Descriptor Type Linear Parameter Dependent System Modeling And Control of Lagrange Dynamics

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Abstract: In this paper, the Lagrange dynamics is studied. A state space representation of Lagrange dynamics and control algorithm based on the state feedback pole placement are presented. The state space model presented is descriptor type linear parameter dependent system. It is shown that the control algorithms based on the linear system theory can be applicable to the state space representation of Lagrange dynamics. To show that the linear system theory can be applicable to the state space representation of Lagrange dynamics, the LMI based regional pole-placement design algorithm is developed and present two examples.

Keywords: Descriptor System, LPD System, Lagrange dynamics. LMI Region, Pole-placement, Regional pole placement.

1. INTRODUCTION

Most of the mechanical systems and physical apparatus are modeled by the Lagrange dynamics. In the robotics literatures, most of researchers on the control of constrained mechanical systems have been focused on the systems in which the constrained motion is modeled holonomic constraints. Since 1980s, the analysis and control of nonholonomic systems have been studied.[1]

The Lagrange dynamics are based on the derivatives of energy with respect to time and coordinates. It is known that, for complex systems, the Lagrange dynamics is easier than the Newton dynamics.[2] There are various physical systems which are subject to some constraints and these constraints should be satisfied during the motion.[3] And for complex systems, which can be modeled easily by Lagrange dynamics, the model equation includes highly coupled nonlinear terms. Because of these reasons, the analysis and control of Lagrange dynamics systems is very complex and the results of works related to the analysis and control of Lagrange dynamics systems are conservative.

In this paper, the Lagrange dynamics is studied. A state space representation of Lagrange dynamics and control algorithm based on the state feedback pole placement are presented. The state space model presented is descriptor type linear parameter dependent system. It is shown that the control algorithms based on the linear system theory can be applicable to the state space representation of Lagrange dynamics. To show that the linear system theory can be applicable to the state space representation of Lagrange dynamics, the LMI based regional pole-placement design algorithm is developed and present two examples.

2. DESCRIPTER LPD SYSTEM AND LAGRANGE DYNAMICS

This section summarizes some definitions of previous works about descriptor system and LPD system. And the Lagrange dynamics are introduced.

2.1 Descriptor LPD System

Before introducing the LPD system, we need to define the set of all admissible parameter trajectories.

Definition 1[4]. Given a compact set $P \subset R^S$, the parameter set F_P denote the set of all piecewise continuous functions mapping R^+ into P with finite number of discontinuities in any interval.

By the definition 1, the parameter value $\rho_i \in F_P$ are differentiable with respect to time. It is assumed in this paper that the parameter value is bounded, i.e.,

$$\left|\rho_i\right| \le \delta \tag{1}$$

The state-space representation of descriptor system is

$$\begin{aligned} E(\rho)\dot{x}(t) &= A(\rho)x(t) + B(\rho)u(t) \\ y(t) &= C(\rho)x(t) \end{aligned} \tag{2}$$

where, $E \in R^{n\times n}$, $A \in R^{n\times n}$, $B \in R^{p\times n}$, $C \in R^{q\times n}$ which are time-varying and u is p dimensional inputs, y is q dimensional outputs. In this paper, the matrix E is assumed to be non-singular for all possible parameter value $\rho \in F_P$. For DLPD system, the quadratic stability is defined by following definition.

Corollary 1. The system matrices E and A are constant matrices. The system described by equation (1) is quadratically stable if there exist a positive definite matrix P and Q such that the following equation is hold.

$$A^T PE + EPA < 0 \tag{3.a}$$

$$\dot{E}^T PE + E^T P\dot{E} < 0 \tag{3.b}$$

where, the matrix E is non-singular.

Proof) The proof of this corollary is very simple. Let $z(t) = E(\rho)x(t)$, then the system is

$$\dot{z}(t) = \left(\dot{E} + EA\right)E^{-1}z(t) + EBu(t) \tag{4}$$

The stability of $z(t)$ is guaranteed by equations (3.a) and (3.b). And the stability of $x(t)$ is guaranteed by the stability of $z(t)$. QED.

The controllability and controllability are summarized by following corollaries.

