feedback linearization, dynamic inversion, or gain scheduling. However these methods rely heavily on knowledge of the plant dynamics. That is, if the mathematical model has uncertainties, the cancellation of the nonlinear dynamics will not be exact. This may have serious consequences since dynamic inversion by itself does not guarantee any robustness to modeling uncertainties. Therefore, stability and performance robustness within the dynamic inversion framework must be addressed by robust control. More recently, the extended-mean assignment (EMA) control technique for linear time-varying (LTV) has emerged as a means of explicitly accounting for uncertainties in the plant dynamics. The EMA control technique is based on the eigenvalue concept for linear time-varying systems, called the SD-eigenvalue. The technique is similar to the conventional pole placement design method for linear time-invariant systems. Closed-loop stability is achieved by the assignment of the extended-mean of these time-varying SD-eigenvalues to the left half complex plane (LHCP).

This paper is concerned with the control allocation using the dynamic inversion with the weighting functions and autopilot design using time-varying control technique (EMA) for an agile missile with aerodynamic fin and thrust vectoring control. The dynamic inversion generates the required nominal control inputs for each control effectors (aerodynamic fin and thrust vectoring control). The proposed control allocation algorithm is capable of extracting the maximum performance from each control effector, aerodynamic fin and thrust vectoring control, by combining the action of them. Time-varying control technique for autopilot design enhance the robustness of the tracking performance for a reference command. Also, The time-varying bandwidth filter approach as command shaping filter is employed to cope with the actuator saturation problem so that the autopilot can be designed without any concern of actuator saturation. The filter greatly reduces the acceleration and rate of an abrupt command trajectory, whereas it has little effect on smooth trajectories that can be tracked within the actuator rate limit. The main results are validated through the nonlinear simulation. Simulation results show that the proposed control system generates the effective control allocation to achieve the command trajectory without the actuator saturation and has the good tracking performance. Entire schematic diagram is shown in the Fig. 1.

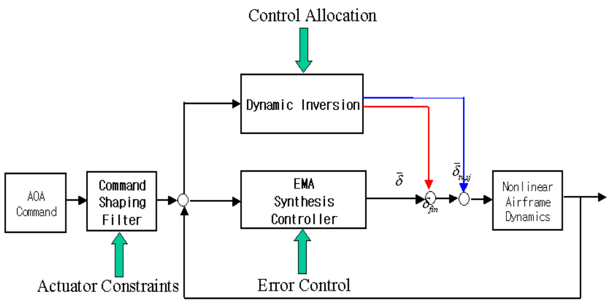

Fig. 1. Schematic diagram

2. Missile Model

This section details the missile model used in this paper. The model with the thrust vectoring control is a nonlinear pitch dynamics model using aerodynamic data as shown Fig. 2. The equation of motion is given by

$$\begin{aligned}\dot{\alpha}(t) &= \frac{\frac{1}{2}\rho V^2 S}{mV}[C_{Z_0}(\alpha(t), M(t)) \\ &\quad + C_{Z_\delta}(\alpha(t), M(t), \delta_{fin}(t))] + q(t) + \frac{T}{mV}\delta_{tvc}(t), \\ \dot{q}(t) &= \frac{\frac{1}{2}\rho V^2 SC}{I_{yy}}[C_{m_0}(\alpha(t), M(t)) \\ &\quad + C_{m_\delta}(\alpha(t), M(t), \delta_{fin}(t)) + \frac{C}{2V}C_{mq}(M(t))] \\ &\quad + \frac{Tl_m}{I_{yy}}\delta_{tvc}(t), \end{aligned} \tag{1}$$

where $\alpha(t)$, $q(t)$, $\delta_{fin}(t)$, $\delta_{tvc}(t)$ are angle of attack, pitch rate, aerodynamic fin deflection angle, and thrust vectoring control deflection angle, respectively, and m, V, ρ, S, C, T, l_m are mass, velocity, air density, reference area, reference length, thrust, and moment arm, respectively. Also, $M(t)$ is Mach number, and C_{mq} is pitch damping.

