Observed Data Oriented Bispectral Estimation of Stationary Non-Gaussian Random Signals – Automatic Determination of Smoothing Bandwidth of Bispectral Windows

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Abstract: Toward the development of practical methods for observed data oriented bispectral estimation, an automatic means for determining the smoothing bandwidth of bispectral windows is proposed, that can also provide an associated optimum bispectral estimate of stationary non-Gaussian signals, systematically only from an observed time series datum of finite length. For the conventional non-parametric bispectral estimation, the MSE (mean squared error) of the normalized estimate is reviewed under a certain mixing condition and sufficient data length, mainly from the viewpoint of the inverse relation between its bias and variance with respect to the smoothing bandwidth. Based on the fundamental relation, a systematic method not only for determining the bandwidth, but also for obtaining the optimum bispectral estimate is presented by newly introducing a MSE evaluation index of the estimate only from an observed time series datum of finite length. The effectiveness and fundamental features of the proposed method are illustrated by the basic results of numerical experiments.

Keywords: Observed data oriented estimation, polyspectral estimation, non-parametric bispectral estimation, automatic smoothing bandwidth determination of bispectral windows, minimum MSE estimation, a real 6th order stationary random signal.

1. Introduction

Since the 1st International Workshop on Higher Order Spectral Analysis was held at Vail, Colorado, U.S.A. in 1989 [1], its applicability to many practical problems has been re-recognized widely in broad areas. Actually, the applications of bispectral and trispectral analyses, i.e., the lowest two of the higher order one with the 3rd and 4th orders, are rapidly evolving in signal processing areas in science and engineering; geophysical, medical, mechanical, acoustical, and optical science and/or engineering fields, and so on [2].

Correspondingly, parametric and non-parametric methods for bispectral estimation of stationary non-Gaussian random signals have been developed [3]. Although both methods respectively have their intrinsic characters, however, each of them has a certain problem to be improved, especially from the viewpoint of observed data oriented estimation, i.e., the automatic optimum estimation only from an observed time series datum of finite length.

This paper is concerned with the conventional non-parametric stationary bispectral estimation based on smoothing the 3rd order periodogram [4]-[7]. Although a few methods for the optimum estimation are proposed, any of them optimizes a certain criterion to determine the optimum shape of bispectral windows under the assumption that the smoothing bandwidth is given beforehand [6],[7]. For actual estimation, however, determination of the optimum bandwidth is considered to be more important than that of the optimum shape, since the bias and variance, and hence, MSE of a bispectral estimate, are usually governed globally by the former. Therefore, the aim of this paper is to propose an automatic means for determining the optimum smoothing bandwidth only from an observed time series datum of finite length.

For the purpose, we propose an evaluation index of the MSE of a bispectral estimate, by focusing on a mutually reverse relation between its bias and variance with respect to the smoothing bandwidth of bispectral windows under a certain mixing condition about the signal BSD (bispectral density function, or simply bispectrum) as well as sufficient data length for observation [4]. The proposed MSE index is devised through normalization to be independent to magnitudes of estimated PSD (power spectrum) and BSD values, since its variance at each point in the 2D-frequency (bispectral) plane is proportional to a triple product of associated 3 PSD values to be estimated. As a result, the proposed index may be expected to be uniform or white over the whole bispectral plane, once a nearly optimum bandwidth is reached under the required conditions. So, simple averaging of the index values within a certain range in the 2D-frequency plane makes a systematic determination of the optimum smoothing bandwidth possible with reduced sampling variation, resulting in the proposed automatic estimation only from an observed time series datum of finite length.

The above reverse relation between the bias and variance of BSD estimates was also used by the conventional methods to determine the optimum shape of bispectral windows [6], [7]. However, it is used for automatic determination of the optimum smoothing bandwidth of the windows toward the final aim of this paper mentioned above. This is the fundamental difference between the proposed optimum BSD estimation and the conventional [6], [7].

In what follows, after a brief review of the conventional BSD estimation by smoothing the 3rd order periodogram, the fundamental reverse relation between bias and variance of a BSD estimate with respect to the smoothing bandwidth is made clear in section 2. Based on the characteristics, a practical index for evaluating the associated MSE only from an observed time series datum is proposed in section 3. In section 4, the effectiveness and fundamental features of the proposed estimation are illustrated by basic results of numerical experiments. Finally, the obtained main results are summarized.