Corollary 1. The descriptor system described by the equation (1) is controllable if the matrix $E(\rho) \in R^{n\times n}$ is nonsingular for all possible ρ and

$$rank\begin{bmatrix} A(\rho) & B(\rho) \end{bmatrix} = n \quad (5)$$

Corollary 2. The descriptor system described by the equation (1) is observable if the matrix $E(\rho) \in R^{n\times n}$ is nonsingular for all possible ρ and

$$rank\begin{bmatrix} A(\rho) & C(\rho) \end{bmatrix} = n \quad (6)$$

These corollaries are important in this paper because the controller presented in this paper is based on the state feedback and the system described by the equation (2) is assumed to be controllable and observable.

2.3 Lagrange Dynamics

The Lagrange dynamics are based on the derivatives of energy with respect to time and coordinates. It is known that, for complex systems, the Lagrange dynamics is easier than the Newton dynamics. The Lagrange dynamics are derived by following steps.

Let the kinetic energy of the system be K and potential energy be P then, the Lagrange matrix is defined by

$$L = K - P \quad (7)$$

The Lagrange dynamics are obtained by following equations

$$F_i = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} \quad (8)$$

$$T_i = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i} \quad (9)$$

where, q_i is axis of generalized coordinates related to directional movement and F_i is sum of all forces related to the directional movement. And $\dot{\theta}_i$ is of generalized coordinates related to revolute movement and T_i is sum of all forces related to the revolute movement. Now, define the axis of the generalized coordinate v as

$$v = \begin{bmatrix} q & \theta \end{bmatrix}^T \quad (10)$$

then, equation (8) and (9) are expressed by the following matrix form

$$M(v)\ddot{v} + C(v,\dot{v})\dot{v} = B_L(v)\tau - A^T(v)\lambda \quad (11)$$

In the equation (11), the matrix $M(v)$ is inertia matrix, $C(v,\dot{v})$ is coriolis and centrifugal forces, $B_L(v)$ is input matrix and $A(v)$ is $n \times m$ Jacobian matrix. The variable λ is Lagrange multiplier the physical meaning of which is constrained force. Thus, the equation (11) is the model of physical system with m order constrained forces.

3. MODELLING AND CONTROL OF LAGRANGE DYNAMICS

3.1 Constraint Equations on the Lagrange Dynamics

There are various physical systems which are subject to some constraints and these constraints should be satisfied during the motion. The Lagrange dynamics described by the equation (11) have m -order force constraints. These constraints can be represented in matrix form as.

$$A(v)\dot{v} = 0 \quad (12)$$

And $A(v)$ is made up of the vector functions $\alpha_i(v)$ as follows

$$A(v) = \begin{bmatrix} \alpha_1(v) & \alpha_2(v) & \cdots & \alpha_m(v) \end{bmatrix} \quad (13)$$

It is clear that is $A(v)$ has a full rank for all v, then the m constraints are independent. Otherwise, by proper row-operations of $A(v)$, a new set of constraints can be found. It is assumed in this paper that $A(v)$ is a full row-rank, i.e., the system has m independent constraints. Then we can found a set of $n-m$ smooth and linearly independent vector fields in the null space of $A(v)$, denoted by $\mathrm{N}(A)$. Let $S(v)$ be the full rank matrix made up of these vectors,

$$S(v) = \begin{bmatrix} s_1(v) & s_2(v) & \cdots & s_{n-m}(v) \end{bmatrix} \quad (14)$$

Because the matrix $S(v)$ is made up of vector fields in the null space of $A(v)$, the following relation must be hold for all v.

$$S(v)A(v) = 0 \quad (15)$$

By pre-multiplication of the matrix $S(v)$ to the equation (11), we can obtain

$$S(v)M(v)\ddot{v} + S(v)C(v,\dot{v})\dot{v} = S(v)B_L(v)\tau \quad (16)$$

In order to obtain the state space representation of the equation (16), let us define the state variables as

$$\begin{aligned} x_1(t) &:= v(t) \\ x_2(t) &:= \dot{v}(t) \end{aligned} \quad (17)$$

and parameters as

$$\begin{aligned} \rho_1(t) &= v(t) \\ \rho_2(t) &= \dot{v}(t) \end{aligned} \quad (18)$$

The state space representation of the equation (16) is

$$\begin{bmatrix} I & 0 \\ 0 & S(\rho_1,\rho_2)M(\rho_1,\rho_2) \end{bmatrix}\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & -S(\rho_1,\rho_2)C(\rho_1,\rho_2) \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ S(\rho_1,\rho_2)B_L(\rho_1) \end{bmatrix}u(t) \quad (19)$$

If we select the outputs as velocities $\dot{v}(t)$ then the output equation is

$$y(t) = \begin{bmatrix} 0 & I \end{bmatrix}x(t) \quad (20)$$

The equation (19) and (20) is the DLPD system for Lagrange dynamic equation.

