3-D Optimal Evasion of Air-to-Surface Missiles against Proportionally Navigated Defense Missiles

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Abstract: In this paper, we investigate three dimensional optimal evasive maneuver patterns for air-to-surface attack missiles against proportionally navigated anti-air defense missiles. Interception error of the defense missile can be generated by evasive maneuver of the attack missile during the time of flight for which the defense missile intercepts the attack missile. Time varying weighted sum of the inverse of these interception errors forms a performance index to be minimized. Direct parameter optimization technique using CFSQP is adopted to get the attack missile's optimal evasive maneuver patterns according to parameter changes of both the attack missile and the defense missile such as maneuver limit and time constant of autopilot approximated by the 1st order lag system. The overall shape of resultant optimal evasive maneuver to enhance the survivability of air-to-surface missiles against proportionally navigated anti-air missiles is a kind of deformed barrel roll.

Keywords: Evasive maneuver, Survivability, Optimization, Deformed barrel-roll

1. INTRODUCTION

Rapid development of recent technology of anti-air defense systems has menaced air-to-surface missiles' survivability. For the enhancement of the survivability of air-to-surface missiles, we can consider a method to augmenting special evasive maneuver into the terminal homing phase. Evasive maneuver of air-to-surface missiles can be defined as a special maneuver not only to increase survivability but also to minimize terminal miss distance without a priori information of the parameters or flight states of the threats.

The weaving motion[1] in two-dimensional space or the barrel roll maneuver[2] in three-dimensional can be a candidate for evasive maneuver. Then, are they optimal? If not, what kind of maneuver pattern is the best for evasion? In Ref. [3], an optimal evasive maneuver policy of anti-ship missiles against the CIWS(Close-In Weapon System) has been discussed, where the optimal evasive trajectories are characterized by a kind of barrel roll. However, it may not be the best for the case of homing threats.

Based on the approach of Ref.[3], in this paper, we investigate three dimensional optimal evasive maneuver patterns for air-to-surface attack missiles against proportionally navigated anti-air defense missiles. The optimal control problem for evasive maneuver of attack missiles considered in this paper is to find an acceleration command which minimizes the performance index given by the time varying weighted sum of the inverse of an interception error of the defense missile with the terminal constraints of zero miss distances. Interception error of the defense missile during the entire engagement can be calculated from the homing loop adjoint [4] of the defense missile. It is assumed that both attack and defense missiles are the 1st order lag systems with different time constant. And we assume that only the attack missile has a command limit. In this paper, the direct input parameter optimization technique using CFSQP [5] is used to find the optimal solution. Optimization results show that optimal evasive trajectory also becomes a kind of the barrel roll whose shape varies according to the time constants of both missiles as well as navigation constant of the defense missile.

The equations of motion of the air-to-surface attack missile and the interception error of the anti-air defense missile in three-dimensional space are formulated in section 2. Section 3 deals with the optimal control problems and their numerical solutions for a typical engagement scenario. And section 4 is conclusion.

2. FORMULATION OF EQUATIONS OF MOTION AND INTERCEPTION ERRORS

Let us consider the engagement scenario between a air-to-surface attack missile and a anti-air defense missile as shown in Fig. 1. In this scenario, it is assumed that the attack missile controls acceleration vector $\vec{a}_m$ normal to its velocity to guide to the target. On the other hand, the target continuously launches anti-air missiles to intercept the attack missile.

Equations of motion of the attack missile

The equations of motion of the attack missile in the three dimensional space are given by

$$\begin{aligned}\dot{\vec{r}}_m = \vec{v}_m,\ \vec{r}_m(t_0) = \vec{r}_{m0} \\ \dot{\vec{v}}_m = \vec{a}_m,\ \vec{v}_m(t_0) = \vec{v}_{m0}\end{aligned} \tag{1}$$

where $r_m(t)$ is denotes the distance of the attack missile from the target, v_m is the velocity of the attack missile. The guidance command vector $\vec{u}_c^V$ is defined in velocity frame as denoted in Fig. 2;

