Optimal Design of Robust Quantitative Feedback Controllers Using Linear Programming and Genetic Algorithms

Vaheed S. Bokharaie*, Ali Khaki-Sedigh+

** Graduate Student, email: v_samadibokharaie@ee.kntu.ac.ir*
+ Full Professor, email: sedigh@eetd.kntu.ac.ir

K.N. Toosi University of Technology, Electrical Eng. Department, Tehran, Iran

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Abstract

Quantitative Feedback Theory (QFT) is one of most effective methods of robust controller design and can be considered as a suitable method for systems with parametric uncertainties. Particularly it allows us to obtain controllers less conservative than other methods like H_∞ and μ-synthesis. In QFT method, we transform all the uncertainties and desired specifications to some boundaries in Nichols chart and then we have to find the nominal loop transfer function such that satisfies the boundaries and has the minimum high frequency gain. The major drawback of the QFT method is that there is no effective and useful method for finding this nominal loop transfer function. The usual approach to this problem involves loop-shaping in the Nichols chart by manipulating the poles and zeros of the nominal loop transfer function. This process now aided by recently developed computer aided design tools proceeds by trial and error and its success often depends heavily on the experience of the loop-shaper. Thus for the novice and First time QFT user, there is a genuine need for an automatic loop-shaping tool to generate a first-cut solution. In this paper, we approach the automatic QFT loop-shaping problem by using an algorithm involving Linear Programming (LP) techniques and Genetic Algorithm (GA).

1. Introduction

The Quantitative Feedback Theory (QFT) method offers a direct, frequency-domain based design approach for tackling feedback control problems with robust performance objectives. In this approach, the plant dynamics may be described by frequency response data, or by a transfer function with mixed (parametric and non-parametric) uncertainty models. One feature that distinguishes QFT from other frequency-domain methods, such as H_∞ and LQG/LTR, is its ability to deal directly with uncertainty models and robust performance criteria. This is achieved by translating robust performance specifications and uncertainty models into so-called QFT bounds. These bounds, typically displayed on a Nichols chart-like plot, then serve as a guide for shaping the nominal loop transfer function which involves the manipulation of gain, poles and zeros. This design process is executed efficiently using computer aided design software, such as the QFT Control Design MATLAB Toolbox [1], and is effective for "simple" problems. Nevertheless, QFT designers are often challenged by such control problem; due to a lack of loop- shaping experience, and could benefit from an algorithm that automatically provides a first-cut solution to the loop-shaping problem. In addition, an automatic loop-shaping facility would enhance the capabilities of the expert QFT designer. Automatic loop-shaping algorithms have been proposed over the past twenty years and this paper reports on a new version.

In this paper we provide an automatic loop-shaping algorithm that builds upon the previous works. We pose the loop-shaping problem as a linear program, and then we optimize the numerator parameters of controller. In contrast to Bryant and Halikias [2], the QFT bounds are tightly described by linear inequalities. This is achieved by first posing the QFT problem in terms of the closed-loop complementary sensitivity function T rather than the nominal loop transfer function as done in the classical QFT approach. Then, since these (closed-loop) QFT bounds are not generally convex, we transform the problem so that they can be exactly described by linear inequalities. These convex QFT bounds arc then evaluated at a finite set of frequencies to form a set or linear inequalities constraining T. Then a linear program is solved where the cost measures high-frequency controller gain and where the linear constraints (non-conservatively) represent the closed-loop QFT bounds.

After optimizing controller numerator, we use genetic algorithm to optimize the denominators of controller. In this step numerator is considered fixed and GA is used just for optimizing the denominator parameters.

In fact, this paper reconciles the method explained in [3] which just makes use of LP and the method explained in [4] which just makes use of random optimization techniques.

2. The QFT Design Technique

The general QFT problem is how to design controller C(s) and pre-filter F(s) such that for a given set of uncertain plants $P \in \{P\}$ with perturbed parameters $\alpha \in \Omega$ the following specifications are satisfied:

(i) Robust Stability:

$$T_R = FT = \frac{FCP}{1+CP} = \frac{FL}{1+L}$$

is exponentially stable $\forall \alpha \in \Omega$.

