Design of Output Regulator for Rejecting Periodic Eccentricity Disturbance in Optical Disc Drive

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Abstract: An add-on type output regulator is proposed in this paper. By an add-on controller we mean an additional controller which operates harmonically with a pre-designed one. The role of the add-on controller is to reject a sinusoidal disturbance of unknown magnitude and phase but with known frequency. Advantages of the proposed controller include that (1) it can be used only when the performance of disturbance rejection needs to be enhanced, (2) when it is turned on or off, unwanted transient can be avoided (i.e., bumpless transfer), (3) it is designed for *perfect* disturbance rejection not just for disturbance reduction, (4) ability for perfect rejection is preserved even with uncertain plant model. This design may be promising for optical disc drive (ODD) systems in which disc eccentricity results in a sinusoidal disturbance. For ODD systems, the sensitivity function obtained by the pre-designed controller, which may have been designed by the lead-lag, H_∞, or DOB (disturbance observer) technique, does not change much with the add-on controller except at the frequency of the disturbance. Since the add-on controller does the job of rejecting major eccentricity disturbance, the gain of the pre-designed controller does not have to be too high.

Keywords: Add-on output regulator, Optical disc drive, Disturbance rejection, Track following

1. Introduction

We consider the problem of rejecting sinusoidal disturbances whose magnitude and phase are unknown but its frequency is known. A solution to this problem has been actively studied since 1970 and coined as 'output regulation' in the literature (for example, in [2,3,6]). Based on the theory, we claim that the output regulator is well applicable to optical disc drive applications—track following problem under disc eccentricity disturbance or focus control under vertical disturbance of a disc media.

In this paper, an 'add-on type' output regulator is developed. By *add-on controller* we mean an additional controller which runs harmonically with a pre-installed controller in the feedback loop. Refer to Fig. 1. For the plant $P(s)$, the controller $C(s)$ is assumed to have already been designed by any design method such as lead-lag compensator design, H_∞ design and DOB (disturbance observer) technique [5, 9, 10, 12] and so on. It is usual to design $C(s)$ so that its performance is satisfactory under general disturbances. But, for a particular situation, its performance may not be satisfactory. This is the case when there is a large sinusoidal disturbance of a specific frequency. For example, a lead-lag compensator design or a DOB design can suppress disturbances on a broad range of frequencies, but in order to reduce a large disturbance of a specific frequency, its gain should be increased to satify a design specification. Instead, we add another controller whose role is just to reject the specific disturbance with relatively small gain.

We summarize advantages of the proposed design as follows.

- The add-on controller can be freely turned on and off without disturbing the overall stability of the closed-loop system. In addition, harmful transient responses, resulted from adding another dynamic controller in the feedback loop, can be avoided. Thus, a kind of bumpless transfer is obtained.
- For optical disc drive (ODD) systems, the transfer function from the disturbance to the error that we want to regulate, which is obtained by the pre-designed controller $C(s)$, does not change much with the add-on controller $R(s)$ except at the frequency of the disturbance. In this sense, we regard that the add-on controller preserves the performance of the pre-designed controller for disturbances other than the specific sinusoidal disturbance under consideration.
- The output regulator achieves asymptotic disturbance rejection (i.e., perfect rejection). In addition, it has been shown in [1] that the perfect rejection can be preserved with uncertain plant model under some conditions. We will also show that, for ODD systems, perfect rejection is guaranteed even under the parametric uncertainty of the plant model. This is beneficial since the ODD plant model is usually obtained experimentally and may have uncertainty.

In Section 2, we propose an add-on design of output regulator. Application to optical disc drive systems is given in Section 3 with simulation and analysis on the frequency domain. Section 4 is devoted to investigation of the robust property of the proposed add-on controller and discusses on the robustness of asymptotic disturbance rejection. In order to avoid messy notation, the size of matrix and the length of vector are not explicitly mentioned throughout the paper, but they are easily understood in the context.

2. Add-on Type Output Regulator

In this section, we construct an add-on type output regulator for generic linear systems written by

$$\begin{aligned} \dot{x} &= Ax + Bu + Pw \\ e &= Cx + Qw \end{aligned} \tag{1}$$

where x is the state, u is the control input and w is the disturbance. We also suppose that the error e can be mea-

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sured while the state x is not measurable. The disturbance w is unknown except that it is (or can be thought to be) generated by

$$\dot{w} = Sw \tag{2}$$

where S is known and neutrally stable (i.e., each eigenvalue is simple and located on the $j\omega$-axis). (Therefore, the disturbance is assumed to be sinusoidal with known frequency, but its magnitude and phase is unknown since we do not assume that the initial condition $w(0)$ is known. By the matrices P and Q, we model how the disturbance vector w affects the system.) This system is called an *exosystem* in the literature of output regulation.