2. A Brief Review of BSD Estimation by Smoothing the 3rd Order Periodogram

Let $x(t)$ be a real 6th order stationary zero-mean non-Gaussian random signal with a non-zero bispectrum $B(f_1,f_2)$, and $x(n)$ (n=0,1,…,N-1) be a sampled time series of $x(t)$ every T seconds over an observed time interval [0,T_0sec], i.e, $T_0=NT$. Then, the 3rd order sample correlation (or cumulant) function $R_3(\tau_1,\tau_2)$ can be estimated in the biased form as

$$R_3(\tau_1,\tau_2) = \frac{1}{N}\sum_{n=N_1}^{N_2} x(n+\tau_1)x(n+\tau_2)x(n),$$
$$(|\tau_1|,\ |\tau_2|,\ |\tau_1+\tau_2| \le N-1), \qquad (1)$$

where $N_1 \equiv \max\{0,-\tau_1,-\tau_2\}$, and $N_2 \equiv N-1+\min\{0,-\tau_1,-\tau_2\}$. The 2D-DTFT (discrete time Fourier transform) of $R_3(\tau_1,\tau_2)$ gives an associated sample BSD or the 3rd order periodogram as

$$B_T(f_1,f_2) = \sum_{\tau_1=-N+1}^{N-1}\sum_{\tau_2=-N+1}^{N-1} R_3(\tau_1,\tau_2)$$
$$\times \exp[-j2\pi T(f_1\tau_1+f_2\tau_2)]. \qquad (2)$$

As is well known, this $B_T(f_1,f_2)$ can be rewritten as [4]

$$B_T(f_1,f_2) = X_T(f_1)X_T(f_2)X_T^*(f_1+f_2)/T_0, \qquad (3)$$

where A^* denotes a complex conjugate A, and $X_T(f)$ is the 1D-DTFT of $x(n)$ defined by

$$X_T(f) = \sum_{n=0}^{N-1} x(n)\exp(-j2\pi nfT). \qquad (4)$$

The equations (2) to (4) suggest that the periodogram $B_T(f_1,f_2)$, and hence, the signal BSD $B(f_1,f_2)$ mathematically depend implicitly on an extra component of frequency $-(f_1+f_2)$ as well as explicitly on those of frequencies f_1 and f_2, because the relation $X_T(-f)=X_T^*(f)$ is valid for a real-valued signal, as is well known from the Fourier theory.

D. R. Brillinger etal. [4] generally evaluated the asymptotic properties of the k-th order periodogram ($k\ge2$), asserting the 3rd order one as a BSD estimate under the mixing condition,

$$T_{cor}^{mn} = (\Delta t)^2 \sum_{\tau_1=-\infty}^{\infty}\sum_{\tau_2=-\infty}^{\infty} |\tau_1|^m |\tau_2|^n |R_3(\tau_1,\tau_2)|, \quad (m+n=2). \quad (5)$$

This means the smooth characteristics of signal BSD $B(f_1,f_2)$, of which partial derivatives up to the 2nd order are finite. Under this condition, the following statistical properties of $B_T(f_1,f_2)$ are derived according to the asymptotic theory :

$$E\{B_T(f_1,f_2)\} = B(f_1,f_2) + O(1/T_0), \qquad (6a)$$

$$Var\{B_T(f_1,f_2)\} = T_0 \cdot P(f_1)P(f_2)P(f_1+f_2) + O(1), \qquad (6b)$$

$$Cov\{B_T(f_1,f_2), B_T(f_3,f_4)\} = O(1), \quad (\text{for all } |f_1-f_3|,$$
$$|f_2-f_4|, \text{ or } |f_1-f_4|,\ |f_2-f_3| > 1/T_0). \qquad (6c)$$

These properties reveal the difficulty for simply using $B_T(f_1,f_2)$ as a BSD estimate. That is, (i) Contrary to the nearly unbiased nature of $B_T(f_1,f_2)$ as observed from Eq. (6a), (ii) Eq. (6b) says that its variance may be subject to a triple product of powers of associated 3 frequency components. Since information of random signals is usually extracted from the estimated PSD peaks, the result of (ii) suggests that large sample variation may be aroused for such values of $B_T(f_1,f_2)$, at least one of the associated 3 frequencies of which being equal to any of the dominant peaks, resulting in the above assertion for use of $B_T(f_1,f_2)$ as a BSD estimate. However, (iii) Eq. (6c) does that $B_T(f_1,f_2)$ and $B_T(f_3, f_4)$ may be uncorrelated (even asymptotically "independent" [4]), when absolute differences of frequency pairs, i.e., $|f_1-f_3|$ and $|f_2-f_4|$, or $|f_1-f_4|$ and $|f_2-f_3|$ are separated by more than an inverse of an observed data length, i.e., $1/T_0$.