Aerodynamic coefficients in Eq. (1) are represented as the function of angle of attack at fixed Mach number $M = 0.95$:

$$C_{Z_0}(\alpha(t)) = a_1\alpha^4(t) + b_1\alpha^3(t) + c_1\alpha^2(t) + d_1\alpha(t)$$

$$C_{Z_\delta}(\alpha(t)) = (a_2\alpha^3(t) + b_2\alpha^2(t) + c_2\alpha(t) + d_2)\delta_{fin}(t) \tag{2}$$

$$C_{m_0}(\alpha(t)) = a_3\alpha^4(t) + b_3\alpha^3(t) + c_3\alpha^2(t) + d_3\alpha(t)$$

$$C_{m_\delta}(\alpha(t)) = (a_4\alpha^3(t) + b_4\alpha^2(t) + c_4\alpha(t) + d_4)\delta_{tvc}(t) \tag{3}$$

where the coefficients a_i, b_i, c_i, d_i of the function are constants obtained from curve-fitting of aerodynamic data.

3. Control Allocation

Control allocation is to determine the amount of the deflection of each control effector to achieve the guidance command. In this paper, dynamic inversion is used as control

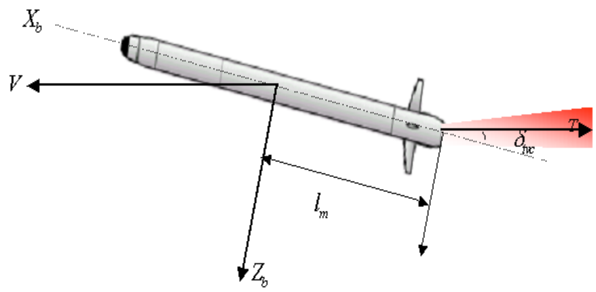

Fig. 2. Agile missile with aerodynamic fin and tvc

allocation strategy. Dynamic inversion is a technique which feedback is used to linearize the system to be controlled and to provide desired dynamic response [3]-[6]. Dynamic inversion of this paper is used to provide the desired angle of attack of the missile which is selected as the controlled output. From Eqs. (1)-(2), the angle of attack dynamic is

$$\begin{aligned}\dot{\alpha}(t) &= \frac{\frac{1}{2}\rho V^2 S}{mV}C_{Z_0}(\alpha(t)) + q(t) \\ &\quad + \frac{\frac{1}{2}\rho V^2 S}{mV}C_{Z_\delta}(\alpha(t))\bar{\delta}_{fin}(t) + \frac{T}{mV}\bar{\delta}_{tvc}(t) \\ &= \frac{\frac{1}{2}\rho V^2 S}{mV}C_{Z_0}(\alpha(t)) + q(t) \\ &\quad + F_a(t)\bar{\delta}_{fin}(t) + F_t(t)\bar{\delta}_{tvc}(t) \end{aligned} \tag{4}$$

where $F_a(t)$ and $F_t(t)$ are the control distribution functions for each control effector, aerodynamic fin and thrust vectoring control, respectively. The force which makes angle of attack have the desired dynamics can be given by

$$F_d(t) = \dot{\alpha}_d(t) - (\frac{\frac{1}{2}\rho V^2 S}{mV}C_{Z_0}(\alpha(t)) + q(t)) \tag{5}$$

where $\dot{\alpha}_d(t)$ is desired angle of attack dynamics and is given by $\dot{\alpha}_d(t) = \omega(\alpha_{cmd}(t) - \alpha(t))$, where $\alpha_{cmd}(t)$ and $\alpha(t)$ are angle of attack command, and measured(estimated) angle of attack, respectively, and ω is design parameter.