$$\vec{u}_c^V = \begin{bmatrix} 0 & u_{yaw} & u_{pth} \end{bmatrix}^T \tag{2}$$

and must be normal to $\vec{v}_m$ such that

$$\vec{u}_c^V \bullet \vec{v}_m = 0 \tag{3}$$

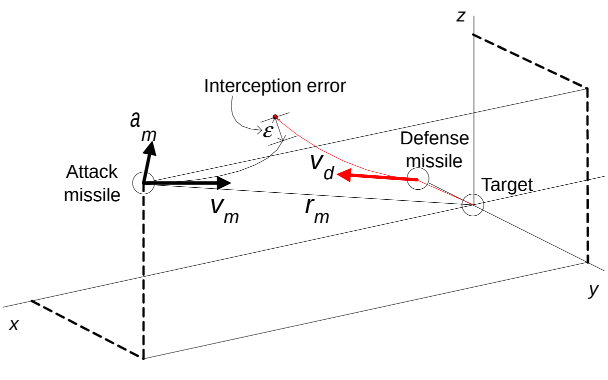

Fig. 1 Three dimensional engagement geometry between the attack missile and the defense missile

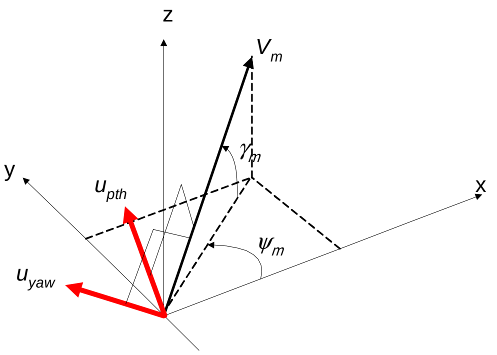

Fig. 2 Definition of guidance command vector

The autopilot of the attack missile is assumed as the first order lag system so that the resultant acceleration $\vec{a}_m^V$ is represented as

$$\dot{\vec{a}}_m^V = \frac{1}{T_a}(\vec{u}_c - \vec{a}_m^V) \tag{4}$$

where T_a denotes the the time constant of the attack missile. Then, $\vec{a}_m^V$ is transformed into inertial reference frame such that

$$\vec{a}_m = \left[C_I^V\right]^T \vec{a}_m^V \tag{5}$$

where C_I^V is calculated by using azimuth ψ_m and elevation angle γ_m of the flight path:

$$C_I^V = T_y(-\gamma_m)T_z(\psi_m) = \begin{bmatrix} c\gamma_m c\psi_m & c\gamma_m s\psi_m & s\gamma_m \\ -s\psi_m & c\psi_m & 0 \\ -s\gamma_m c\psi_m & -s\gamma_m s\psi_m & c\gamma_m \end{bmatrix} \tag{6}$$

where

$$\psi_m = \tan^{-1}\frac{v_y}{v_x}, \qquad \gamma_m = \tan^{-1}\frac{v_z}{\sqrt{v_x^2+v_y^2}} \tag{7}$$

<u>Interception error of the defense missile</u>

we assume that neither gravity nor aerodynamic forces affect the ballistics of the defense missiles. Then, the speed of the defense missile $v_d (= |\vec{v}_d|)$ remains in constant during the entire engagement. And we also assume the attack missile lies on the collision path so that it does not deviate much from the reference x-axis. Then, we approximate the interception time for the defense missile to intercept the attack missile as

$$\tau(t) \approx \frac{r_m(t)}{v_m + v_d} = \frac{r_m(t)}{v_c} \tag{8}$$

where v_c denotes the closing velocity between the attack missile and the defense missile. For homing missiles, it is intuitively true that τ is an implicit function of time and has its maximum value at $t=0$ and then monotonically decreases until becoming 0.

System lag and command limit are the major factors to cause miss distance of the defense missile. In most cases, the command limit of the defense missile can be neglected since it is enough high compared to that of the attack missile. Therefore, the interception error of the defense missile is caused by the evasive maneuver of the attack missile during the interception time. We can calculate the interception error of the defense missile using the method of adjoint [4]. If the defense missile is guided by PNG(Proportional Navigation Guidance), the homing loop adjoint can be represented as shown in Fig. 3.