The rather large sample variation described in (ii) may be considered as a result of lack of time averaging accuracy when at least one of the two absolute lags $|\tau_i|$s (i=1,2) becomes nearly equal to its maximum, i.e., N-1. This fact and the uncorrelated nature of different $B_T(f_1,f_2)$s described in (iii) suggest that the more precise estimate can be expected by truncating the low accuracy portion of thus calculated $R_3(\tau_1, \tau_2)$ with a 2D-lag window $w_2(\tau_1,\tau_2)$, and 2D-discrete time Fourier transforming the resultant as follows:

$$\hat{B}(f_1,f_2) = (\Delta t)^2 \sum_{\tau_1=-M}^{M}\sum_{\tau_1=-M}^{M} w_2(\tau_1,\tau_2)R_3(\tau_1,\tau_2)$$
$$\times \exp[-j2\pi T(f_1\tau_1+f_2\tau_2)], \qquad (7)$$

where M denotes the maximum lag used for the estimation, and should be chosen to be less than a squared root of T_0 [4]. Equivalently, Eq. (7) can be rewritten in Fourier domain as

$$\hat{B}(f_1,f_2) = \int_{-f_s/2}^{f_s/2}\int_{-f_s/2}^{f_s/2} W_2(\mu,\nu)B_T(f_1-\mu, f_2-\nu)\mathrm{d}\mu\mathrm{d}\nu, \qquad (8)$$

where $W_2(f_1,f_2)$ is the bispectral window defined by the 2D-DTFT of $w_2(\tau_1,\tau_2)$.

Usually, the 2D-lag window $w_2(\tau_1,\tau_2)$ is chosen so as to have the properties bellow [6],[7].

$$W_2(\tau_1,\tau_2) = w_2(\tau_2,\tau_1) = w_2(-\tau_1,\tau_2-\tau_1),$$
$$= w_2(\tau_1-\tau_2,-\tau_2) \ge 0, \qquad (9a)$$
$$w_2(0,0) = \int_{-f_s/2}^{f_s/2}\int_{-f_s/2}^{f_s/2} W_2(f_1,f_2)df_1df_2 = 1,$$

where the last equation is the normalization condition derived from the Fourier theory, and means that integral of the bispectral window $W_2(f_1,f_2)$ in the whole 2D-frequency plane being equal to a unity. Similarly, $W_2(f_1,f_2)$ is assumed to satisfy the same symmetry as that of $B(f_1,f_2)$, and be non-negative [6],[7], because the bias of estimated BSD might be controlled freely if negative values would be allowed for it, and hence, the bias itself loses its intrinsic meaning [6]. That is,

$$W_2(f_1,f_2) = W_2(f_2,f_1) = W_2(-f_1-f_2,f_2),$$
$$= W_2(f_1,-f_1-f_2) = W_2(-f_1,-f_2) \ge 0. \quad (9b)$$

Under the mixing condition for the BSD given by Eq. (5), and sufficient data length compared with an inverse of the band-width of the narrowest peak, the bias of the smoothed BSD estimate is evaluated in the followings: From the unbiased nature of $B_T(f_1,f_2)$, we have

$$\hat{B}(f_1, f_2) \simeq \int_{-f_s/2}^{f_s/2}\int_{-f_s/2}^{f_s/2} W_2(\mu,\nu)B(f_1-\mu, f_2-\nu)d\mu d\nu. \quad (10a)$$