3.2 Controller Structure

The most important control strategy of physical systems is reference tracking. To achieve this objective, the control structure is shown by figure 1.

In figure 1, parameters in the block are all parameter dependent. It is shown in figure 1 that the controller has two control parameters one of which is state feedback and the other is control gain with integrator. The input signal is described by

$$u(t) = -F(\rho)x(t) + K(\rho)\int e(t)dt \quad (21)$$

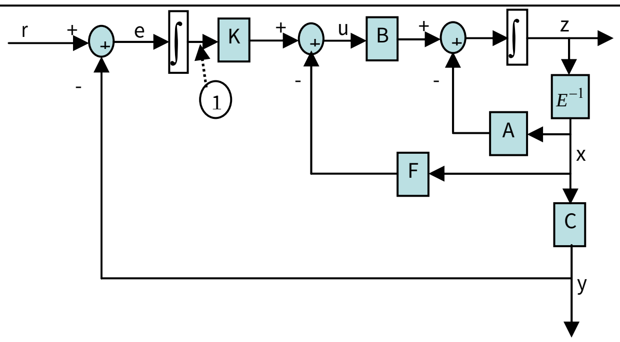

Fig. 1. Controller structure

where $F(\rho)$ is a parameter dependent state feedback gain matrix and $K(\rho)$ is a integrator gain matrix. Note that because of controller parameters $F(\rho)$ and $K(\rho)$ are time-varying parameter dependent, it is very complicated and complex to computing controller gains. In order for obtain controller gains $F(\rho)$ and $K(\rho)$, it is needed to simplify control input or controller structure. The new state x_{n+1} can be defined at Ⅰ in the figure 1. Then the dynamic equation becomes

$$\begin{bmatrix} E(\rho) & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \dot{x}(t) \\ \dot{x}_{n+1}(t) \end{bmatrix} = \begin{bmatrix} A(\rho) & 0 \\ -C(\rho) & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ x_{n+1}(t) \end{bmatrix} + \begin{bmatrix} B(\rho) \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ I \end{bmatrix} r(t) \quad (22.a)$$

$$y(t) = \begin{bmatrix} C(\rho) & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ x_{n+1}(t) \end{bmatrix} \quad (22.b)$$

and, the control input is

$$u(t) = \begin{bmatrix} F(\rho) & K(\rho) \end{bmatrix}\begin{bmatrix} x(t) \\ x_{n+1}(t) \end{bmatrix} \quad (23)$$

It is known by the equation (23) that the control input is state feedback for the system described by the equation (22). The closed loop dynamics is

$$\begin{bmatrix} E(\rho) & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \dot{x}(t) \\ \dot{x}_{n+1}(t) \end{bmatrix} = \begin{bmatrix} A(\rho)-B(\rho)F(\rho) & -B(\rho)K(\rho) \\ -C(\rho) & 0 \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ x_{n+1}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} r(t) \quad (24.a)$$

$$y(t) = \begin{bmatrix} C(\rho) & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ x_{n+1}(t) \end{bmatrix} \quad (24.b)$$

By substituting parameters in the equation (19) into the equation (22.a), we can obtain the state space representation of Lagrange dynamics as

$$\begin{bmatrix} I & 0 & 0 \\ 0 & S(\rho)M(\rho) & 0 \\ 0 & 0 & I \end{bmatrix}\begin{bmatrix} \dot{v} \\ \ddot{v} \\ \dot{x}_{n+1} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \\ 0 & -S(\rho)C(\rho) & 0 \\ 0 & -I & 0 \end{bmatrix}\begin{bmatrix} v \\ \dot{v} \\ x_{n+1} \end{bmatrix} + \begin{bmatrix} 0 \\ S(\rho)B_L(\rho) \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} r(t) \quad (25.a)$$

$$y(t) = \begin{bmatrix} 0 & I & 0 \end{bmatrix}\begin{bmatrix} v(t) & \dot{v}(t) & x_{n+1}(t) \end{bmatrix}^T \quad (25.b)$$

