Autopilot Design for Agile Missile with Aerodynamic Fin and Side Thruster

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Abstract: This paper is concerned with a mixed control with aerodynamic fin and side thrusters applied to an agile missile using two-time scale dynamic inversion and linear time-varying control technique. The nonlinear dynamic inversion method with the weighting function allocates the desired control inputs (aerodynamic fin and side thrusters) to track a reference trajectory, and the time-varying control technique guarantees the robustness for the uncertainties. Closed-loop stability is achieved by the assignment of the extended-mean of these linear time-varying eigenvalues to the left half complex plane. The proposed schemes are validated by nonlinear simulations.

Keywords: two-time scale dynamic inversion, linear time-varying control, control allocation, aerodynamic fin, side thrusters

1. Introduction

Advances in fighter aircraft technology continue to create new challenges for designer of weapon systems. The introduction of an agile missile is one of the new challenges. The modern control system of the agile missile has the many challenges due to the stringent required performance such as fast time response, high angle of attack, and high maneuver. These have led to various control problems associated with air-breathing propulsion, reduced aerodynamic control surface area, and low dynamic pressures. Similarly, the development of super-maneuverable aircraft has motivated efforts to extend a missile's flight envelope. Usually, these missiles combine the new control effectors (thruster vectoring, side thrusters) with the conventional control surface(aerodynamic fin). Thruster vectoring and side thrusters can provide additional moments and forces to achieve the reference command. However, managing each of a group of control devices with the independent control logic sometimes can result in reduced missile controllability and efficiency. Therefore, for the super-maneuverability of the agile missile, control allocation for control effector family is needed [1]-[3].

On the other hand, the dynamics of an agile missile are inherently nonlinear and may vary rapidly with time. Furthermore, these dynamics are highly uncertain since aerodynamic data for vehicles operating under such conditions are difficult to obtain and may in fact be a poor approximation to the actual dynamics. These and other concerns have prompted researchers to look beyond the classical methods. Most nonlinear control techniques are based on linearizing the equations of motion at each equilibrium point or by the application of nonlinear feedback as known variously as feedback linearization, dynamic inversion, or gain scheduling, these methods rely heavily on knowledge of the plant dynamics. More recently, the extended-mean assignment (EMA) control technique for linear time-varying (LTV) has emerged as a means of explicitly accounting for uncertainties in the plant dynamics. The linear EMA control technique is based on the eigenvalue concept for linear time-varying systems, called the SD-eigenvalue [4]. The technique is similar to the conventional pole placement design method for linear time-invariant systems. Closed-loop stability is achieved by the assignment of the extended-mean of these time-varying SD-eigenvalues to the left half complex plane (LHCP).

This paper is concerned with a mixed control with aerodynamic fin and side thrusters applied to an agile missile using dynamic inversion and the LTV control technique. In the proposed autopilot, a simple dynamic inversion autopilot generates the required nominal control inputs; aerodynamic fin and side thrusters. This autopilot is augmented by the LTV controller which acts to guarantee the robustness for the uncertainties. The LTV controller features a stable on-line algorithm derived from the EM value of the SD-eigenvalue stability criterion [5]. In general, the stability of linear time-invariant systems is equivalent to the confinement of all eigenvlaues to the left-half-plane (LHP). The SD-eigenvalue stability criterion is one that the LHP stability criterion carries over to time-varying case using the SD-eignevalue. The work described in this paper adapts these previous efforts for use in the autopilot of the agile missile and proposes the weighting function to account for the actuator constraints.