By expanding the 2nd term of the integrand in the right side of Eq. (10a) into a Taylor series including up to the 2nd order partial derivatives of $B(f_1,f_2)$, with aids of the assumed symmetry and normalization condition of $W_2(f_1,f_2)$, the bias of the smoothed BSD estimate can be evaluated by the next equation,

$$Bias[\hat{B}(f_1, f_2)] \approx B_{2,0}(f_1, f_2)B_w^{2,0} + B_{0,2}(f_1, f_2)B_W^{0,2} + B_{1,1}(f_1, f_2)B_W^{1,1}, \quad (10b)$$

where $B_{i,j}(f_1,f_2)$ $(i+j=2)$ denotes the the ith and jth partial partial derivatives of $B(f_1,f_2)$ with respect to f_1 and f_2, respect-tively, with the total 2nd order, and $B_W^{i,j}$ s $(i+j.=2)$ are the similar 2nd order moments of $W_2(f_1,f_2)$ defined as

$$B_{W2,0} = B_{W0,2} \equiv \frac{1}{2}\int_{-f_s/2}^{f_s/2}\int_{-f_s/2}^{f_s/2} \nu^2 W_2(\mu,\nu)d\mu d\nu, \quad (11a)$$

$$B_{W1,1} \equiv -\int_{-f_s/2}^{f_s/2}\int_{-f_s/2}^{f_s/2} \mu\nu W_2(\mu,\nu)d\mu d\nu. \quad (11b)$$

On the other hand, from the uncorrelated nature of $B_T(f_1,f_2)$ given by Eq. (6c) and the physical meaning of BSD $B(f_1,f_2)$, the variance of the smoothed estimate given by Eq. (7) or (8) is also evaluated [6],[7] as

$$Var[\hat{B}(f_1, f_2)] \approx E_W T_0 P(f_1)P(f_2)P(f_1+f_2), \quad (12)$$

where E_W is the energy of $W_2(f_1,f_2)$ defined by

$$E_w \equiv (\Delta t)^2 \sum_{\tau_1=-M}^{M}\sum_{\tau_2=-M}^{M} \left|w_2(\tau_1,\tau_2)\right|^2, = \int_{-f_s/2}^{f_s/2}\int_{-f_s/2}^{f_s/2} \left|W_2(\mu,\nu)\right|^2 d\mu d\nu. \quad (13)$$

By taking into account the normalization condition of $W_2(f_1,f_2)$, Eq. (10a) means that the magnitudes of $B_W^{i,j}$ s in Eqs. (11a) and (11b) increase with the bandwidth of $W_2(f_1,f_2)$, while the energy in Eq. (12) decreases with it. These facts clearly reveal the fundamental reverse relation between the bias and variance of the smoothed BSD estimate with respect to the smoothing bandwidth, as mentioned previously.

3. Proposed MSE Evaluation Index of Smoothed BSD Estimate

Eqs. (12) and (10b) imply that the variance of the normalized smoothed BSD estimate is proportional to a ratio of the triple product of associated 3 PSD values to the squared magnitude of BSD, the energy E_W of a bispectral window $W_2(f_1,f_2)$, and a given data length T_0, while its bias is the summation of a half of a product of the 2nd order moment of $W_2(f_1,f_2)$, the 2nd order partial derivative of the signal BSD with respect to a single or double frequency indices, and an inverse of the BSD magnitude. Thus, replacing the signal PSD and BSD values by the corresponding estimates, and approximating the 2nd order partial derivatives with the corresponding differences give the next indices to evaluate the normalized statistics.

$$\begin{aligned} NB[\hat{B}(f_1, f_2)] = &\frac{1}{\hat{B}(f_1, f_2)(\Delta f)^2} \\ &\times\{B_{W2,0}[\hat{B}(f_1+\Delta f, f_2) - 2\hat{B}(f_1, f_2) + \hat{B}(f_1-\Delta f, f_2)] \\ &- B_{W1,1}[\hat{B}(f_1+\Delta f/2, f_2+\Delta f/2) \\ &\quad - \hat{B}(f_1+\Delta f/2, f_2-\Delta f/2) \\ &\quad - \hat{B}(f_1-\Delta f/2, f_2+\Delta f/2) \\ &\quad + \hat{B}(f_1-\Delta f/2, f_2-\Delta f/2)] \\ &+ B_{W0,2}[\hat{B}(f_1, f_2+\Delta f) - 2\hat{B}(f_1, f_2) + \hat{B}(f_1, f_2-\Delta f)]\}, \end{aligned} \quad (14a)$$