Last two terms of Eq. (4) mean the force generated by aerodynamic fin and thrust vectoring control, and Eq. (5) means the force required to achieve the desired angle of attack dynamics. Therefore, for angle of attack to accomplish the desired command, the following equality must be satisfied with

$$\begin{aligned}F_d(t) &= F_a(t)\bar{\delta}_{fin}(t) + F_t(t)\bar{\delta}_{tvc}(t) \\ &= \begin{bmatrix} F_a(t) & F_t(t) \end{bmatrix}\begin{bmatrix} \bar{\delta}_{fin}(t) \\ \bar{\delta}_{tvc}(t) \end{bmatrix} \\ &= g(\underline{x})\underline{u}(t) \end{aligned} \tag{6}$$

where $g(\underline{x})$ is (1×2) control distribution matrix, and $\underline{u}(t)$ is (2×1) control input vector.

From Eq. (6), the amount of the deflection of each control effector can be determined by matrix inversion as follows:

$$\begin{bmatrix} \bar{\delta}_{fin}(t) \\ \bar{\delta}_{tvc}(t) \end{bmatrix} = g^{-1}(\underline{x})F_d(t) \tag{7}$$

where $g^{-1}(\underline{x})$ is not unique because of rank redundancy. Hence the control allocation function of each control effector can be obtained from using the pseudo-inverse property minimizing the following object function:

$$\min J = \underline{u}^T(t)W(t)\underline{u}(t) \quad \text{subject to } g(\underline{x})\underline{u}(t) = v(t) \tag{8}$$

where $\underline{u}(t)$, $W(t)$, $v(t)$ are control input vector, positive definite symmetric weighting matrix, and scalar pseudocontrol, respectively. The pseudocontrol $v(t)$ is distributed in such a way that the weighted energy of the actual control input $\underline{u}(t)$ is minimized. The above optimization problem has an explicit solution which can be using several technique. But, by using the Lagrange multipliers, the optimal $u(t)$ is given by

$$\underline{u}(t) = [W^{-1}(t)g^T(\underline{x})\{g(\underline{x})W^{-1}(t)g^T(\underline{x})\}^{-1}]v(t). \tag{9}$$

In Eq. (9), the effective control allocation algorithm can be designed by adjusting the weighting matrix $W(t)$ according to the flight conditions. Applying Eq. (9) to the given angle of attack dynamics Eq. (4) results in the following nominal control trajectories:

$$\begin{bmatrix} \bar{\delta}_{fin}(t) \\ \bar{\delta}_{tvc}(t) \end{bmatrix} = \begin{bmatrix} \frac{F_a(t)}{(F_a(t))^2 + (\frac{w_1(t)}{w_2(t)})(F_t(t))^2} \\ \frac{(\frac{w_1(t)}{w_2(t)})F_t(t)}{(F_a(t))^2 + (\frac{w_1(t)}{w_2(t)})(F_t(t))^2} \end{bmatrix} \times F_d(t) \tag{10}$$

where $w_1(t)$, $w_2(t)$ are the weighting values of each control effector, respectively.

4. LTV Control Techniques

In this section, the time-varying eigenvalue concepts, SD- and PD-eigenvalue, are introduced into the Extended-Mean Assignment, which is the time-varying version of pole placement for LTI system, and the time-varying bandwidth command shaping filter, which effectively reduces the actuator rate while maintaining good tracking response for both smooth and abrupt command trajectories.

4.1. EMA Control

The EMA synthesis control technique is exemplified here with a generic second-order LTV system

$$\ddot{y}(t) + p_2(t)\dot{y}(t) + p_1(t)y(t) = u(t) \tag{11}$$

This LTV system can be written in an operator form $\mathcal{D}_p\{y(t)\} = u(t)$, where

$$\begin{aligned} \mathcal{D}_p &= D^2 + p_2(t)D + p_1(t) \\ &= (D - \lambda_2(t))(D - \lambda_1(t)) \\ &= D^2 - [\lambda_1(t) + \lambda_2(t)]D + \lambda_1(t)\lambda_2(t) - \dot{\lambda}_1(t) \end{aligned} \tag{12}$$

is known as a polynomial differential operator and the factorization is known as Cauchy-Floquet factorization. The scalar functions $\lambda_1(t)$ and $\lambda_2(t)$ are called SD-eigenvalues for the LTV system (11), and $\rho_1(t) = \lambda_1(t)$, $\rho_2(t) = \lambda_1(t) + \dot{q}(t)q^{-1}(t)(q(t) = \int e^{\int(\lambda_2(t)-\lambda_1(t))dt})$ are called PD-eigenvalues [7]-[10].