The agile missile with aerodynamic fin and side thrusters needs to combine the actions of two types of actuators in order to cope with variations in dynamic pressure, and saturations in aerodynamic fin actuator and side thruster actuator. They will require a highly coordinated actuator management scheme. This paper proposes a simple weighting function in order to allocate the control inputs appropriately. This weighting function is an effective algorithm to specify the aerodynamic fin deflections and to select the side thrusters ignition. If the aerodynamic fin deflection increases toward a saturation value, the weighting function of aerodynamic fin becomes infinity. Then aerodynamic fin deflection becomes zero. In that case, the missile autopilot ignites the side thrusters since the control allocation algorithm encourages the side thrusters. If the control law needs the aerodynamic fin input during the side thrusters burning, the weighting function of aerodynamic fin encourages the aerodynamic fin input to the missile. The weighting function of side thrusters discourages side thruster firings except when absolutely necessary. For other cases, the missile uses aerodynamic fin as a control input. Fig. 1 shows the schematic diagram of the designed autopilot.

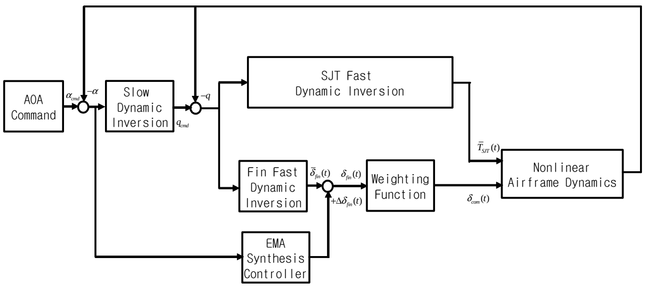

Fig. 1. Schematic diagram

2. Missile model

A nonlinear pitch-axis missile model is used for the present research. This model is derived from the previous research efforts [1]-[3]. A body coordinate system is used to derive the equation of motion. The coordinate system is illustrated in Fig. 2. The origin of the body axis system is assumed to be at the missile center of gravity. The X_b-axis of the body frame points in the direction of the missile nose, and the Z_b-axis completes the right-handed triad.

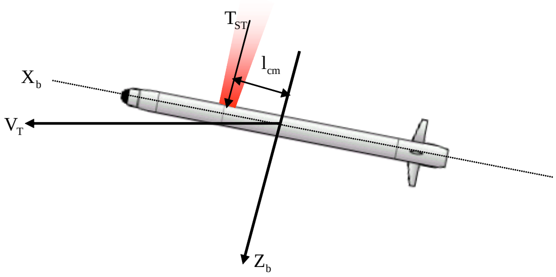

Fig. 2. Missile pitch plane

The pitch-axis dynamics of the missile are described by the following differential equations:

$$\dot{\alpha}(t) = \frac{\rho V_T \bar{S}}{2m}\left(C_Z(\alpha) + \Delta C_Z(\alpha)\delta_{fin}\right) + q(t) + \frac{1}{mV_T}T_{ST} \tag{1}$$

$$\dot{q}(t) = \frac{\rho V_T^2 \bar{S}\bar{C}}{I_{yy}}\left(C_{m_0}(\alpha) + \Delta C_m(\alpha)\delta_{fin} + \frac{\bar{C}}{2V_T}C_{mq+m\alpha}(M)\cdot q(t)\right) - \frac{l_{cm}}{I_{yy}}T_{ST} \tag{2}$$

where the parameters are given in Table 1. The missile is a rigid body of constant mass, the time rates of changes the moments and products of inertia are zero. The products of inertia are zero, because the body axes coincide with the missile's principal axes. The aerodynamic force and moment coefficients $C_{Z_0}(\alpha)$, $\Delta C_Z(\alpha)$, $C_{m_0}(\alpha)$, $\Delta C_m(\alpha)$ are given