And by substituting parameters in the equation (19) into the equation (24.a) and (24.b), we can obtain the state space representation of closed loop Lagrange dynamics as

$$E_{CL}(\rho)\dot{x}(t) = A_{CL}(\rho)x(t) + B_{CL}r(t) \quad (26.a)$$

$$y(t) = C_{CL}x(t) \quad (26.b)$$

where, $x(t) = \begin{bmatrix} v(t) & \dot{v}(t) & x_{n+1} \end{bmatrix}^T$ and

$$E_{CL}(\rho) = \begin{bmatrix} I & 0 & 0 \\ 0 & S(\rho)M(\rho) & 0 \\ 0 & 0 & I \end{bmatrix}$$

$$A_{CL}(\rho) = \begin{bmatrix} 0 & I & 0 \\ -SB_LF_1(\rho) & -S[C(\rho)+B_LF_2](\rho) & -SB_LK(\rho) \\ 0 & -I & 0 \end{bmatrix}$$

$$B_{CL} = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}, \quad C_{CL} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

The equation (26.a) shows the closed loop poles of the Lagrange dynamics is the eigen-values of the matrix $E_{CL}^{-1}A_{CL}$. The algorithm obtaining controller gains $F(\rho)$ and $K(\rho)$ are derived following subsection.

3.3 Regional Pole Placement of Lagrange Dynamics

The LMI region is defined following definition[5].

Definition 1. LMI regions are convex subset D of the complex plan characterized by

$$D = \left\{ z \in C : L + Mz + M^T z^* \right\} \quad (27)$$

where M and L are fixed real matrices, and z and z^* are complex valued scalar and its complex conjugate pair.

The matrix valued function

$$f_D(z) \triangleq L + Mz + M^T z^* \quad (28)$$

is called the characteristic function of the region D.

Following tree theorems describe the regional pole-placement conditions and the main results of this paper.

Theorem 1[5]: The closed loop poles lie in the LMI region D

$$D = \left\{ z \in C : L + Mz + M^T z^* \right\} \quad (29)$$

where,

$$L = L^T = \left[\lambda_{jk} \right]_{1 \le j,\, k \le m}, \quad M = \left[m_{jk} \right]_{1 \le j,\, k \le m}$$

if and only if there exists a symmetric matrix X satisfying following four inequalities.

$$\left[\lambda_{jk} X + m_{jk} A_{cl} X + m_{kj} A_{cl}^T \right]_{1 \le j,\, k \le m} < 0 \quad (30)$$

$$X > 0$$

proof) Proof of this theorem is omitted and refer Chilali and Gahinet's work [5]. QED.

We are now state a local pole placement. The *i*-th parameter ρ_i is sampled by k which denoted as ρ_i^k. Let $M^{i,j,\cdots,l}(E,A,B,C)$ be the model obtained by substituting the first parameter values as i-th sample, the second parameter as j-th sample, etc.. And select a function $\mu(\rho^{i,j,\cdots,l})$, a local convex function, then the model can be approximated by

$$M(\rho) = M_{ap}(\rho) + \Delta M(\rho) \quad (31.a)$$

$$M_{ap}(\rho)=\begin{bmatrix}\mu(\rho^{i,j,\cdots l})M^{i,j,\cdots l}+\mu(\rho^{i-1,j,\cdots l})M^{i-1,j,\cdots l}\\ +\mu(\rho^{i,j-1,\cdots l})M^{i,j-1,\cdots l}\\ +\cdots+\mu(\rho^{i,j,\cdots l-1})M^{i,j,\cdots l-1}\end{bmatrix} \quad (31.b)$$

where

$$\mu(\rho^{i,j,\cdots l})+\mu(\rho^{i-1,j,\cdots l})+\mu(\rho^{i,j-1,\cdots l}) +\cdots+\mu(\rho^{i,j,\cdots l-1})=1 \quad (32)$$

The approximation described by the equation (31.b) is reasonable because the parameter value is assumed to be continuous function of time and parameters (velocities) of the Lagrange dynamic system can be measured. For appropriately selected function $\mu(\rho^{i,j,\cdots l})$, which is a convex function between [(i,j,⋯l)~(i-1,j,⋯l)], the approximation error is small enough. The following theorem states the algorithm of obtaining the controller gain matrix for $M^{i,j,\cdots,l}(E,A,B,C)$.