*(ii) Robust tracking performance: T*wo time functions a(t) and b(t), are given and a command input r(t) (for example a step function) that specify the output tolerance of y(t) in the form:

$$a(t) \le y(t) \le b(t) \qquad \forall P \in \{P\} \qquad (1)$$

These tracking specifications in the time domain can be translated into the frequency domain upper and lower bounds for $T_R(j\omega)$, as shown in figure 2, that satisfies:

$$a(\omega) \le |T_R(j\omega)| \le b(\omega) \quad \textit{in dB units} \qquad (2)$$

(iii) Disturbance rejection specification: A function $D(\omega)$ is given that specifies the output specifications of $T_d(j\omega)$, in then form:

$$|T_d(j\omega)| \le D(\omega) \qquad \forall P \in \{P\} \qquad (3)$$

where

$$T_d(s) = \frac{y_d(s)}{d(s)} = \frac{1}{1+G(s)P(s)} \equiv S(s)$$

and *d(s)* is the output disturbance function.

In the classical QFT design, the above specifications transforms into boundaries for some pre-specified frequencies in Nichols chart and we have to derive an open-loop transfer function $L_0(s) = P_0(\alpha_0, s).C(s)$ such that L_0 lie above the boundaries of all frequencies (note that $P_0(\alpha_0, s)$ is the nominal plant). In this paper we will introduce a method for automating this process. The pre-filter F(s) can then be calculated easily.

3. Automatic Loop-Shaping

In this paper we provide an automatic loop-shaping algorithm that includes two steps. In both steps we transform the open loop boundaries into close loop boundaries in complex plane and we find a nominal closed loop transfer function instead of nominal open loop transfer function. In fact these two formulations are the same and there is no inherent difference between them. But It will be shown that this new formulation will able us to transform the problem into a linear problem setup. In the first step (which is just like the method introduced in [3]) we pose the loop-shaping problem as a LP problem which yields a stabilizing controller of prescribed order and minimal high-frequency gain, although in this step we just optimize the numerator of closed loop transfer function and its denominator must be a priori defined. This formulation is achieved by first posing the QFT problem in terms of the complementary sensitivity function T rather than the nominal loop transfer function as done in the classical QFT approach. Although these new bounds might not be convex, in these cases we have to use their convex hulls. These convex QFT bounds are then approximated with a polyhedral to form a set of linear inequalities constraining T. Next, Closed-loop stability is imposed by fixing the poles of T (to be stable). Finally, a linear program is solved where the linear constraints (non-conservatively) represent the closed-loop QFT bounds.

A key limitation of the above procedure is that the poles of T are fixed with only the zeros taken as optimization variables. So in the second step, we optimize the denominator coefficients where the cost function is the quadratic sum of Euclidean distance between the open-loop response and the bounds in the Nichols plane. This minimization tries to reach the optimal loop-shaping defined by I.M. Horowitz and M. Sidi [5, 6]. Now we describe these two steps in more detail:

STEP 1:

1. Convert the open-loop bounds into closed-loop bounds.
2. Check for convexity, if not: compute the convex hull.

3. Compute a set of linear inequalities for each bound.
4. Define the poles of T(s).
5. Compute the matrices A and B for the linear inequalities Ax<B from steps 3 and 4 and this equality:

$$T(s)=\sum_{j=1}^{n}\frac{\alpha_j}{s+p_j}+\sum_{k=1}^{m}\frac{\beta_k s+\gamma_k}{s^2+c_k s+d_k} \quad (4)$$

   Note that T(s) is linear in its residuals (α, β, γ) and is not linear in its denominator coefficients (or poles).

6. Solve the linear program to find the appropriate values for α, β, γ. If there is no solution allow user to either select a new set of poles or increase order of complementary sensitivity function T.