Now define the Extended-Mean(EM) value of an integrable function $\sigma(t)$ by

$$\text{EM}\{\sigma(t)\} = \limsup_{T\to\infty,\, t_0\geq 0} \frac{1}{T}\int_{t_0}^{t_0+T} \sigma(\tau)d\tau. \tag{13}$$

Then the LTV sytem (11) with the bounded piecewise smooth coefficients $p_i(t)$ is exponentially stable for all $t_0 \geq 0$ if $\mathcal{D}_p$ has a bounded SD-eigenvalues $\{\lambda_1(t), \lambda_2(t)\}$ with EM values in the left half plane(LHP); i.e., for some $M > 0$,

$$|\lambda_i(t)| < M, \quad \text{EM}\{\text{Re}[\lambda_i(t)]\} < 0, \quad i = 1, 2 \tag{14}$$

This statement follows from a necessary and sufficient criterion for exponential stability of a LTV system based on the EM of PD-eigenvalues given in [11]. If the LTV system (11) is unstable, a feedback control law

$$u(t) = k_1(t)y(t) + k_2(t)\dot{y}_2(t) \tag{15}$$

can be synthesized so that SD-eigenvalues $\gamma_1(t)$ and $\gamma_2(t)$ of the closed-loop system $\mathcal{D}_h\{y(t)\} = 0$, where

$$\begin{aligned} \mathcal{D}_h &= D^2 + h_2(t)D + h_1(t) \\ &= (D - \gamma_2(t))(D - \gamma_1(t)) \end{aligned} \tag{16}$$

has the desired EM values in the LHP.

Now implementing the control law (15) on the LTV plant (11) and comparing coefficients with the desired closed-loop system (16) yield

$$h_i(t) = a_i(t) - k_i(t). \tag{17}$$

Because $h_i(t)$ are related to $\gamma_i(t)$ by

$$\begin{aligned} h_1(t) &= \gamma_1(t)\gamma_2(t) - \dot{\gamma}_1(t) \\ h_2(t) &= -[\gamma_1(t) + \gamma_2(t)], \end{aligned} \tag{18}$$

the feedback control gains $k_i(t)$ can then be synthesized as

$$\begin{aligned} k_1(t) &= p_1(t) + \dot{\gamma}_1(t) - \gamma_1(t)\gamma_2(t) \\ k_2(t) &= p_2(t) + \gamma_1(t) + \gamma_2(t). \end{aligned} \tag{19}$$

4.2. EMA Autopilot Design

Let

$$\xi(t) = \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} = \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix} \tag{20}$$

be the state vector of the missile. Then, from Eq. (1), the state equation is given by

$$\dot{\xi}(t) = f(\xi(t), \delta_{fin}(t)) = \begin{bmatrix} f_1(\xi_1(t), \xi_2(t), \delta_{fin}(t) \\ f_2(\xi_1(t), \xi_2(t), \delta_{fin}(t) \end{bmatrix}. \tag{21}$$

Now, for a given angle of attack command $\alpha_{cmd}(t)$, let $\bar{\delta}_{fin}(t)$ be the nominal fin deflection and $\bar{\xi}(t)$ be the nominal state trajectory such that

$$\dot{\bar{\xi}}(t) = f[\bar{\xi}(t), \bar{\delta}_{fin}(t)]. \tag{22}$$

Define the tracking errors by

$$x(t) = \xi(t) - \bar{\xi}(t), \quad (23)$$

and the tracking error control by

$$v(t) = \delta(t) - \bar{\delta}_{fin}(t). \quad (24)$$

Then the linearized tracking error dynamics is given by

$$\dot{x}(t) = A(t)x(t) + B(t)v(t) \quad (25)$$

where

$$A(t) = \left.\frac{\partial f}{\partial \xi}\right|_{\bar{\xi},\bar{\delta}_{fin}} = \begin{bmatrix} a_{11}(t) & 1 \\ a_{21}(t) & a_{22}(t) \end{bmatrix},$$
$$B(t) = \left.\frac{\partial f}{\partial \delta}\right|_{\bar{\xi},\bar{\delta}_{fin}} = \begin{bmatrix} b_1(t) \\ b_2(t) \end{bmatrix}. \quad (26)$$