Table 1. Parameters

α	angle of attack	q	pitch rate
ρ	air density	V_T	missile velocity
$\bar{S}$	reference area	$\bar{C}$	reference length
I_{yy}	momnet of inertia	m	missile mass
T_{ST}	side thruster	l_{cm}	moment arm

as look-up tables with respect to the angle of attack a, pitch fin deflection δ_{fin}, and Mach number M. These coefficients must have a polynomial form in order to design the EMA controller. These coefficients of polynomial have been curve-fitted from the coefficients lookup tables. The curve-fitted coefficients functions are as follows:

$$C_{Z_0}(\alpha(t)) = a_z\alpha^4(t) + b_z\alpha^3(t) + c_z\alpha^2(t) + d_z\alpha(t)$$
$$C_{m_0}(\alpha(t)) = a_m\alpha^4(t) + b_m\alpha^3(t) + c_m\alpha^2(t) + d_m\alpha(t)$$

$$\Delta C_Z(\alpha(t)) = a_{zd}\alpha^3(t) + b_{zd}\alpha^2(t) + c_{zd}\alpha(t) + d_{zd}$$
$$\Delta C_m(\alpha(t)) = a_{md}\alpha^3(t) + b_{md}\alpha^2(t) + c_{md}\alpha(t) + d_{md}$$

where, Mach number M holds on constant because of the assumption that the flight is short-period.

3. Control allocation

The design methodology used in this study is a dynamic inversion with weighting function approach using a two-time scale assumption to separate the dynamics [6]-[9]. The inner-loop inversion uses the aerodynamic fin deflection and the side thruster to control the fast states $q(t)$. The outer-loop inversion uses the fast states as inputs to control the slow state $\alpha(t)$. The two-time scale dynamic inversion structure is show in Fig. 3.

3.1. Fast dynamic inversion: inner-control loop

The inner loop of the dynamic inversion control law controls the fast state $q(t)$. This loop calculates an aerodynamic fin deflection command and a side thruster command from the rate command $q_c(t)$ given by the show inversion in Fig. 3. The desired dynamic used is given by $\dot{q}_d(t) = \omega_q(q_c(t) - q(t))$, where ω_q is design parameter. The aerodynamic fin nominal input and the side thruster nominal input are calculated by dynamic inversion, respectively. The fast dynamic inversion equations are given by

$$\bar{\delta}_{fin}(t) = \left(K_\alpha \Delta Cm(\alpha(t))\right)^{-1}\left(\dot{q}_d(t) - K_\alpha\left[C_{m0}(\alpha(t)) + \frac{\bar{C}}{2V_T}C_{mq+m\alpha}(M)\cdot q(t)\right]\right) \tag{3}$$

$$\bar{T}_{ST}(t) = \left(-\frac{l_{cm}}{I_{yy}}\right)^{-1}\left(\dot{q}_d(t) - K_\alpha\left[C_{m0}(\alpha(t)) + \frac{\bar{C}}{2V_T}C_{mq+m\alpha}(M)\cdot q(t)\right]\right) \tag{4}$$

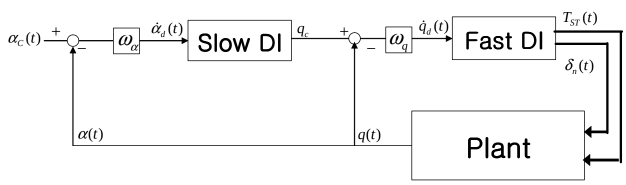

Fig. 3. Two-time scale dynamic inversion structure

where

$$K_\alpha = \frac{\rho V_T^2 \bar{S}\bar{C}}{2I_{yy}}$$

3.2. Slow dynamic inversion: outer-control loop

A second inversion is applied to the dynamic of the slow state $\alpha(t)$. The slow dynamic inversion assumes that the fast states track their commanded values instantly. The slow dynamic inversion attempts to replace the actual $\alpha(t)$ dynamics with the desired dynamic $\dot{\alpha}_d(t) = \omega_\alpha(\alpha_c(t) - \alpha(t))$, where ω_α is design parameter. The Slow dynamic inversion equation is as follows

$$q_c(t) = \dot{\alpha}_d(t) - K_q\{Cz0\,(\alpha(t)) + Czdz\,(\alpha(t)) \cdot \delta_{fin}(t)\} \quad (5)$$

where

$$K_q = \frac{\rho V_T^2 \bar{S}}{2mV_T}$$

Since the velocity is very fast, the $T_{ST}(t)$ has little effect on this dynamic inversion equation. So the side thruster's term is ignored.