Our control goal is to design an error feedback controller (using only the error e) so that the closed-loop system is asymptotically stable and that the error $e(t)$ goes to zero as time goes to infinity. In our approach, the first goal of closed-loop stability is achieved by the controller $C(s)$ in Fig. 1 while the second goal of asymptotic disturbance rejection will be gained by the add-on controller $R(s)$. In particular, we propose a design method for $R(s)$ assuming that the controller $C(s)$ is pre-installed so that we do not know any information about $C(s)$ except that it stabilizes the plant $P(s)$ when there's no disturbance. This is a useful feature of our design for industry when a feedback system has been already established but we want to enhance its performance even without any knowledge on the existing controller. In the following, we summarize our situation.

Assumption 1: For the plant (1) with $w \equiv 0$, there exists a dynamic controller $C(s)$, whose realization is given by

$$\begin{aligned} \dot{z} &= Fz + Ge \\ u_c &= Hz + Je, \end{aligned} \tag{3}$$

which stabilizes the closed-loop system. In other words, the matrix

$$\begin{bmatrix} A + BJC & BH \\ GC & F \end{bmatrix}$$

is Hurwitz. ◇

Now to design the output regulator $R(s)$, we assume the following.

Assumption 2: The following two conditions hold.

1. There exist matrices Π and Γ such that

$$\Pi S = A\Pi + B\Gamma + P \tag{4}$$

$$0 = C\Pi + Q. \tag{5}$$

2. The matrix pair

$$\left(\begin{bmatrix} C & Q \end{bmatrix}, \begin{bmatrix} A & P \\ 0 & S \end{bmatrix} \right)$$

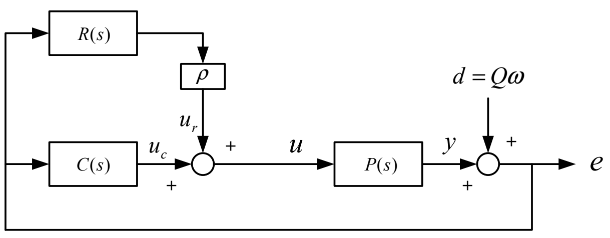

Fig. 1.

is detectable.

◇

Remark 1: For output regulation, Assumption 2 is quite standard in the literature (e.g., [2, 6, 8]), although some relaxed version of detectability (Assumption 2.2) is also available in, for example, [1, 6]. Assumption 2.1 implies that the subspace $\{(x, w) : x = \Pi w\}$ (in the state-space of x and w) can be made invariant by the feedback $u = \Gamma w$ (Eq. (4)) and that on the subspace the error e is zero since $e = Cx + Qw = (C\Pi + Q)w = 0$ (Eq. (5)). A subspace on which the error e is zero is called an *error zeroing manifold* and Assumption 2.1 is concisely expressed that the error zeroing manifold of (1) and (2) is controlled-invariant. It is actually well-known that Assumption 2, with stabilizability of (A, B), is enough to design a stabilizing output regulator, while our concern in this paper is to design an add-on output regulator on top of Assumption 1. It will be seen in Section 3 that Assumption 2 is satisfied for optical disc drive systems. ◇

Implementation of add-on output regulator is rather simple because it consists of a state observer and a state feedback gain Γ obtained in Assumption 2.1. Since we do not assume the knowledge about $C(s)$, we also measure the output of $C(s)$ and use it as in Fig. 2. The proposed add-on controller (which we denote by $\rho R(s)$ where $R(s) = [R_e(s), R_c(s)]$ according to the convention of Fig. 2) is given by

$$\dot{\xi} = \begin{pmatrix} A - K_1 C & P - K_1 Q \\ -K_2 C & S - K_2 Q \end{pmatrix} \xi + Ke + \begin{pmatrix} B \\ 0 \end{pmatrix} u \tag{6}$$

$$= \begin{pmatrix} A - K_1 C & \rho(t) B\Gamma + P - K_1 Q \\ -K_2 C & S - K_2 Q \end{pmatrix} \xi + Ke + \begin{pmatrix} B \\ 0 \end{pmatrix} u_c \tag{7}$$

$$u_r = \begin{pmatrix} 0 & \rho(t)\Gamma \end{pmatrix} \xi \tag{8}$$

where u_c is the output of $C(s)$, i.e.,

$$u_c = C(s)e, \tag{9}$$

and K_1 and K_2 are chosen such that

$$\left\{ \begin{bmatrix} A & P \\ 0 & S \end{bmatrix} - \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \begin{bmatrix} C & Q \end{bmatrix} \right\} \quad \text{is Hurwitz.}$$