$$NV[\hat{B}(f_1, f_2)] = E_W \cdot NT \cdot \frac{\hat{P}(f_1)\hat{P}(f_2)\hat{P}(f_1+f_2)}{|\hat{B}(f_1, f_2)|^2}, \quad (14b)$$

where Δf denotes a certain frequency increment for difference calculation, say, a half-valued bandwidth of a bispectral window $W_2(f_1,f_2)$. As clearly observed from Eq. (14b), the variance of the estimate also includes the estimated PSD values at associated 3 frequencies. For the PSD estimation, the observed data oriented automatic estimation proposed previously by smoothing the periodogram [8] can also be used separately.

To evaluate these indices, and hence, the MSE index only from an observed time series datum, any averaging is required to suppress the sampling variation. Since the normalized estimate discussed above may be expected to become white, averaging over a certain 2D-frequency range available is considered in this paper, resulting in the following proposed indices:

$$Bias = A_{\nu,f}\{|NB[\hat{B}(f_1, f_2)]|\}, \quad (15a)$$

$$Var = A_{\nu,f}\{NV[\hat{B}(f_1, f_2)]\}, \quad (15b)$$

$$MSE = Var + Bias^2, \quad (15c)$$

where $A_{\nu,f}$ represents the associated averaging in the available 2D-frequency range.

By following to the above scheme, once a sample time series is given, with a form of the bispectral window being assumed a priori, the minimum MSE makes an automatic determination of the optimum smoothing bandwidth of a bispectral window possible, so that the optimum BSD estimate may be obtained. This is the principle of the automatic determination of the smoothing bandwidth to establish an observed data oriented BSD estimation proposed in this paper.

4. Results of Numerical Experiments

4. 1 Experimental conditions

To confirm the effectiveness and fundamental characteristics of the proposed automatic BSD estimation, basic numerical experiments are carried out by using a zero-mean stationary non-Gaussian time series, the theoretical PSD and BSD characteristics of which are clearly known.

Throughout the following experiments, sampling frequency f_s=1/T is assumed to be fixed as 22.05 kHz, and an observed time series $x(n)$ is numerically generated by the stable AR-model of the 6-th order,

$$x(n) = -\sum_{i=1}^{6} a_i x(n-i) + \varepsilon(n), \qquad (15a)$$

where $\varepsilon(n)$ is a zero-mean white noise of uniform BSD [9], [10]. That is, its 3rd order cumulant function is given by

$$c_{\varepsilon,3}(\tau_1,\tau_2) = \delta_2(\tau_1,\tau_2) = \delta(\tau_1)\delta(\tau_2), \qquad (15b)$$

where $\delta_2(\cdot,\cdot)$, and $\delta(\cdot)$ denote the 2D- and 1D- Kronecker's deltas, respectively. Furthermore, the system function of the AR-filter is assumed to simply consist of 3 elementary stable 2nd order oscillatory AR-filters as follows:

$$H(z) = \frac{1}{1+\sum_{i=1}^{6} a_i z^{-i}} = \prod_{k=1}^{3} \frac{1}{(1-z_k z^{-1})(1-z_k^{*} z^{-1})}, \qquad (15c)$$

where 3 pairs of complex conjugate poles (z_k and z_k^*) (k=1,2, 3) being located at such points in the z-plane that the associated center frequencies have 2kHz, 4kHz, and 8kHz for a sampling frequency f_s=22.05kHz, and the corresponding radii of the poles from the origin are chosen as 0.6, 0.7, and 0.8, respectively, each value of which controls the half power bandwidth of the PSD peak of the corresponding 2nd order AR-oscillatory system.