3.3. Weighing function

Control allocation is the process of assigning control responsibility amongst redundant. Generally, a dynamic inversion technique uses the pseudo-inverse for control allocation. But this agile missile is difficult to use the pseudo-inverse, because aerodynamic fin deflection and side thrusters are have different physical propensity. The commanded aerodynamic fin input is defined by

$$\delta_{com} = \frac{\delta_{fin}}{1 + G\,(\delta_{fin})} \quad (6)$$

where δ_{fin} is nominal command input calculated by the dynamic inversion and the LTV controller. The weighting function of the aerodynamic fin $G\,(\delta_{fin})$ is specified by the following Equation:

$$G\,(\delta_{fin}) = \tan\left[\frac{\pi}{2}\left((1-\zeta)\left[\frac{\delta_{fin}}{|\delta_{sat}|}\right] + \zeta\right)\right] - \tan\left(\frac{\pi}{2}\zeta\right) \quad (7)$$

ζ : Steepness parameter; $0 < \zeta < 1$
δ_{sat} : Fin deflection limit

The fin weighting function $G\,(\delta_{fin})$ has a small value for low δ_{fin}, however, as $\delta_{fin}/\delta_{sat}$ approaches unity, $G\,(\delta_{fin})$ diverges to infinity. One may control the breakpoint at which $G\,(\delta_{fin})$ diverges by adjusting the "ζ" parameter in Eq. (7). For large ζ , the function begins to contribute at lower δ_{fin} and slowly diverges as δ_{fin} increases. If ζ is brought below 0.9, $G\,(\delta_{fin})$ begins to diverge more sharply at higher δ_{fin}, until $\zeta \to 0$, $G\,(\delta_{fin})$ can approximate a delta function peaking when the actuator is against its limit. $G\,(\delta_{fin})$ is plotted for several values of ζ in Fig. 4 [10]. The side thruster weighting algorithm is shown in Fig. 5, where W_i, $(i = 1, \cdots, n)$ are critical values. n is the number of side thrusters which can ignite at once. The number of side thrusters n is determined by the comparison of the nominal command inputs

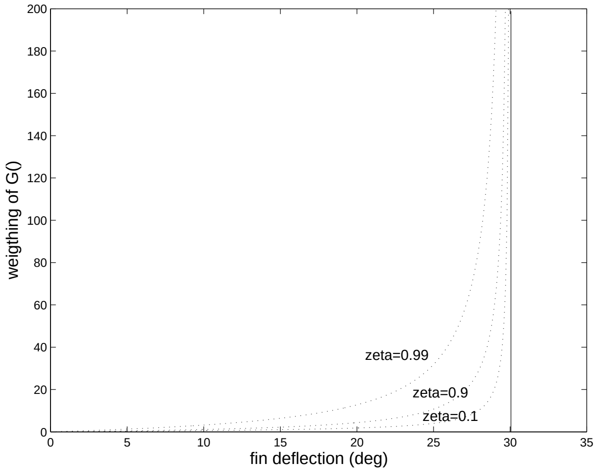

Fig. 4. Weighting function

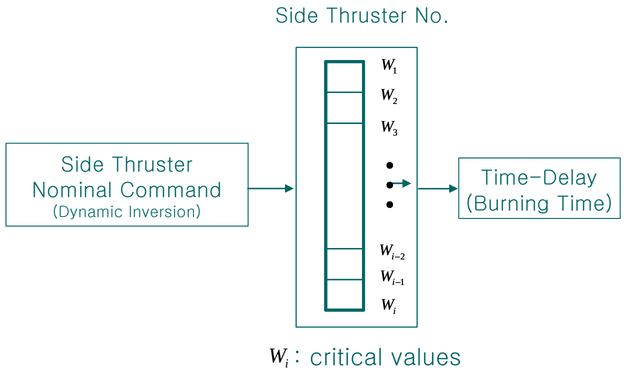

Fig. 5. Weighting algorithm for side thrusters

obtained by the dynamic inversion with the critical values. The critical values have to be decided on the considering missile system.