The overall control is written as

$$u = u_r + u_c. \tag{10}$$

Here, the scalar variable ρ is a switching function whose role is to turn on and off the output regulator (more rigorously, it determines whether the output regulator is included in the feedback loop or not). In particular, when $\rho = 0$, only $C(s)$ is running and if $\rho = 1$, the add-on controller also takes part in the feedback as well as $C(s)$. It will be shown shortly that the overall closed-loop system is stable, in fact, for *any* value of $\rho(t)$. We can take advantage of this fact to suppress transient response which could be caused by abruptly incorporating the output regulator into the feedback loop. Indeed, in a typical situation, the general purpose controller $C(s)$ starts first with $\rho(t) = 0$. After a while, if residual vibration on the error variable is not satisfactory, $\rho(t)$ is switched to 1 for the output regulator to do its job. This transition can be smooth if we interpolate $\rho(t)$ from 0 to 1 by a slowly varying continuous signal. We will illustrate its effect in Section 3 through simulation.

Now we turn to the stability and convergence issue. By Assumption 1, it is clear that the closed-loop is stable when $\rho = 0$ (although the error $e(t)$ is not guaranteed to converge to zero). It is still left to show the stability for nonzero ρ and the error convergence with the output regulator.

Theorem 1: Under Assumptions 1 and 2, all the states of the closed-loop system (1)–(3), (6)–(10) are bounded for any time-varying bounded function $\rho(t)$. In particular, when $\rho(t) = 1$, the output error $e(t)$ converges to zero. $\diamond$

Proof: We first note that the system (6) is a state observer for the plant (1) and the exosystem (2). Indeed, by taking $e_x := \xi_x - x$ and $e_w := \xi_w - w$ (where $\xi^T = [\xi_x^T, \xi_w^T]$), we have

$$\begin{pmatrix} \dot{e}_x \\ \dot{e}_w \end{pmatrix} = \begin{pmatrix} A - K_1 C & P - K_1 Q \\ -K_2 C & S - K_2 Q \end{pmatrix} \begin{pmatrix} e_x \\ e_w \end{pmatrix},$$

which is exponentially stable. Therefore, it follows that

$$e_w(t) \to 0 \qquad \text{as} \qquad t \to \infty.$$

Now the plant (1), the controller (3) and the exosystem (2) can be written as

$$\begin{aligned}
\dot{x} &= (A + BJC)x + BHz + (P + BJQ)w + \rho B\Gamma w + \rho B\Gamma e_w \\
\dot{z} &= GCx + Fz + GQw \\
\dot{w} &= Sw.
\end{aligned}$$

With the matrix Π of Assumption 2, we define $\tilde{x} := x - \Pi w$. Then, in a new coordinates $(\tilde{x}, z, w)$ the above system becomes

$$\begin{aligned}
\dot{\tilde{x}} &= (A + BJC)\tilde{x} + (A + BJC)\Pi w + BHz + (P + BJQ)w \\
&\qquad + B\Gamma w + \rho B\Gamma e_w - (1 - \rho)B\Gamma w - \Pi S w \\
&= (A + BJC)\tilde{x} + BHz \\
&\qquad + [A\Pi + B\Gamma - \Pi S + P + BJ(C\Pi + Q)]w \\
&\qquad + B\Gamma[\rho e_w - (1 - \rho)w] \\
&= (A + BJC)\tilde{x} + BHz + B\Gamma[\rho e_w - (1 - \rho)w] \\
\dot{z} &= GC\tilde{x} + Fz + G(C\Pi + Q)w = GC\tilde{x} + Fz \\
\dot{w} &= Sw,
\end{aligned}$$

in which, (4) and (5) have been used. Here, the first two equation can be rewritten as

$$\begin{pmatrix} \dot{\tilde{x}} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} A + BJC & BH \\ GC & F \end{pmatrix} \begin{pmatrix} \tilde{x} \\ z \end{pmatrix} + \begin{pmatrix} B\Gamma \\ 0 \end{pmatrix} (\rho e_w - (1 - \rho)w).$$