The autopilot design task amounting to finding a control law such that the tracking error becomes zero exponentially for any admissible angle of attack command. This can be achieved using an EMA controller. However, to use prototype EMA controller, it is necessary to transform the linearized tracking error dynamics into the phase-variable canonical form. This can be done via Silverman's coordinate transformation, provided that $[A(t), B(t)]$ is uniformly completely controllable [12]. Whereas this approach will result in a minimal realization, the resulting system coefficients are very complicated. To simplify the matter, a nonminimal realization is adopted, which yields a phase-variable canonical form with very simple coefficients. To that end, apply the state coordinate transformation

$$x(t) = L(t)z(t) \quad (27)$$

where $L(t)$ is a time-varying coordinate transformation matrix given by

$$L(t) = \begin{bmatrix} 1 & 0 \\ -a_{11}(t) & 1 \end{bmatrix}. \quad (28)$$

Then the linearized system (25) in the $z(t)$ coordinates becomes

$$\dot{z}(t) = A_c(t)z(t) + B_c(t)v(t) \quad (29)$$

where $A_c(t) = L^{-1}(t)[A(t)L(t) - \dot{L}(t)]$ is of the companion form

$$A_c(t) = \begin{bmatrix} 0 & 1 \\ -p_1(t) & -p_2(t) \end{bmatrix} \quad (30)$$

with $-p_1(t) = \dot{a}_{11}(t) + a_{21}(t) - a_{11}(t)a_{22}(t)$, and $-p_2(t) = a_{11}(t) + a_{22}(t)$, and

$$B_c(t) = \begin{bmatrix} b_1(t) \\ a_{11}(t)b_1(t) + b_2(t) \end{bmatrix}. \quad (31)$$

Note that $z_1(t) = x_1(t) = \alpha(t) - \alpha_{cmd}(t)$. By eliminating $z_2(t)$ from Eq. (29), it is seen that this state equation is equivalent to a scalar equation

$$\ddot{z}_1(t) + p_2(t)\dot{z}_1(t) + p_1(t)z_1(t) =$$
$$b_1(t)\dot{v}(t) + (\dot{b}_1(t) + b_2(t) - a_{22}(t)b_1(t))v(t) \quad (32)$$

To render this equation into the phase-variable form, the angle of attack zero dynamics is introduced as follows:

$$\dot{v}(t) + \frac{\dot{b}_1(t) + b_2(t) - a_22(t)b_1(t)}{b_1(t)}v(t) = \frac{1}{b_1(t)}u(t) \quad (33)$$

Combining Eqs. (32) and (33) yields the desired form

$$\ddot{z}_1(t) + p_2(t)\dot{z}_1(t) + p_1(t)z_1(t) = u(t) \quad (34)$$

Now an EMA control law $u(t)$ can be designed for the angle of attack tracking error dynamics (34) using the outlined in section 4.1.

4.3. Time-Varying Bandwidth Filter

In practice, fast tracking performance is always constrained by the physical limit of the actuator rate. A common practice in coping with this dilemma is to use an actuator rate limiter. A major drawback of this method is that the system becomes unpredictable when the actuator rate is saturated. It may result in limit cycle, or even in instability.

An alternative approach is to use a tracking command shaping filter. The filter should greatly reduce the rate of abrupt command trajectory, whereas it should have little effect on smooth trajectories that can be tracked within the actuator limits. These two requirements cannot be achieved with a fixed-parameter filter.