Therefore, the theoretical PSD and BSD of thus generated time series are correctly known and given by [3]

$$P(f) = \sigma_2 T\,|H[\exp(j2\pi fT)]|^2, \quad |f| \le f_s/2 \qquad (16a)$$

$$B(f_1,f_2) = T^2 \prod_{i=1}^{3} H[\exp(j2\pi f_i T)], \qquad (16b)$$

where $f_3 = -f_1 - f_2, \quad (|f_1|,|f_2|,|f_1+f_2| \le f_s/2).$ (16c)

For the power and bispectral windows of non-negative values in both time and frequency domains and the required symmetries, Bohman's standard window for PSD estimation is used. That is, the lag and spectral windows are given by [6], [8]

$$w(\tau) = \frac{1}{\pi}\left|\sin\left(\frac{\pi\tau}{M}\right)\right| + \left(1-\frac{|\tau|}{M}\right)\cos\left(\frac{\pi\tau}{M}\right), \qquad (17a)$$

$$W(f) = \frac{4\pi^2 MT[1+\cos(2\pi fMT)]}{[(2\pi fMt)^2 - \pi^2]^2}, \qquad (17b)$$

By using them, as a class of 2D-windows which satisfy the previous requirements, simple types of 2D-lag and bispectral windows are considered by the next construction [6],

$$w_2(\tau_1,\tau_2) = w(\tau_1)w(\tau_2)w(\tau_2-\tau_1), \qquad (18a)$$

$$W_2(f_1,f_2) = [W(f_1)W(f_2)] \otimes [W(f_2)\delta(f_1+f_2)], \qquad (18b)$$

where $\otimes$ denotes a 2D-conboluion integral.

4.2 Method of numerical experiments

As stated previously, the optimum maximum lag or smoothing bandwidth of the windows may be different for the PSD and BSD estimation. So, the automatic PSD estimation is first applied, and the proposed automatic BSD one is carried out by using the estimated optimum PSD.

At that time, because the accuracy to estimate the 2nd and 3rd order sample correlation functions given by Eq. (1) is essentially important and heavily depends on an observed data length, basic experiments are carried out by parametrically changing the number of observed time series datum N, or equivalently observed data length T_0=NT, since sampling period T is fixed as constant. Moreover, the FFT based method with adequate zero-padding is used to keep a frequency interval between estimated consecutive PSD or BSD values constant against the change of N. The optimum maximum lag of the window is automatically searched to get the minimum MSE value within a certain range of lags designated beforehand.

4.3 Basic results of numerical experiments

As an example, the results for the automatic PSD estimation, when the number of time series datum N being changed from 2048 to 16384, are shown in Fig. 1, where broken, broken with a dot, and solid lines represent the bias, variance, and MSE indices, respectively, and the minimum point of the MSE index is dsignated by an asterisk for all the cases examined.

From these results, it is confirmed that the proposed evaluation indices of the PSD estimate works well for the stationary time series of non-zero BSD, too. Moreover, the obtained opti-

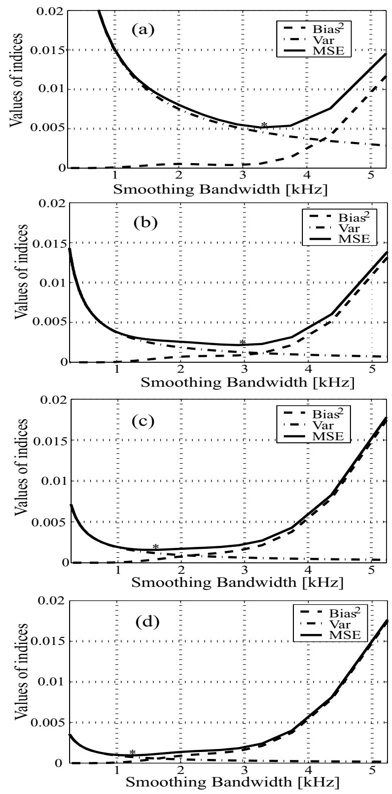

Fig. 1 Characteristics of the proposed indices against the smoothing bandwidth of the power spectral window, where (a), (b), (c), and (d) represents the results for N=2048, 4096, 8192, and 16384, respectively, with a sampling period T=0.045ms, and an asterisk denotes the optimmum bandwidth corresponding to the minimum of the MSE index.

mum smoothing half-power bandwidth gradually decreases from about 3.28kHz to 1.19kHz with increase of N from 2048 to 16384. In this case, a half-power bandwidth of the narrowest peak can be predicted as $|\ln 0.8|/(\pi T)\cong 1.56$kHz, which nearly coincides with that of the optimum smoothing bandwidth 1.54kHz for an observed data length $T_0=8192T=732$ms.

Correspondingly, Fig. 2 graphically shows a few typical examples of the estimated PSDs in a case of N=8192, where broken lines denote the theoretically expected PSD, a solid line in (a) represents the periodogram, and those in (b), (c), and (d) are the results estimated by too narrow, optimum, and too broad smoothing bandwidths, respectively.