Theorem 2: For $M^{i,j,\cdots,l}$ (E,A,B,C), the closed loop poles lie in the LMI region D

$$D=\left\{z\in C: L+Mz+M^Tz^*\right\}$$

where,

$$L=L^T=\left[\lambda_{jk}\right]_{1\le j,\,k\le m},\quad M=\left[m_{jk}\right]_{1\le j,\,k\le m}$$

if and only if there exists a symmetric matrix X satisfying following four inequalities.

$$\left[\lambda_{jk}X+m_{jk}A_{cl}^{i,j,\cdots l}X+m_{kj}A_{cl}^{i,j,\cdots l^T}\right]_{1\le j,\,k\le m}<0 \quad (33)$$

$$X>0$$

proof) Proof of this theorem is simple extension of theorem 1. QED

Theorem 2 states the local regional pole placement of the $M^{i,j,\cdots,l}(E,A,B,C)$. Because the equation (33) is not convex, we cannot obtain the controller gain matrix. Define $Y^{i,j,\cdots l}\triangleq F^{i,j,\cdots l}X$, then conditions of local pole placement is summarized by theorem 3.

Theorem 3: The closed loop poles lie in the LMI region D if and only if there exists a symmetric matrix X satisfying following inequalities.

$$\left[\begin{array}{l}\lambda_{jk}X+m_{jk}\begin{pmatrix}E^{i,j,\cdots l^{-1}}A^{i,j,\cdots l}X\\ +E^{i,j,\cdots l^{-1}}B^{i,j,\cdots l}Y^{i,j,\cdots l}\end{pmatrix}\\ +m_{kj}\begin{pmatrix}E^{i,j,\cdots l^{-1}}A^{i,j,\cdots l}X\\ +E^{i,j,\cdots l^{-1}}B^{i,j,\cdots l}Y^{i,j,\cdots l}\end{pmatrix}^T\end{array}\right]_{1\le j,\,k\le m}<0 \quad (35)$$

$$X>0$$

the i-th state-feedback gain matrix is

$$\left[F^{i,j,\cdots l}\quad K^{i,j,\cdots l}\right]=Y^{i,j,\cdots l}X^{-1} \quad (36)$$

proof) The proof of this theorem is very simple extension of the results of Chilali and Gahinet's work [5]. QED.

The theorem 2 and theorem 3 shows the local regional pole-placement condition and the way of finding local controller gains. The global pole-placement condition and global controller gain can be achieved by using approximated plant. In order for global pole-placement, the control input, made up of local controller gain, is selected by

$$u(t)=-\begin{bmatrix}\mu^{i,j,\cdots l}(\rho)F^{i,j,\cdots l}+\mu^{i-1,j,\cdots l}(\rho)F^{i-1,j,\cdots l}\\ +\mu^{i,j-1,\cdots l}(\rho)F^{i,j-1,\cdots l}\cdots+\mu^{i,j,\cdots l-1}(\rho)F^{i,j,\cdots l-1}\end{bmatrix}x(t) +\begin{bmatrix}\mu^{i,j,\cdots l}(\rho)K^{i,j,\cdots l}+\mu^{i-1,j,\cdots l}(\rho)K^{i-1,j,\cdots l}\\ +\mu^{i,j-1,\cdots l}(\rho)K^{i,j-1,\cdots l}\cdots+\mu^{i,j,\cdots l-1}(\rho)K^{i,j,\cdots l-1}\end{bmatrix}\int e(t)dt \quad (37)$$

By noting the equation (37), the controller gain is made up of local controller gains and which is convex combination of local controller gains between [(i,j,⋯l)~(i-1,j,⋯l)]. The following theorem states the global regional pole-placement.

Theorem 4. Assume that the plant model is approximated by the equation (31) and local controller gains are obtained by the equation (36) for all parameter sampled points. Then the closed loop poles are lie in the desired region.

Proof). The proof of this theorem is very simple extension of the results of Chilali and Gahinet's work. QED.