7. If the problem is solved, extract the controller from:

$$C=\frac{1}{P_0}\frac{T}{1-T}$$

STEP 2: The first step can only determine optimal zeros with denominator which is a priori defined. The second step is implemented to optimize the closed-loop poles. I.M. Horowitz and M. Sidi [5, 6] demonstrated that if an optimal controller exist, it is unique and the associated open-loop or closed-loop lies on the boundary of each trial frequency $w_1,\ldots,w_t$. To take into account this property, we consider a cost function based on the quadratic sum of Euclidean distance between the open-loop and QFT bound in the Nichols plane at each frequency:

$$f(d_1,d_2,\ldots)=\sum_{k=1}^{t}|\,dist(C(j\omega_k)P_0(j\omega_k),bound\,of\,\omega_k)\,|$$

This function depends a priori on the numerator and denominator coefficients of T, but the first step of optimization has determined the numerator coefficients so we would just optimize the denominator coefficients.

It must be noted that second step don't eliminate the results taken in first step. For understanding that, note that we closed loop transfer function is generally considered as follows that is another formulation of (4):

$$T=\frac{a_p s^p+\ldots+a_1 s+a_0}{s^q+b_{q-1}s^{q-1}+\ldots+b_1 s+b_0}$$

By applying some ordinary mathematics we can conclude that the high-frequency gain is equal to the a_m , so in the second step when we consider the numerator of T(s) fixed, the high-frequency gain won't change and just the distance between T(jω) in each trial frequency and its related bounds will decrease.

4. Results

We show the effects of using this method on a benchmark problem of QFT theory. The problem is designing a controller for a DC motor whose uncertain transfer function is:

$$P(s)=\frac{K}{s(1+\tau_m s)(1+\tau_e s)}$$

in which $150 \le K \le 300$ and $0.012 \le \tau_m \le 0.020$ and $\tau_e = 0.001$s.
The closed-loop objectives are:

- Robust stability specifications:

$$\forall P(j\omega)\in P,\forall\omega>0;\left|\frac{C(j\omega)P(j\omega)}{1+P(j\omega)C(j\omega)}\right|\le 1.1$$

- Tracking Specifications:

$$\forall P(j\omega)\in P,\forall\omega>0;T_d(\omega)\le\left|F\frac{CP}{1+PC}\right|\le T_u(\omega)$$

with:

$$T_d(\omega)=\left|\frac{1}{1+\frac{3(j\omega)}{50}+\frac{3(j\omega)^2}{50^2}+\frac{3(j\omega)^3}{50^3}}\right|$$

and

$$T_u(\omega)=\left|\frac{1}{1+\frac{(j\omega)}{120}+\frac{(j\omega)^2}{120^2}}\right|$$

- Input disturbance rejection specifications:

$$\forall P(j\omega) \in P, \forall \omega > 0; \left| \frac{P(j\omega)}{1+P(j\omega)C(j\omega)} \right| \leq \left| \frac{\frac{(j\omega)}{10}}{1+\frac{(j\omega)}{10}} \right|$$

- Output disturbance rejection specifications:

$$\forall P(j\omega) \in P, \forall \omega > 0; \left| \frac{1}{1+P(j\omega)C(j\omega)} \right| \leq \left| \frac{(j\omega)}{10} \right|$$

And in the first step, we imposed the fixed denominator as follows:

$$D(s) = \left(1+\frac{2}{80}s+\frac{1}{80^2}s^2\right)\left(1+\frac{1}{1000}s\right)\left(1+\frac{1}{10000}s\right)$$

The optimization only composed by the first step leads to results shown in figures 1 to 4 in terms of frequency and transient response. Tracking and output disturbance rejection specifications are satisfied but the input disturbance rejection specification is not satisfied.

The two step optimization allows us to obtain a controller satisfying all specifications as shown in figures 5 to 8. It is completely obvious that the input disturbance rejection specification is also satisfied.

5. Conclusions:

This paper proposes an automatic QFT controller design with a two step optimization algorithm following the optimal controller defined by I.M. Horowitz and M. Sidi [5, 6]. The first step is a convex closed-loop numerator optimization and the second one is a convex closed-loop denominator optimization. This method is tested on some benchmark problems and has given good results.