From comparison of these results, the following features are clearly observed. Although the periodogram in (a) suffers from heavy sample variation, it comes to be suppressed in the estimated PSDs shown from (b) to (d) as the smoothing bandwidth of the spectral window becomes broad. However, the estimated PSD in (d) shows rather reasonable bias due to the excess smoothing, while that in (b) also suffers from reasonable sampling variation yet due to the lack of smoothing.

As a result, the PSD estimated with the optimum smoothing bandwidth in (c) may provide the most precise estimate.

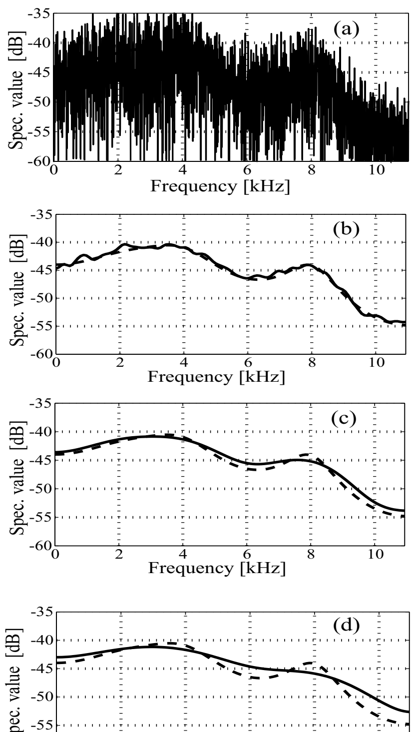

Fig. 2 Typical examples of estimated PDSs in a case of N=8192, where broken lines represent the theoretically expected PSD, a solid line in (a) denotes the sample periodogram, and (b), (c), and (d) are the PSDs estimated with too narrow, optimum, and too broad smoothing bandwidths, respectively. The half-power smoothing bandwidths in (b), (c), and (d) are 0.266kHz, 1.54 kHz, and 2.62kHz, respectively.

These results illustrate well the fundamental characteristics of the previously proposed observed data oriented PSD estimation by automatic determination of the smoothing bandwidth of power spectral windows.

Correspondingly, Fig. 3 shows the characteristics of the proposed evaluation indices of the BSD estimate against the smoothing bandwidth of a bispectral window under the same condition as that in Fig. 1. And typical examples of contour displays of the estimated BSDs for N=8192 are shown in Fig. 4, where (a) represents the theoretical BSD, (b) is the sample BSD, and (c), (d), and (e), are those estimated with too narrow, optimum, and too broad smoothing bandwidths of the bispectral window, respectively.

From comparison of these results and those in Figs. 1 and 2 for the PSD estimation, the following aspects may be observed: (i) As a whole, the proposed automatic BSD estimation works well, almost equivalently to that for the PSD shown above. (ii) However, when the number of observed time series datum N is large, the optimum bandwidth of the window results in rather broad values compared with those for the PSD estima-

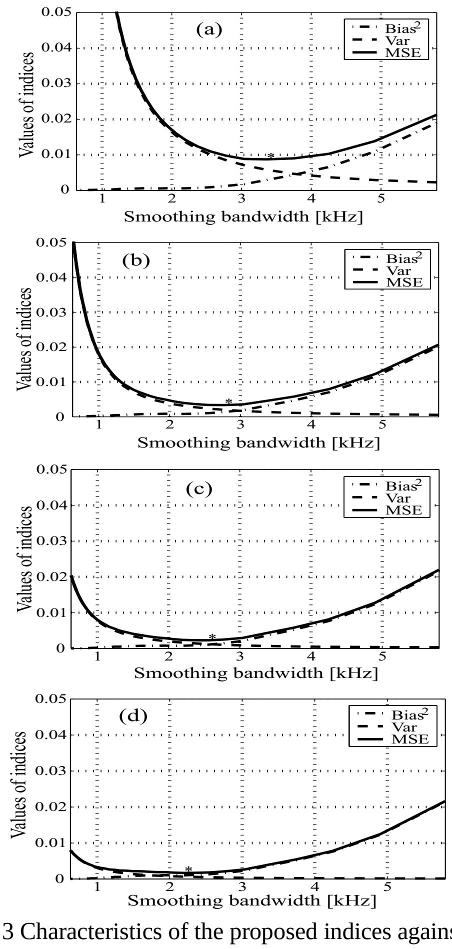

Fig. 3 Characteristics of the proposed indices against the smoothing bandwidth of the bispectral window, where (a), (b), (c), and (d) represents the results for N=2048, 4096, 8192, and 16384, respectively, with a sampling period T=0.0454ms, and an asterisk denotes the optimmum bandwidth giving the minimum of the MSE index