The theorem 4 states the global global-placement condition and controller design procedure is summarized as 1) sampliing model 2) design local controller 3) combine it.

4. EXAMPLES

In this section we show an example which is inverted pendulum system. Inverted pendulum system is shown Fig. 2 and its dynamic equation is

$$\begin{aligned}&(J+ml^2)\ddot{\theta}+ml\cos\theta\,\ddot{x}-mgl\sin\theta=0\\ &ml\cos\theta\ddot{\theta}+M\ddot{x}-ml\dot{\theta}\sin\theta\dot{\theta}=F\end{aligned} \quad (38)$$

Matrix form of dynamic equation is,

$$\begin{bmatrix}(J+ml^2) & ml\cos\theta\\ ml\cos\theta & M\end{bmatrix}\begin{bmatrix}\ddot{\theta}\\ \ddot{x}\end{bmatrix}-\begin{bmatrix}0 & 0\\ 0 & ml\dot{\theta}\sin\theta\end{bmatrix}\dot{\theta}=\begin{bmatrix}0\\ F\end{bmatrix}-\begin{bmatrix}mgl\sin\theta\\ 0\end{bmatrix}$$

Because the angle θ is measurable, we can select parameter value as $\rho_1=\cos\theta$, $\rho_2=\sin\theta$ and $\rho 3=\dot{\theta}$. The state space realization is

$$\begin{bmatrix}1 & 0 & 0\\ 0 & M & ml\rho_1\\ 0 & ml\rho_1 & J+ml^2\end{bmatrix}\begin{bmatrix}\ddot{x}\\ \dot{\theta}\\ \ddot{\theta}\end{bmatrix}=\begin{bmatrix}0 & mgl\rho_2/\rho_3 & 0\\ 0 & 0 & 1\\ 0 & mgl\rho_2\rho_3 & 0\end{bmatrix}\begin{bmatrix}\dot{x}\\ \theta\\ \dot{\theta}\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}F$$

Table 1 is the result of pole placement algorithm for possible parameter values. The parameter value is selected as the range $-\frac{\pi}{6}\le\theta\le\frac{\pi}{6}, -1\le\dot{\theta}\le 1$

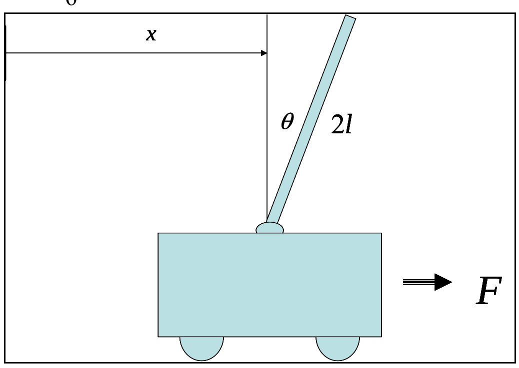

Fig. 2. Inverted pendulum

Table 1. State feedback gains

	$\theta = -\frac{\pi}{6}$	$\theta = 0-$	$\theta = \frac{\pi}{6}$
$\dot{\theta} = -1$	1.0e+005 * -0.0050 2.6317 2.7376	1.0e+003 * -0.4357 -3.4929 -1.5452	1.0e+005 * 0.0050 2.6317 2.7376
$\dot{\theta} = 0$	1.0e+005 * 0.0000 2.6318 2.7377	1.0e+003 * -0.4357 -3.4929 -1.5452	1.0e+005 * 0.0000 2.6351 2.7412
$\dot{\theta} = 1$	1.0e+005 * -0.0050 2.6317 2.7376	1.0e+003 * -0.4357 -3.4929 -1.5452	1.0e+005 * 0.0050 2.6317 2.7376

In the Table 1, the state feedback gains are shown for each sampled parameters. As shown in the table, state feedback gains are similar. These similarities are due to the structure of the plant.

5. CONCLUSION

In this paper, the state space model of the Lagrange dynamics is presented. The presented state space model is descriptor type linear parameter dependent system. Main result of this paper is that the easy way of treating the Lagrange dynamics is developed and controller design algorithms in the linear system theory can be applicable to the Lagrange dynamics by using presented state space model. Because of uncontrollable modes which included in state-space modeling, some conservatism is esist.

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