Since the system matrix is Hurwitz by Assumption 1, this system is ISS (input-to-state stable) [4]. Therefore, the states $\tilde{x}$ and z are bounded for any bounded ρ since e_w is bounded and w is also bounded thank to the property of the exosystem. Finally, when $\rho(t) = 1$, the state $\tilde{x}(t)$ and $z(t)$ go to zero since the input to the system decays to zero. When, $\tilde{x}(t)$ is zero, the error $e(t)$ is also zero because

$$e(t) = Cx(t) + Qw(t) = C\tilde{x}(t) + (C\Pi + Q)w(t) = C\tilde{x}(t).$$

$\blacksquare$

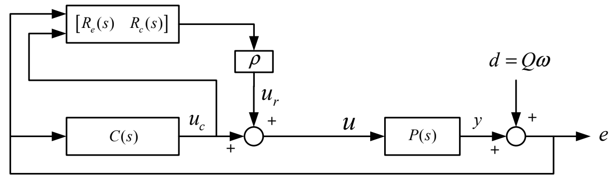

Fig. 2.

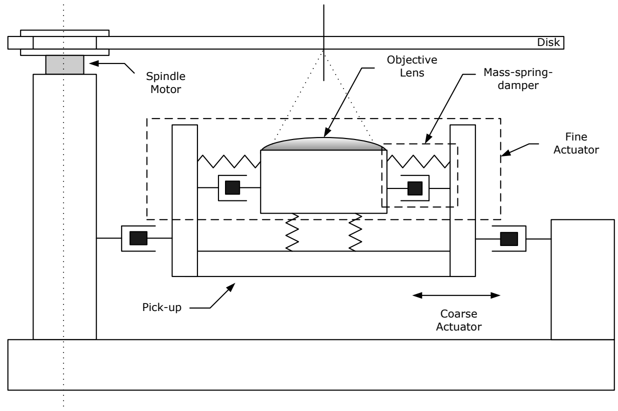

Fig. 3. Diagram of optical disk drive

3. Track Following Problem and Solution for Optical Disc Drive

Track following problem for Optical Disc Drives such as CD-ROM or DVD is to control the position of optical pick-up (more precisely, optical spot) so that it follows the desired track of optical disc media which is usually deviated from the concentric circles due to the disc eccentricity. The position of the pick-up is controlled by two cooperative actuators; a fine actuator and a coarse actuator, which are briefly depicted in Fig. 3. While the coarse actuator moves slowly across the entire disk radius, the fine actuator has faster response for a small displacement. For CD-ROM drive, the optical spot must follow the track within 0.1μm while the displacement error caused by the disc eccentricity amounts to more than 280μm in the worst case. Although the disturbance is relatively large, the fine actuator should take care of it because the frequency of the disturbance is synchronized with the disc rotation that is too fast for the coarse actuator. Therefore, the fine actuator plays a central role for track following and we thus consider the fine actuator only.

The optical pick-up (that is, the fine actuator) is effectively modeled by a mass-spring-damper system that is a second order system having full relative degree. Hence, it is always represented by

$$\dot{x} = Ax + Bu = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix} x + \begin{bmatrix} 0 \\ b \end{bmatrix} u \tag{11}$$

$$y = Cx = \begin{bmatrix} 1 & 0 \end{bmatrix} x \tag{12}$$

where u is the force and y is the position. For example, we have obtained a model of LG $\times 52$ CD-ROM drive experimentally using LDV (Laser Doppler Velocimeter), which is

$$P(s) = \frac{818.22}{s^2 + 64.73s + 166800} (m/V), \tag{13}$$

in which, the natural frequency (ω_n) is 65Hz. This is a transfer function of a VCM (Voice Coil Motor) actuator from voltage input to position output. In fact, a VCM drive circuit is used to drive the actuator, but its dynamics is ignored (except its gain) since its bandwidth is sufficiently high. Note that (13) is realized in the form of (11) and (12).

The ODD system measures the position of the pick-up by a relative position error between the desired track and the actual position of the pick-up. Therefore, the disc eccentricity affects this measure as a disturbance. We can model it as $e = y + d$ where d is the disturbance, but since this disturbance is sinusoidal whose frequency is the frequency σ of the disc spindle motor, it can be expressed by

$$e = Cx + Qw = \begin{bmatrix}1 & 0\end{bmatrix} x + \begin{bmatrix}-1 & 0\end{bmatrix} w \qquad (14)$$

$$\dot{w} = Sw = \begin{bmatrix}0 & 1\\ -\sigma^2 & 0\end{bmatrix} w, \qquad (15)$$

for the control goal that $e(t) = x_1(t) - w_1(t) \to 0$. Note that the state x and w are not measurable but the only measure is e and that the initial condition $w(0)$ is unknown which determines the magnitude and phase of disturbance. Here, the equations (11) and (14) are now in the form of (1) (with $P = 0$).