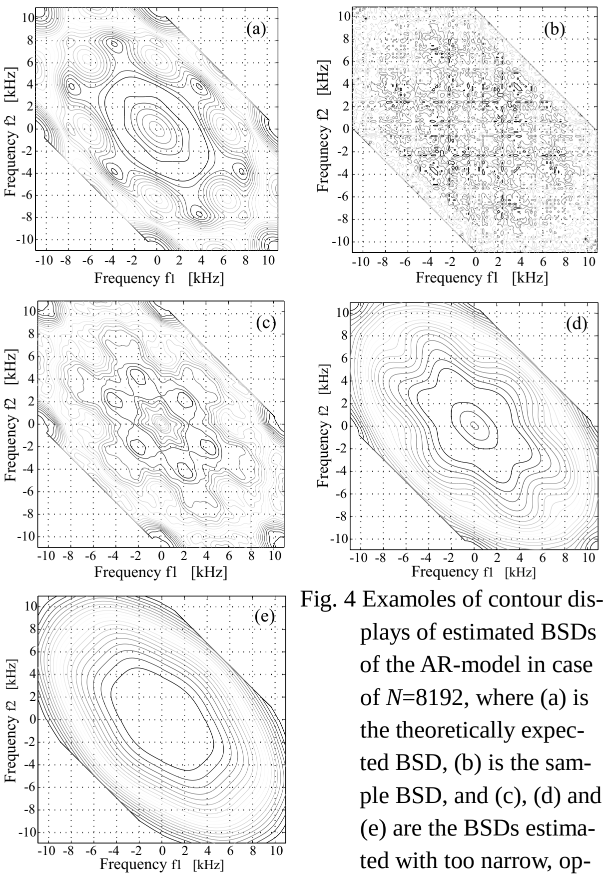

Fig. 4 Examoles of contour displays of estimated BSDs of the AR-model in case of N=8192, where (a) is the theoretically expected BSD, (b) is the sample BSD, and (c), (d) and (e) are the BSDs estimated with too narrow, optimum, and too broad bandwidth of the window, respectively.

tion, and (iii) the minimum MSE index also stays at rather large value. (iv) Although the minimum point of the MSE is more clear than that for the PSD estimation, its peak value at (4kHz,4kHz) in the fundamental bispectral plane is not clearly observed.

The reason for the aspects of (ii) to (iv) is surmised as the result that the magnitude and bandwidth of the BSD peak is generally determined by a triple product of associated frequency components, so that the small peak at the above 2D-frequencies are obscured by rather a broadband peak at 4kHz of the PSD as seen from the PSD characteristics in Fig. 2.

5. Conclusions

Toward the development of observed data oriented BSD estimation methods, an automatic determination of the smoothing bandwidth of bispectral windows is proposed by newly introducing the MSE evaluation index of BSD estimates, based on the mutually reverse relation between the bias and variance of the conventional non-parametric BSD estimate by smoothing the 3rd order periodogram. And the effectiveness of the propo- sed method is confirmed through the basic results of numerical experiments by using a numerically generated stationary non- Gaussian time series of non-zero BSD, the theoretical charac- teristics of which are known clearly.

However, not only the obtained minimum value of the MSE index is relatively large, but also its bandwidth of the optimum BSD peak is rather broad compared with those resulted by the previously proposed automatic PSD estimation due to the triple product effect of the BSD characteristics, resulting in missing of the relatively small peak in the signal BSD even with the obtained optimum smoothing bandwidth. These aspects show the difficulties and/or limitations of the BSD estimation based on the non-parametric method by smoothing the 3rd order periodogram. The possibility to overcome these problems by additional use of the parametric BSD estimation method is now under study. The detailed results will be reported in near future.

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