This weighting function is an effective algorithm to specify the aerodynamic fin deflection and to select the side thrusters ignition.

4. Linear time-varying controller

At the initial scheme, we simplify the problem by neglecting the side thruster. Because, the aerodynamic fin deflection input is continuous but the side thruster input is discrete event. We can easily use the aerodynamic fin deflection for the fine tuning and the guaranteed robustness. The tracking error dynamics is linearized from the nonlinear dynamic Eqs. (1) and (2). Let

$$\xi(t) = \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} = \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix} \quad (8)$$

be the state vector of the reduced-order nonlinear model. The the state equation can be writtern as

$$\dot{\xi}(t) = f(\xi, \delta) = \begin{bmatrix} f_1(\xi_1, \xi_2, \delta) \\ f_2(\xi_1, \xi_2, \delta) \end{bmatrix} \quad (9)$$

where

$$f_1(\xi_1, \xi_2, \delta) = \frac{\rho V_T S}{2m}\left(C_{Z_0}\,(\xi_1) + \Delta C_Z\,(\xi_1, \delta)\right) + \xi_2$$

$$f_2(\xi_1,\xi_2,\delta) = \frac{\rho V_T^2 SC}{I_{yy}}\left(C_{m_0}(\xi_1) + \Delta C_m(\xi_1,\delta)\right. \quad (10)$$
$$\left. + \frac{C}{2V_T} C_{mq+m\alpha}(M)\cdot\xi_2\right)$$

Now, let $\bar{\delta}(t)$ be the nominal aerodynamic fin, and $\bar{\xi}(t)$ be the nominal state trajectory such that

$$\dot{\bar{\xi}}(t) = f(\bar{\xi},\bar{\delta}) \quad (11)$$

Define the tracking errors by

$$x(t) = \left[\begin{array}{cc} \alpha(t) - \bar{a}(t) & q(t) - \bar{q}(t) \end{array}\right]^T \quad (12)$$

and the tracking error control input by

$$v(t) = \delta(t) - \bar{\delta}(t) \quad (13)$$

The linearized tracking error dynamics are given by

$$\dot{x} = A(t)x(t) + B(t)v(t) \quad (14)$$

where

$$A(t) = \left.\frac{\partial f}{\partial \xi}\right|_{\bar{\xi},\bar{\delta}} = \begin{bmatrix} a_{11}(t) & 1 \\ a_{21}(t) & a_{22}(t) \end{bmatrix} \quad (15)$$

$$B(t) = \left.\frac{\partial f}{\partial \delta}\right|_{\bar{\xi},\bar{\delta}} = \begin{bmatrix} b_1(t) \\ b_2(t) \end{bmatrix} \quad (16)$$

A nonlinear tracking controller is based on a tracking error stabilization controller, which drives the tracking errors $x(t)$ to zero exponentially as $t \to \infty$. This can be achieved by using the EMA controller. However, to use the prototypes EMA controller, it is necessary to transform the linearized tracking error dynamics into the phase-variable canonical form. This can be done via the Silverman's coordinate transformation [11], provided that $\left[\begin{array}{cc} A(t) & B(t) \end{array}\right]$ is uniformly completely controllable. Applying the state coordinate transformation $x(t) = L(t)z(t)$ to Eq. (14) wirh a time-varying coordinate transformation matrix $L(t)$ given by

$$L(t) = \begin{bmatrix} 1 & 0 \\ -a_{11}(t) & 1 \end{bmatrix} \quad (17)$$

Then the linearized system (14) in the $z(t)$ coordinates becomes

$$\dot{z}(t) = A_c(t)z(t) + B_c(t)v(t) \quad (18)$$

where $A_c(t) = L^{-1}(t)\left[A(t)L(t) - \dot{L}(t)\right]$ is of the companion from

$$A_c(t) = \begin{bmatrix} 0 & 1 \\ -p_1(t) & -p_2(t) \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 1 \\ \dot{a}_{11}(t) + a_{21}(t) - a_{11}(t)a_{22}(t) & a_{11}(t) + a_{22}(t) \end{bmatrix}$$
$$B_c(t) = L^{-1}(t)B(t)$$
$$= \begin{bmatrix} b_1(t) \\ a_{11}(t)b_1(t) + b_2(t) \end{bmatrix}$$