Remark 2: There is, in fact, a sensor gain K_{opt} in the feedback loop (see Fig. 4), which converts the position displacement into voltage. Our experiment shows $K_{opt} \approx 1.25 \times 10^6 V/m$, but we regarded this value as 1 for simple discussion in the above. In order to take K_{opt} into account, one may consider the plant transfer function as $K_{opt}P(s)$ instead of (13) and realize it with $K_{opt}b$ instead of b. In this case, the initial condition of disturbance is multiplied by K_{opt}, which, however, is still unknown (thus, nothing is changed). ◇

It is interesting to see that the ODD system meets Assumption 2. To see this, we have to show that there exist matrices Π and Γ such that, when the input $u = \Gamma w$ is applied, the subspace $\{(x, w) : x = \Pi w\}$ is invariant on which the error $e(t) = Cx(t) + Qw(t)$ is identically zero. If $e(t)$ is zero, it follows for (11) and (15) that

$$\begin{aligned} y(t) &= Cx(t) = x_1(t) = -Qw(t)\\ \dot{y}(t) &= x_2(t) = -QSw(t)\\ \ddot{y}(t) &= a_1x_1(t) + a_2x_2(t) + bu(t) = -QS^2w(t). \end{aligned}$$

This implies that the subspace $\{(x, w) : x_1 = -Qw, x_2 = -QSw\}$ is an invariant error zeroing manifold if $u = \frac{1}{b}(a_1Q + a_2QS - QS^2)w$ is applied. Therefore, we have

$$\Pi = \begin{bmatrix}-Q\\ -QS\end{bmatrix}, \qquad \Gamma = \frac{1}{b}(a_1Q + a_2QS - QS^2). \qquad (16)$$

This is actually thanks to the fact that the plant has full relative degree (i.e., relative degree = system order).

We also assume that a pre-installed stabilizing controller $C(s)$ exists (Assumption 1). Here, we simply assume that the following lead-lag compensator has been designed:

$$C(s) = -\frac{1.253s^2 + 6454s + 939950}{s^2 + 69713s + 5222000}. \qquad (17)$$

Remark 3: The controller $C(s)$ is usually obtained by a lead-lag compensator design, loop shaping, H_∞ and/or disturbance observer (DOB) technique [5,9,10,12]. It should be noted that *perfect* disturbance rejection is not usually achieved with these designs only. In fact, in order for perfect disturbance rejection, the closed-loop system

$$\begin{bmatrix}\dot{x}\\ \dot{z}\end{bmatrix} = \begin{bmatrix}A + BJC & BH\\ GC & F\end{bmatrix}\begin{bmatrix}x\\ z\end{bmatrix} + \begin{bmatrix}BJQ\\ GQ\end{bmatrix} w$$
$$e = Cx + Qw, \qquad \dot{w} = Sw$$

must satisfy the condition of Lemma 1 in the Appendix. Since this is usually not the case, we continue our design of add-on output regulator. ◇

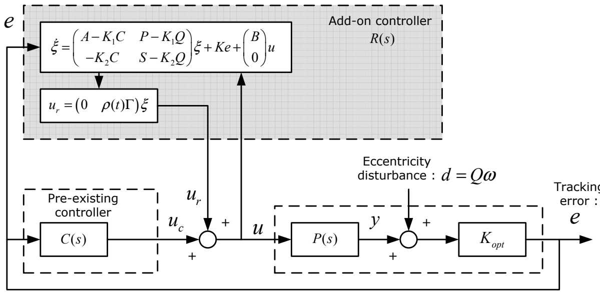

Fig. 4. System configuration

Fig. 4 describes the overall system where $R(s)$ is the add-on output regulator of (6). (Note that Fig. 4 is an implementation of (6) while Fig. 2 is with (7) which is equivalent.) In order to illustrate the effectiveness of the proposed controller, a computer simulation is carried out. For the simulation, the spindle motor frequency is chosen as 62Hz (3720 rpm; $\sigma = 2\pi \cdot 62$) and the observer gain K_1 and K_2 are selected as

$$K_1 = \begin{bmatrix}29.3\\ -4798\end{bmatrix}, \qquad K_2 = \begin{bmatrix}-99.7\\ -3122\end{bmatrix}. \qquad (18)$$

Also, to be realistic, all the controllers are discretized by Tustin's method with 88.2kHz sampling rate and $\pm 10V$, 16-bit A/D and D/A are included in the simulation. Finally, the plant parameters a_1, a_2 and b have been perturbed by $\pm 20\%$ (for example, a_1 and b are increased and a_2 is decreased). The results are illustrated in Fig. 5. It should be noted that the add-on controller smoothly enters the stage by a ramp-type signal $\rho(\cdot)$ and the output of $C(s)$ diminishes as well as the tracking error e. Perfect rejection of disturbance can be observed even for the *perturbed* plant. Justification of this nice property will be given in Section 4.