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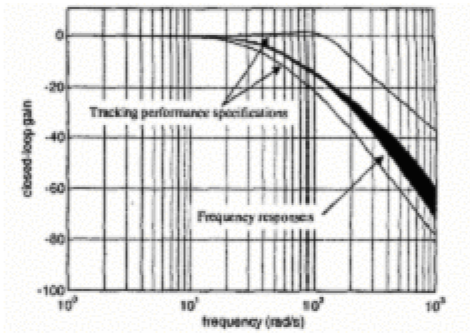

Figure 1 : Closed-loop response frequency

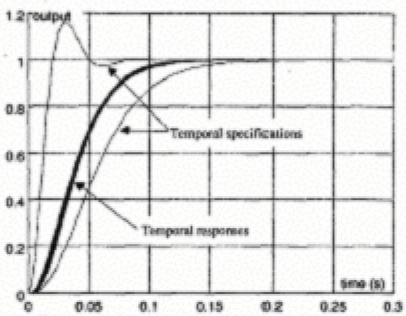

Figure 2 : Closed-loop step response

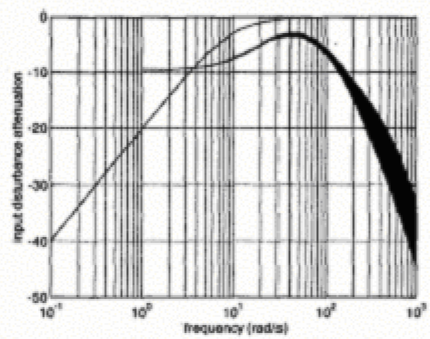

Figure 3 : Input disturbance response frequenc

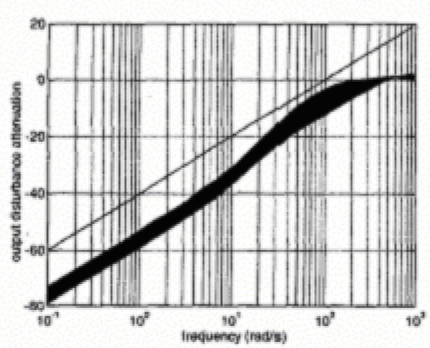

Figure 4 : Sensitivity response frequency

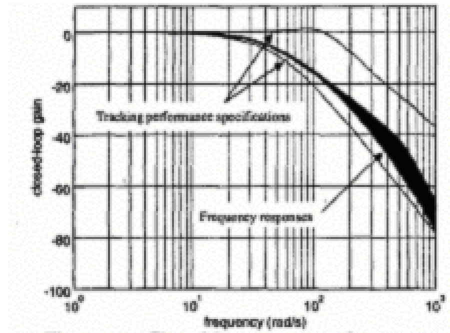

Figure 5 : Closed-loop response frequency

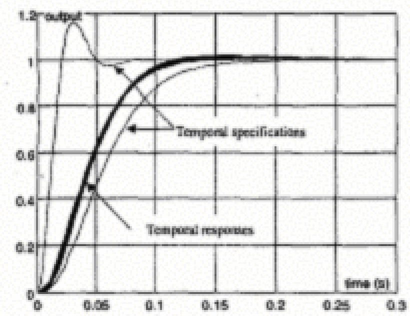

Figure 6 : Closed-loop step response

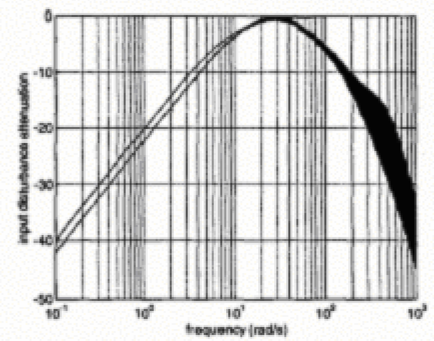

Figure 7 : Input disturbance response frequency

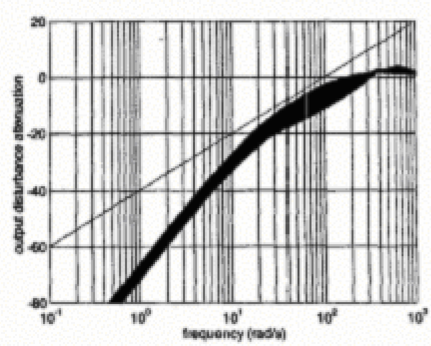

Figure 8 : Sensitivity response frequency