Taking advantage of the EM stability criterion, a novel second-order LTV filter with a variable bandwidth is designed to deal with the conflicting for a command shaping filter [13]. Let $c_{in}(t)$ be the guidance command to be tracked bythe missile autopilot, and $c_{out}(t)$ be the command shaping output. Then the governing equation for the filter is given by

$$\ddot{c}_{out}(t) + [2\zeta\omega_n(t) - \frac{\dot{\omega}_n(t)}{\omega_n(t)}]\dot{c}_{out}(t)$$
$$+\omega_n^2(t)c_{out}(t) = c_{in}(t) \quad (35)$$

where ζ is a constant damping coefficient and $\omega_n(t) > 0$ determine the effective bandwidth which is adjusted by the TVB(Time-Varying Bandwidth) logic. The TVB logic consists of two parts. (i) The command shaping logic given by

$$r_\omega(t) = \bar{\omega}_n - a \cdot \text{sat}\{b \cdot |\text{ddzone}[\ddot{c}_{out}(t)]|\} \quad (36)$$

where sat$(\cdot)$ is the saturation function defined by

$$\text{sat}(x) = \begin{cases} -c & x < -c \\ x & |x| \le c \\ c & x > c \end{cases}, \quad (37)$$

and ddzone$(\cdot)$ is the dead-zone function defined by

$$\text{ddzone}(x) = \begin{cases} x + d & x < -d \\ 0 & |x| \le d \\ x - d & x > d \end{cases}. \quad (38)$$

(ii) The filter bandwidth profile $\omega_n(t)$ and its rate $\dot{\omega}_n(t)$ generator

$$\ddot{\omega}_n(t) + 2\zeta_0\omega_0\dot{\omega}_n(t) + \omega_0^2\omega_n(t) = \omega_0^2 r_\omega(t) \quad (39)$$

which smoothes out the crude bandwidth profile reference $r_\omega(t)$. There are several constant design parameters in the TVB logic which determine the behavior and performance of the command shaping functionality. In Eq. (36), the parameter $\bar{\omega}_n$ is the nominal (maximum) bandwidth, and the minimum bandwidth is determined by $\bar{\omega}_n - a$. The threshold for the TVB mechanism to be activated by $\ddot{c}_{out}(t)$ is set by d, and b affects the promptness of the activation. The damping coefficient ζ_0 and natural frequency ω_0 of the LTI smoothing filter (39) determine the promptness of the activation of the TVB mechanism. The design parameters are determined empirically.

5. Simulation Results

Simulations are performed to validate the proposed schemes. The parameters of the missile considered in this paper and the flight conditions are shown in the Table 1 and Table 2, respectively.

Table 1. Parameters of missile.

$T = 13800N$	$S = 0.826m$	$C = 0.15kg/m^3$
$m = 384.7kg$	$I_{yy} = 692.3$	$l_m = 2.0m$
$\delta_{fin,max} = 30°$	$\delta_{tvc,max} = 5°$	$\tau_{fin} = 150\ 1/s$
$\tau_{tvc} = 150\ 1/s$		

Table 2. Flight conditions.

$M = 0.95$	$H = 1000m$	$\rho = 1.112kg/m^3$
$V_s = 336.4m/s$		

Fig.3 shows a 3-s piecewise constant angle of attack tracking command, TVB filtered command, and the angle of attack output. As shown in Fig. 3, the angle of attack EMA control provides remarkable results for the piecewise constant step command. Fig. 4 shows the pitch rate. The allocated control inputs to achieve the angle of attack command are depicted in Fig. 5. It shows that the dynamic inversion effectively generates the nominal control inputs. Fig. 6 shows the time-varying control gains $k_1(t)$ and $k_2(t)$ by EMA control.

6. Conclusions

In this paper, the control allocation algorithm using dynamic inversion and weighted pseudo-inverse property, and autopilot design of a agile missile angle of attack using the EMA control technique are presented. The features of the proposed schemes include (1)effective control allocation for each control effector(aerodynamic fin, thrust vectoring control) to achieve the angle of attack command (2)good tracking performance for angle of attack command without scheduling of any constant design parameters throughout a wide range of angle of attack (3)time-varying control gains to improve the robustness for the model uncertainty (4)TVB command shaping filter that effectively reduces the actuator rate while maintaining good tracking response for both smooth and abrupt trajectories. The proposed schemes are validated by nonlinear simulations.