Now for ODD systems, we analyze the performance of the add-on controller on the frequency domain. When $\rho = 0$, the transfer function from the disturbance d to the error e is clearly given by (see Fig. 2)

$$S_{\rho=0}(s) = \frac{1}{1 - P(s)C(s)}.$$

The transfer function from d to e for $\rho = 1$ can also be calculated. Indeed, by referring to (7), two transfer functions $R_e(s)$ and $R_c(s)$ are given by

$$\left[\begin{array}{c|c} \begin{pmatrix}A - K_1C & B\Gamma - K_1Q\\ -K_2C & S - K_2Q\end{pmatrix} & * \\ \hline \begin{pmatrix}0 & \Gamma\end{pmatrix} & 0\end{array}\right]$$

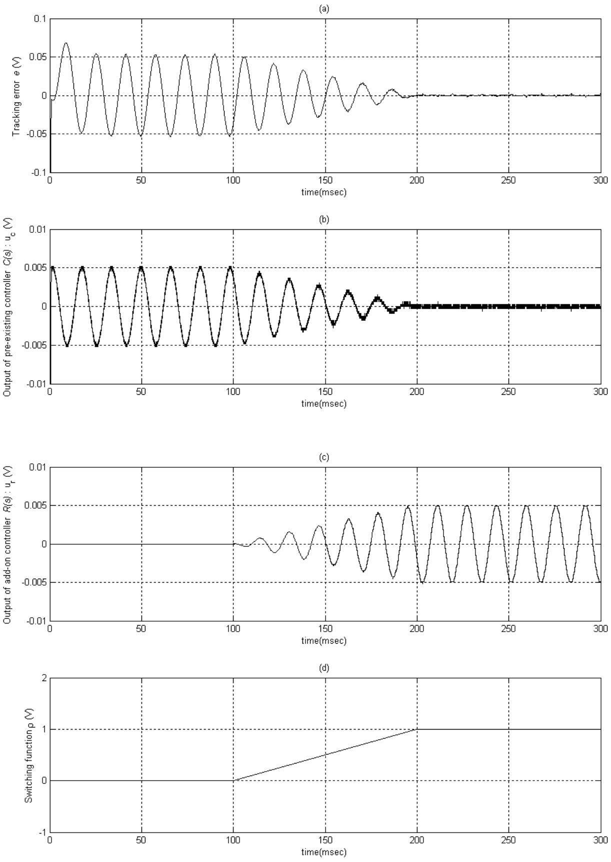

Fig. 5. Simulation results. (a) Tracking error e. (b) Output of the pre-installed controller $C(s)$. (c) Output of the add-on controller $R(s)$. (d) The signal ρ which starts increasing at 100$m$sec. This result is for a plant whose parameters are perturbed by $\pm 20\%$ from its nominal value.

where $* = [K_1^T, K_2^T]^T$ for $R_e(s)$ and $* = [B^T, 0]^T$ for $R_c(s)$, respectively. (Note that P of (7) is zero for ODD systems.) Again from Fig. 2, we have

$$S_{\rho=1}(s) = \frac{1}{1 - (C(s) + C(s)R_c(s) + R_e(s))\, P(s)}.$$

Fig. 6 shows the comparison between $S_{\rho=0}(s)$ and $S_{\rho=1}(s)$ when the observer gain of (18) is used. It can be noted that two functions are not very different except at the spindle frequency σ (where perfect rejection of disturbance is achieved). This means our design does not alter the sensitivity function obtained by the pre-designed controller for high and low frequencies, which is another advantage of our design for ODD systems.