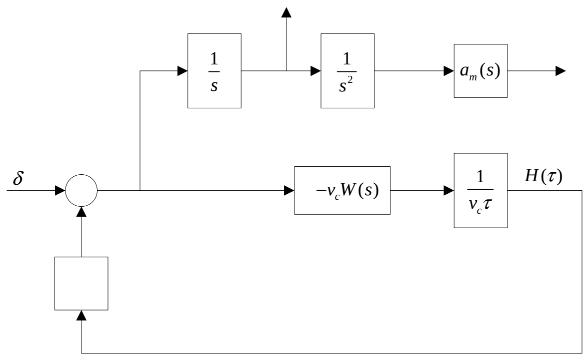

Fig. 3 Generalized homing loop adjoint of a defense missile

The guidance system of the defense missile is represented in the time domain by $W(\tau)$. A single-lag guidance system can be represented by

$$W(s) = \frac{N'}{s(1+sT_d)} \tag{9}$$

where T_d is the effective time constant of the guidance system and N' is the navigation constant. And $H(\tau)$ denotes an adjoint signal of interest and calculated as

$$H(\tau) = \frac{1}{\tau}\int W(x)[\delta(\tau - x) - H(\tau - x)]dx \tag{10}$$

Converting from the time domain to the frequency domain using Laplace transform, we can express Eq. (10) as

$$\frac{-dH(s)}{ds} = W(s)[1 - H(s)] \tag{11}$$

Recall also that

$$\frac{d}{ds}[1 - H(s)] = \frac{-dH(s)}{ds} \tag{12}$$

Substitute Eq. (12) into Eq. (11) and take integral to obtain

$$\int \frac{d(1 - H(s))}{1 - H(s)} = \int W(s)ds \tag{13}$$

Then, Eq. (13) becomes

$$1 - H(s) = \exp\left(\int W(s)ds\right) \tag{14}$$

Now let us find the miss due to a step attack missile maneuver for a single lag guidance system. We can obtain new expression of Eq. (14) by substituting Eq. (9) into Eq. (14);

$$\begin{aligned} 1 - H(s) &= \exp\left(\int \frac{N'}{s(1+sT_d)}ds\right) \\ &= \left[s \middle/ \left(s + \frac{1}{T_d}\right)\right]^{N'} \end{aligned} \tag{15}$$

Then, from the Fig. 3, the interception error of the defense missile due to the evasive maneuver of the attack missile is given by

$$\begin{aligned} MNM(s) &= \frac{1 - H(s)}{s^3} a_m(s) \\ &= \frac{1}{s^3}\left[s \middle/ \left(s + \frac{1}{T_d}\right)\right]^{N'} a_m(s) \end{aligned} \tag{16}$$

If $F(s)$ is defined as

$$F(s) = \frac{1 - H(s)}{s^3} = \frac{1}{s^3}\left[s \middle/ \left(s + \frac{1}{T_d}\right)\right]^{N'} \tag{17}$$

then, the interception error in time domain can be obtained by taking the inverse Laplace transform of Eq. (16);

$$\begin{aligned} MNM(\tau) &= \mathcal{L}^{-1}\{MNM(s)\} \\ &= \mathcal{L}^{-1}\{F(s)a_m(s)\} \\ &= \int_0^\tau f(x)a_m(\tau - x)dx \\ &= f * a_m \end{aligned} \tag{18}$$

where f is the inverse Laplace transform of F and " * " denotes the convolution operator. And, τ is the adjoint time of the defense missile and can be interpreted the time of flight for the defense missile to intercept the attack missile.

For each navigation constant, $F(s)$ and its inverse Laplace transform $f(x)$ are given by

For $N' = 3$

$$F\big|_{N'=3}(s) = \frac{1}{\left(s + \frac{1}{T_d}\right)^3} \tag{19}$$

$$f\big|_{N'=3}(x) = 0.5x^2 e^{-x/T_d} \tag{20}$$

For $N' = 4$

$$F\big|_{N'=4}(s) = \frac{s}{\left(s + \frac{1}{T_d}\right)^4} \tag{21}$$

$$f\big|_{N'=4}(x) = x^2 e^{-x/T_d}\left(0.5 - \frac{x}{6T_d}\right) \tag{22}$$

For $N' = 5$

$$F\big|_{N'=5}(s) = \frac{s^2}{\left(s + \frac{1}{T_d}\right)^5} \tag{23}$$

$$f\big|_{N'=5}(x) = x^2 e^{-x/T_d}\left(0.5 - \frac{x}{3T} + \frac{x^2}{24T_d^{\,2}}\right) \tag{24}$$