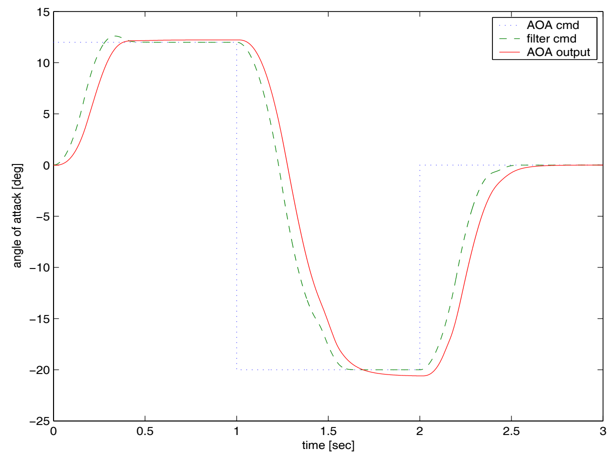

Fig. 3. Command tracking performance

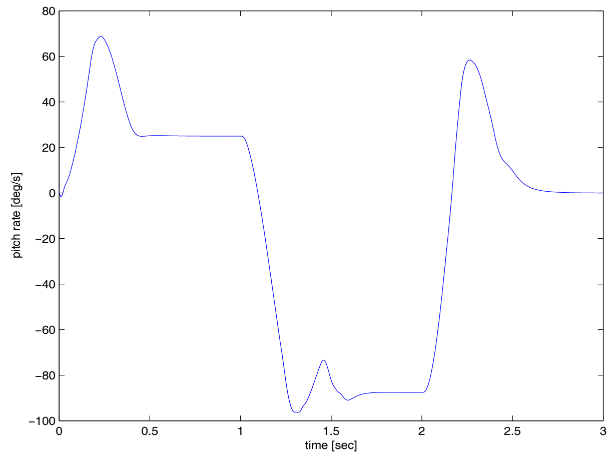

Fig. 4. Pitch rate output

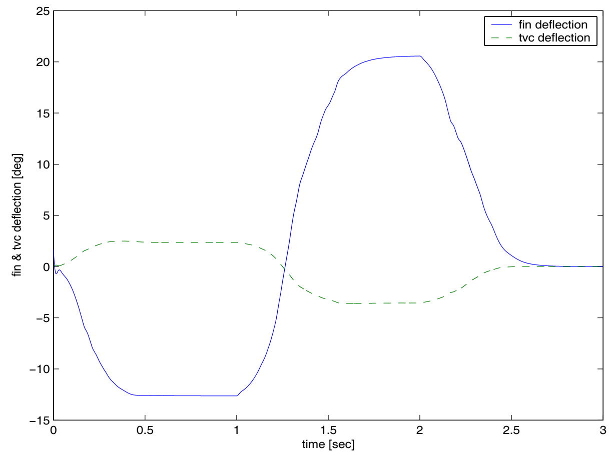

Fig. 5. Fin and tvc deflection time history

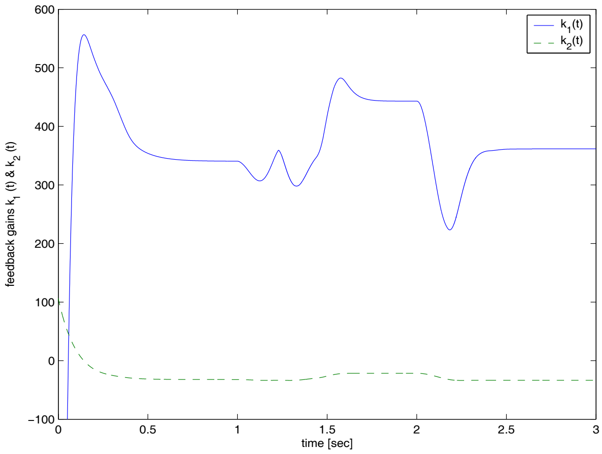

Fig. 6. Time-varying feedback gains

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