4. Output Regulation for Uncertain ODD Plant

Up to now, we have studied stability and performance of disturbance rejection with a *nominal* plant. However, some uncertainty in the plant model is unavoidable because the ODD plant model is usually obtained experimentally. We suppose that the real ODD system is given by

$$\dot{x} = A_\mu x + B_\mu u = \begin{bmatrix} 0 & 1 \\ a_1 + \mu_1 & a_2 + \mu_2 \end{bmatrix} x + \begin{bmatrix} 0 \\ b + \mu_0 \end{bmatrix} u$$
$$y = Cx = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

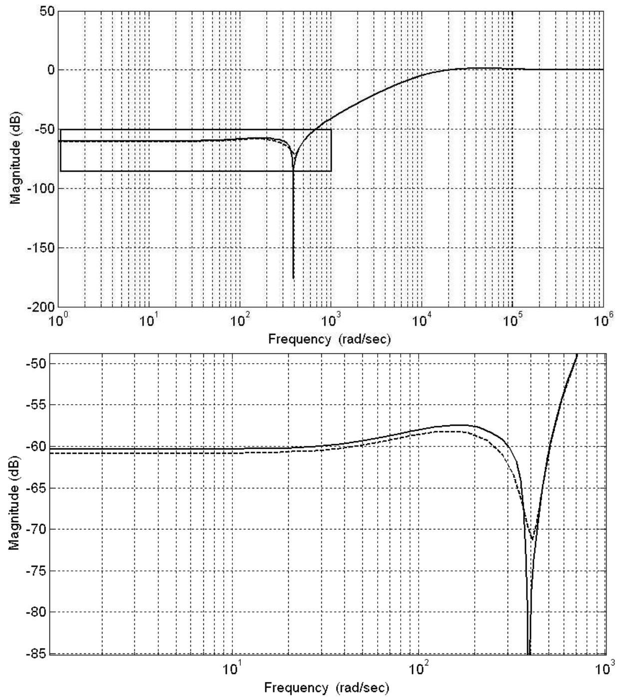

Fig. 6. Bode magnitude plot of $S_{\rho=0}(s)$ (dashed) and $S_{\rho=1}(s)$ (solid). The bottom one is the enlarged version of the top.

where the unknown constants (μ_1, μ_2, μ_0) belong to an admissible parameter set $\mathcal{P} \subset \mathbb{R}^3$ which contains the origin. It is assumed that b and $b + \mu_0$ have the same sign for all admissible μ_0. Then, the overall closed-loop system with the real plant when $\rho = 1$ is written as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{z} \\ \dot{\xi}_x \\ \dot{\xi}_w \end{bmatrix} = \begin{bmatrix} A_\mu + B_\mu JC & B_\mu H & 0 & B_\mu \Gamma \\ GC & F & 0 & 0 \\ K_1C + BJC & BH & A - K_1C & -K_1Q + B\Gamma \\ K_2C & 0 & -K_2C & S - K_2Q \end{bmatrix} \times \begin{bmatrix} x \\ z \\ \xi_x \\ \xi_w \end{bmatrix} + \begin{bmatrix} B_\mu JQ \\ GQ \\ K_1Q + BJQ \\ K_2Q \end{bmatrix} w \qquad (19)$$
$$\dot{w} = Sw$$
$$e = Cx + Qw.$$

In order to ensure robust stability, the system matrix in the above should be Hurwitz for every μ_i's in $\mathcal{P}$. Unfortunately, we don't have much to say about the robust stability because it is affected by the pre-designed controller $C(s)$ (i.e., by the values of F, G, H and J) and we assumed in this paper that any information on $C(s)$ is not known except that it stabilizes the nominal plant. However, for a discussion about the robust output regulation, it is assumed in the following that the above system matrix is Hurwitz for admissible parameters of (μ_1, μ_2, μ_0).

Now, we investigate the question whether the proposed add-on controller can regulate the error $e(t)$ even for the real ODD plant that may be different from the nominal one. This question is answered by applying Lemma 1 in the Appendix to the overall system (19). Indeed, Lemma 1 tells us that if (and only if) there exist matrices Π_1, Π_2, Π_3 and Π_4 such that

$$
\begin{aligned}
\Pi_1 S &= A_\mu \Pi_1 + B_\mu H \Pi_2 + B_\mu \Gamma \Pi_4 && (20)\\
\Pi_2 S &= F \Pi_2 && (21)\\
\Pi_3 S &= BH\Pi_2 + (A - K_1 C)\Pi_3 + (-K_1 Q + B\Gamma)\Pi_4 && (22)\\
\Pi_4 S &= -K_2 C \Pi_3 + (S - K_2 Q)\Pi_4 && (23)\\
0 &= C\Pi_1 + Q && (24)
\end{aligned}
$$

for every (μ_1, μ_2, μ_0), then robust output regulation is achieved. (In fact, we have used (24) to derive a simple expression as above.)