Note that $z_1(t) = x_1(t) = \alpha(t) - \bar{\alpha}(t)$. By eliminating $z_2(t)$ from Eq. (18), it is seen that this state equation is equivalent to a scalar equation of the form

$$\ddot{z}_1 + p_2(t)\dot{z}_1 + p_1(t)z_1$$
$$= b_1(t)\dot{v}(t) + (\dot{b}_1(t) + b_2(t) - a_{22}(t)b_1(t))v(t) \quad (19)$$

To render this equation into the phase-variable form, we introduce the angle of attack zero dynamics

$$\frac{1}{b_1(t)}u(t) = \dot{v}(t) + \frac{1}{b_1(t)}(\dot{b}_1(t) + b_2(t) - a_{22}(t)b_1(t))v(t) \quad (20)$$

Combining Eqs. (19) and (20) yields the desired form

$$\ddot{z}_1(t) + p_2(t)\dot{z}_1(t) + p_1(t)z_1 = u(t) \quad (21)$$

Note that $v(t)$ is now a hidden state in the augmented system $u \to z_1$. The zero of the angle of attack dynamics in the augmented system has been canceled by its inverse. To make the cancellation valid, we need verify that the zero dynamics is exponentially stable. This is indeed the case because it is readily verified that the SD eigenvalue

$$\lambda_2(t) = -\frac{b_2(t) + \dot{b}_1(t) - a_{22}(t)b_1(t)}{b_1(t)} \quad (22)$$

for Eq. (22) has a negative EM for all $|\alpha(t)| < \pi/2$ and $2M(t) > 0$ [4].

5. Simulation study

Simulation studies have been performed to validate the proposed schemes using an arbitrary angle of attack command trajectory. The missile plant model used in the simulations includes the nonlinear second-order actuator dynamics. Parameters of the nonlinear dynamic equations for this missile control problem are given in Table. 2. We assume that the maximum number of missile's ignited side thrusters at a time is 10.

Table 2. Parameters for simulations

$M = 6.00$	Mach number
$\Gamma = 45°$	bank angle (deg)
$H = 20000$	altitude (m)
$\rho = 0.08803$	air density at H, (kg/m^3), ISA
$V_s = 295.1$	speed of sound at H,m/s, ISA
$T = 13800$	thrust (N)
$S = 0.070685$	reference area (m^2)
$C = 0.3$	reference length (m)
$m = 168.7$	mass (kg)
$I_{yy} = 491.3$	moment of inertia
$l_m = 1.6$	moment arm (m)
$\delta_{fin,\max} = 30$	maximum fin deflection (deg)
$T_{ST} = 4700$	side thruster (N); burning time $30ms$

Fig. 6 illustrates the time histories of the angle of attack step trajectory tracking performance. Fig. 5 shows a 3-piecewise constant angle of attack tracking command, and the angle of attack output. The designed autopilot accurately tracks step commands with a settling time less than 0.5sec. Fig. 7 shows the pitch rate command and pitch rate response. The time historise of the pitch rate command is calculated by outer-loop inversion (slow dynamic inversion). Fig. 8 depicts the time histories of the aerodynamic fin deflection and side thrusters. It shows two command input combinations; the aerodynamic fin and side thrusters. The dot line is the aerodynamic fin deflection and the symbol "x" is the number of the side thruster. The aerodynamic fin deflection is chattering during side thruster igniting. But the autopilot uses the aerodynamic fin within the actuator saturation limit 30 deg. The simulation results indicate that the fast time response can be easily achieved by the proposed autopilot with aerodynamic fin and side thrusters without the actuator saturation.