Finally, by substituting Eq. (20), (22) and (24) into Eq. (18), we can obtain the miss distances for adjoint time τ, which is caused by the evasive maneuver of the attack missile;

$$\begin{aligned} \varepsilon\big|_{N'=3}(\tau) &= MNM\big|_{N'=3}(\tau) \\ &= \int_0^\tau \left(0.5x^2 e^{-x/T_d}\right) a_m(\tau - x)dx \end{aligned} \tag{25}$$

$$\begin{aligned} \varepsilon\big|_{N'=4}(\tau) &= MNM\big|_{N'=4}(\tau) \\ &= \int_0^\tau \left(x^2 e^{-x/T_d}\left(0.5 - \frac{x}{6T_d}\right)\right) a_m(\tau - x)dx \end{aligned} \tag{26}$$

$$\begin{aligned} \varepsilon\big|_{N'=5}(\tau) &= MNM\big|_{N'=5}(\tau) \\ &= \int_0^\tau \left(x^2 e^{-x/T_d}\left(0.5 - \frac{x}{3T_d} + \frac{x^2}{24T_d^{\,2}}\right)\right) a_m(\tau - x)dx \end{aligned} \tag{27}$$

Note that the interception error ε becomes 0 as τ goes to 0 and if the attack missile does not maneuver, then the interception error is always zero. We also note that for non-zero $a_m(t)$ for $0 \le t \le t_f$, by using the convolution integral of Eqs. (25), (26) and (27) we can calculate the interception errors for the adjoint time without multiple run of nonlinear simulation. Here, t_f denotes the flight time of the attack missile to intercept the target. These analytic expressions on the interception error, then, can be used to assess the evasive performance or to evaluate the cost of survivability of the attack missile. In following section, an optimal control problem for evasive maneuver of the attack missile against the PNG guided defense missile will be discussed.

3. 3-D OPTIMAL EVASIVE MANEUVER PROBLEM

Now, let us consider following optimal control problem;

Find $\vec{u}_c$ *which minimizes*

$$J = \int_0^{t_f} \frac{\tau^2(t)}{\varepsilon(t)} dt \tag{28}$$

subject to Eq. (1)
with terminal constraint

$$\vec{r}_m(t_f) = 0 \tag{29}$$

and inequality input constraint

$$|\vec{u}_c| \le U_c \tag{30}$$

where U_c denotes the maximum permissible acceleration command. Since the interception error ε converges to 0 as the defense missile approaches to the attack missile, the cost function becomes so large in the terminal flight phase that a lot of numerical effort to minimize the cost may be concentrated more on this phase than on initial/midcourse phase. For the stability of the solution finding algorithm, $\tau^2(t)$ is considered into the performance index to reduce the large weighting effect due to the inverse of the interception error in the terminal phase. The closed-form solutions of this optimal control problem might not be easily derived due to the nonlinearities included in the performance index and the inequality constraints. To find the policy of three dimensional evasive maneuver of the air-to-surface missile against proportionally navigated the anti-air missile, we should adopt numerical optimization techniques to solve the problem. This optimal control problem is converted into parameter optimization problem with unknown parameter vector composed of discretized control and flight time such that

$$X = \left[u_{yaw}(i),\ u_{pth}(i),\ t_f\right]^T, \quad i = 0, ..., N \tag{31}$$

Therefore, the number of unknown parameter is $2(N+1)+1$. As a tool for parameter optimization, CFSQP [5] which is an open code for constrained optimization problems based on sequential quadratic programming is used. Integration of equations of motion to evaluate the value of performance index and the violation of terminal constraints is performed by the 4th order Runge-Kutta method. During integration, the controls $u_{yaw}(t)$ and $u_{pth}(t)$ are assumed to be linearly changed between adjacent nodes.

The initial conditions of engagement scenario between the attack missile and the defense missile are given as

1) For the attack missile;

At the beginning of the evasive maneuver, it is assumed that the attack missile lies on the near collision path to target so that the initial acceleration is very small.

$$\vec{r}_m(t_0) = [4000 \quad 0 \quad 0]^T \quad (m)$$

$$\vec{v}_m(t_0) = [-300 \quad 0.1 \quad 0.1]^T \quad (m/s)$$

$$\vec{a}_m^V(t_0) = [0 \quad 0.1 \quad 0.1]^T \quad (m/s^2)$$

Guidance command is highly limited and realized by the 1st order lag approximation;

$$U_c = 30 \ \ (m/s^2) \ (\approx 3g)$$

$$T_a = 1.0 \ \ (rad/s)$$

2) For the defense missile;

We assume that the defense missile launches at the target and its initial velocity is more than twice when it is compared to that of the attack missile. It is also assumed that velocity of the defense missile does not vary during the entire engagement.

$$\vec{r}_d(t_0) = [0 \ \ 0 \ \ 0]^T \ \ (m)$$

$$\vec{v}_d = 700 \ \ (m/s)$$

Time constant of the defense missile is the same as the attack missile.

$$T_d = 1.0 \ \ (rad/s)$$

It is assumed that the defense missile has perfect measurement on the attack missile and no command limit.