We first suppose that F does not have $\pm j\sigma$ as its eigenvalue. Then, the Lyapunov equation (21) has a unique solution $\Pi_2 = 0$. Equation (20), therefore, becomes

$$\Pi_1 S = A_\mu \Pi_1 + B_\mu \Gamma \Pi_4. \qquad (25)$$

Note that A_μ and B_μ have full relative degree for any μ_i's. From (24) and (25), it is seen that $\Pi_1 = I$ by the same argument as when we get (16). Keeping these in mind, we now show that there exists a matrix Π_* so that $(\Pi_1, \Pi_2, \Pi_3, \Pi_4) = (I, 0, \Pi_*, \Pi_*)$ is the unique solution for (20)–(24). Indeed, with this candidate Π_i's, it is easily seen that (21) and (24) are trivially met and (22) and (23) become the same one $\Pi_* S = S\Pi_*$. Thus, they are reduced to

$$
\begin{aligned}
B_\mu \Gamma \Pi_* &= S - A_\mu && (26)\\
\Pi_* S - S\Pi_* &= 0. && (27)
\end{aligned}
$$

By converting (27) to a linear equation using stacking operator and Kronecker product (see, e.g., Appendix of [7]), we get

$$\Pi_* = \begin{bmatrix} \alpha & -\beta \\ \sigma^2\beta & \alpha \end{bmatrix}$$

where α and β are undetermined yet. Finally, by converting (26) to a linear equation with the above Π_*, it boils down (through a tedious calculation) to

$$(b + \mu_0)\begin{bmatrix} \Gamma_1 & \sigma^2\Gamma_2 \\ \Gamma_2 & -\Gamma_1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} -\sigma^2 - a_1 - \mu_1 \\ -a_2 - \mu_2 \end{bmatrix}.$$

This equation has the solution if and only if $\Gamma_1^2 + \sigma^2\Gamma_2^2 \neq 0$ which is always true. Therefore, it is concluded that robust disturbance rejection is achieved for any μ_i's in $\mathcal{P}$.

5. Conclusions

For the proposed approach to be more practical in the ODD industry, it would be better if the assumption is removed that the frequency of the sinusoidal disturbance is known. In the current paper, the estimation of the disturbance is only valid when it has a constant frequency, so that the add-on controller can only be applied when the spindle motor is at its steady-state. Since the speed of rotation usually varies in the prevalent optical disc drives, the proposed theory needs to be extended. Indeed, some theoretical works are already available in this respect (see for example [11]), but a more practical solution have to be further investigated.

Appendix

Lemma 1: Consider a system given by

$$
\begin{aligned}
\dot{x} &= \overline{A}x + \overline{P}w\\
\dot{w} &= Sw && (28)\\
e &= \overline{C}x + Qw
\end{aligned}
$$

where $\overline{A}$ is Hurwitz and the eigenvalues of S are distinct and located at the $j\omega$-axis. For every initial condition $(x(0), w(0))$,

$$e(t) \to 0 \qquad \text{as} \qquad t \to \infty$$

if and only if there exists a matrix Π such that

$$\Pi S = \overline{A}\Pi + \overline{P} \qquad \text{and} \qquad 0 = \overline{C}\Pi + Q. \qquad (29)$$

◇

Proof: The proof is well-known, but is included here for completeness. Since $\overline{A}$ is Hurwitz and S is neutrally stable, the solution $(x(t), w(t))$ does not go unbounded for each initial condition $(x(0), w(0))$.

For sufficiency, take a new coordinate $(\tilde{x}, w)$ as $(x - \Pi w, w)$. Then, we have

$$
\begin{aligned}
\dot{\tilde{x}} &= \overline{A}x + \overline{P}w - \Pi S w\\
&= \overline{A}\tilde{x} + (\overline{A}\Pi - \Pi S + \overline{P})w = \overline{A}\tilde{x} && (30)\\
e &= \overline{C}x + Qw = \overline{C}\tilde{x} + (C\Pi + Q)w = \overline{C}\tilde{x},
\end{aligned}
$$

from which it follows that $e(t) \to 0$ as time goes to infinity.

For necessity, suppose that the assumption (29) does not hold. Since the Lyapunov equation $\Pi S = \overline{A}\Pi + \overline{P}$ always has the unique solution Π, this implies that $\overline{C}\Pi + Q \neq 0$. Then, from (30), $e(t) \to (\overline{C}\Pi + Q)w(t)$, which is not identically zero. ■

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