6. Conclusions

A nonlinear missile autopilot based on the two-time scale dynamic inversion with weighting function and the LTV control technique has been proposed. With the aid of the LTV controller, this autopilot tracks a command well in angle of attack while holding the bank angle to be 45°. The simulation results show that the designed autopilot with dynamic inversion and the EMA controller has the fast time response without the actuator saturation. Additionally, the use of the LTV controller enables the autopilot to effectively adapt online to uncertain nonlinear time-varying aerodynamic phenomena, which are difficult to model for the purposes of contorl design and simulations. Each of these improvement represents a step toward the creation of a powerful design technique for the next generation high-performance missile autopilot applications.

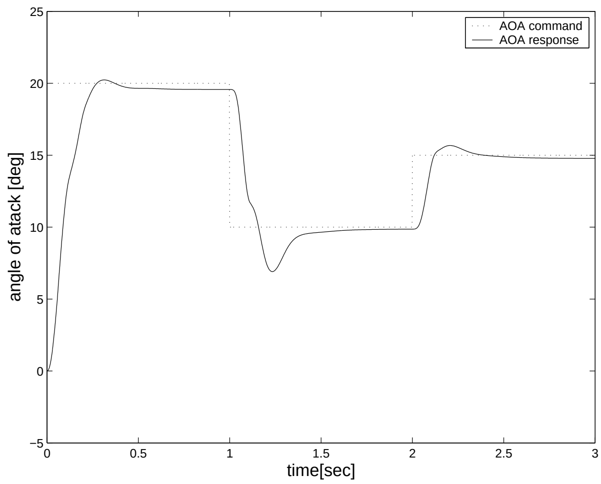

Fig. 6. AOA step trajectory tracking performance

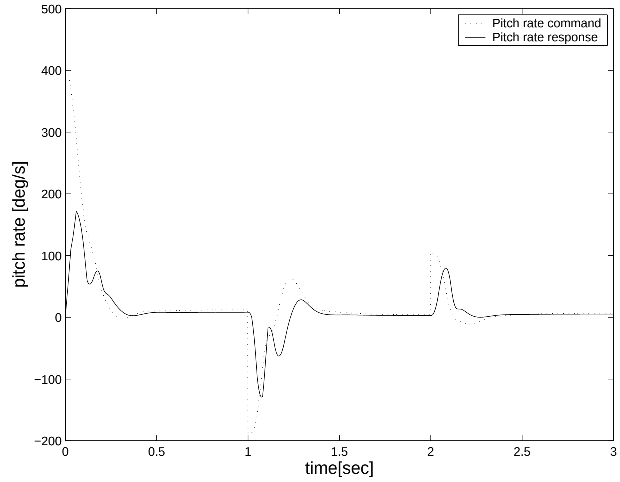

Fig. 7. Pitch rate

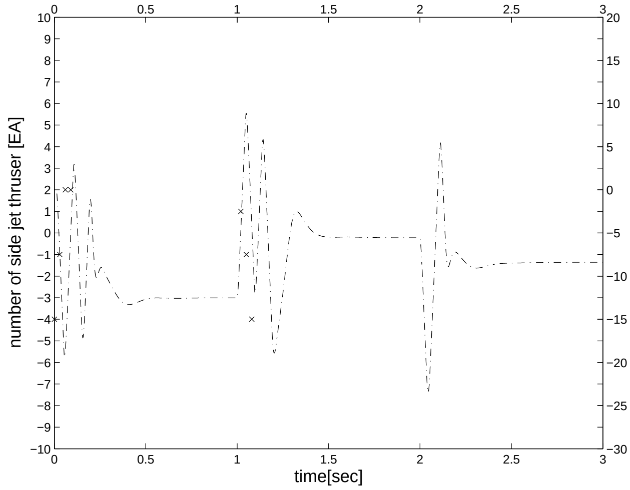

Fig. 8. Aerodynamic fin deflection and side thruster

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