Based on the above engagement conditions, we investigate the three-dimensional optimal evasion of the attack missile and its evasive performance for the case of that the navigation constant of the defense missile is 3 and 4, i.e., $N' = 3$ and $N' = 4$. Fig. 4 and 5 show the optimal trajectories of the attack missile to maximize its survivability for both navigation constants of the defense missile, respectively. Both trajectories have kinds of deformed barrel roll due to homing characteristics of the attack missile in the last part of the flight. The evasive maneuver of the attack missile for $N' = 4$ is more radical and forms larger lateral displacement. Although lateral trajectory displacement is larger, the interception error for the defense missile with $N' = 4$ is rather small as shown in Fig. 9 and 10. For both cases, the interception error approaches to zero as the attack missile approaches to the target. Performance index for both navigation constants is given as

$$J\,|_{N'=3} = 1.170e3 \quad , \quad J\,|_{N'=4} = 3.513e3$$

From the optimization results, it is obvious that the survivability of the attack missile is weakened for the attack missile with higher navigation constant. In general, PNG shows that higher navigation constant is more effective for maneuvering targets. However, PNG with higher navigation constant tends to generate larger guidance command. Not considered in this paper, if the guidance command of the defense missile is limited, the interception error will be increased. In this case, we cannot assert that higher navigation constant of the defense missile always deteriorates the survivability of the attack missile.

4. CONCLUSION

In this paper, three-dimensional optimal evasive maneuver of air-to-surface attack missiles against proportionally navigated anti-air defense missiles are investigated. We use the homing loop adjoint of the defense missiles to generate the interception error of the defense missiles. And then we apply the direct input parameter optimization technique using CFSQP to minimize the performance index given by the time varying weighted sum of the inverse of the interception error. Numerical results for typical navigation constants of the defense missile show that the survivability of the attack missile is weakened as the navigation constant of the defense missile is increased. Optimal evasive trajectories of the attack missile are kinds of deformed barrel roll shape due to the homing characteristics.

Optimization for various combinations of time constant of both attack and defense missile under the consideration of the command limit of the defense missile should be carried out as a further study.

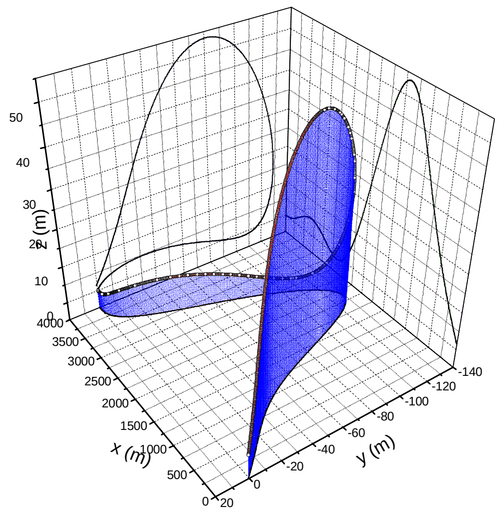

Fig. 4. Optimal trajectory of the attack missile against the defense missile with PNG with $N' = 3$

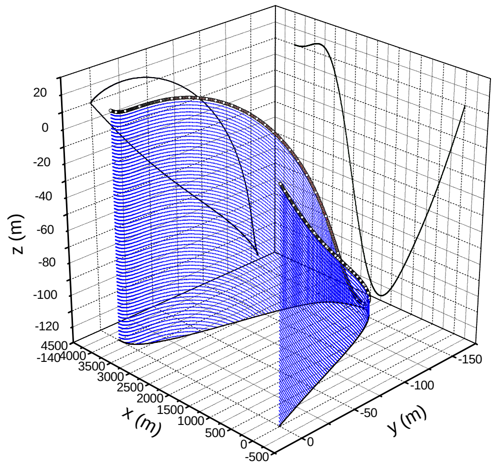

Fig. 5. Optimal trajectory of the attack missile against the defense missile with PNG with $N' = 4$

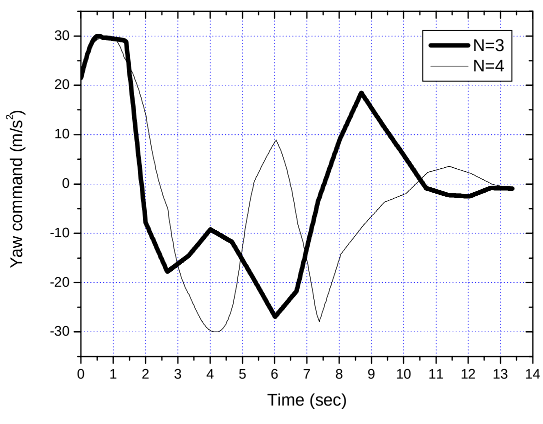

Fig. 6. Yaw command histories of the attack missile against the defense missile with PNG

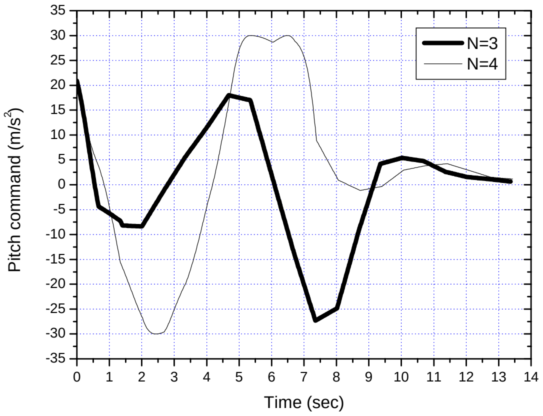

Fig. 7. Pitch command histories of the attack missile against the defense missile with PNG

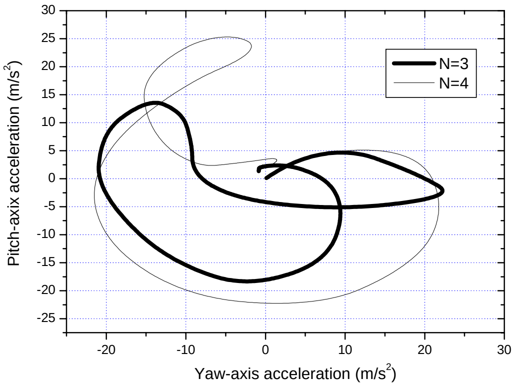

Fig. 8 Projected acceleration patterns of the attack missile against the defense missile with PNG

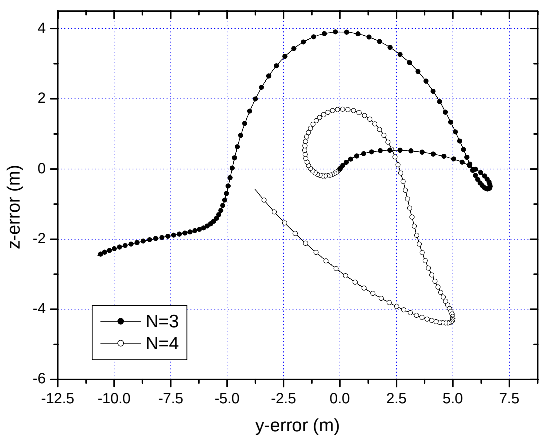

Fig. 9 Projected interception errors of the defense missile

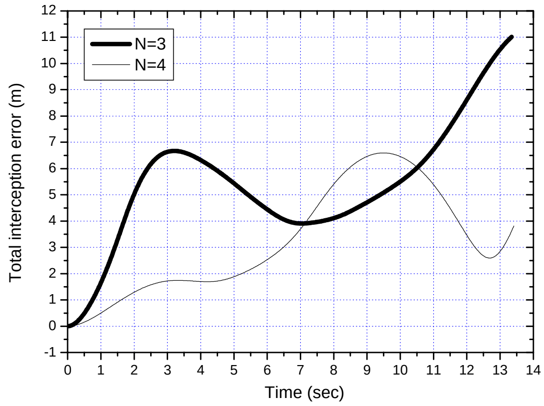

Fig. 10 Time history of the interception errors of the defense missile

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