Robust H_∞ Control for Bilinear Systems with Parameter Uncertainties via output Feedback

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Abstract: This paper focuses on robust H_∞ control for bilinear systems with time-varying parameter uncertainties and exogenous disturbance via output feedback. H_∞ control is achieved via separation into a H_∞ state feedback control problem and a H_∞ state estimation problem. The suitable robust stabilizing output feedback control law can be constructed in term of approximated solution to x-dependent Riccati equation using successive approximation technique. Also, the H_∞ filter gain can be constructed in term of solution to algebraic Riccati equation. The output feedback control robustly stabilizes the plant and guarantees a robust H_∞ performance for the closed-loop systems in the face of parameter uncertainties and exogenous disturbance.

Keywords: Robust H_∞ Control, Bilinear System, Parameter Uncertainty, Output Feedback, Successive Approximation

1. INTRODUCTION

Many real physical systems are described by bilinear systems, from the practical point of view there is a need for the application oriented controller design technique for bilinear system [1],[2],[3],[4],[5]. Many researchers design state feedback control laws for bilinear systems using iterative algebraic Riccati equation [1],[2], but they did not consider the exogenous disturbance and uncertainty. Recently, robust control and filter are issued and developed by many researchers for linear systems [6],[7],[8],[9],[10]. But in the class of nonlinear systems including bilinear systems, because conditions for the solvability of the robust H_∞ control design problem and H_∞ filter design problem are hardness, still there are a lot of problems to be developed. Therefore, we consider the bilinear systems with time-varying parameter uncertainties and exogenous disturbance in class of measurement feedback control. A popular method to solve the standard H_∞ control problem is the state-space approach developed in [6],[7], which is based on the relation between the H_∞ norm bound of linear system and the Riccati equation or inequality. The background of these results is that the robust stability of the system with parameter uncertainty can be guaranteed by a positive definite solution of the Riccati equation [6],[7].

In this paper, this state-space approach is extended to bilinear H_∞ control problem. As the result of H_∞ control for linear system [6],[7], H_∞ control is achieved via separation into a H_∞ state feedback control problem and a H_∞ state estimation problem. Also, we extended the results of [8],[9] to bilinear systems. Those are quadratic stabilization methods for linear systems. The quadratic stabilization x-dependent Riccati equation that is associated with full information problem and the quadratic filter algebraic Riccati equation that is associated with robust filter problem are presented. Hence, a x-dependent Riccati equation can be hardly found, because it includes a state vector x. Therefore, we present a new algorithm for solving x-dependent Riccati equation using successive approximation. Thus we can find the approximated solution using proposed method. Also, we deal with the smoothness conditions for the solvability of the robust H_∞ design problem.

In section 2, we consider bilinear systems with parameter uncertainties and exogenous disturbance and develop a H_∞ state feedback control problem and a H_∞ state estimation problem. The smooth conditions for the solvability of the robust H_∞ output feedback control design problem for bilinear systems are provided. In section 3, a new algorithm for solving x-dependent Riccati equation using successive approximation is proposed. To prove the efficiency of the proposed method, we simulate a numerical example in section 4. Finally, section 5 gives our conclusion.

2. ROBUST H_∞ CONTROL FOR BILINEAR SYSTEMS VIA OUTPUT FEEDBACK

Consider the bilinear systems with time-varying parameter uncertainties and exogenous disturbance described by state-space representation of the form

$$\dot{x}(t) = [A+\Delta A(t)]x(t) + B_1 w(t) + [B_2 + \Delta B_2(t) + \{x(t)M\} + \{x(t)\Delta M(t)\}]u(t) \quad (1)$$

$$x(t_0) = x_0$$

$$z(t) = C_1 x(t) + D_{12} u(t) \quad (2)$$

$$y(t) = [C_2 + \Delta C_2(t)]x(t) + D_{21} w(t) \quad (3)$$

where $x(t)\in R^n$ is the state, $w(t)\in R^p$ is the disturbance input which belongs to $L_2[0,\infty)$, $u(t)\in R^m$, is the control input, $z(t)\in R^p$ is the controlled output and $y(t)\in R^q$ is measured output. Generally the nonlinear terms in (1) are described by the forms:

$$\{x(t)M\} = \sum_{j=1}^{n} x_j(t) M_j \tag{4}$$

$$\{x(t)\Delta M\} = \sum_{j=1}^{n} x_j(t) \Delta M_j \tag{5}$$

$A, B_1, B_2, C_1, C_2, D_{12}, D_{21}$ and M_j are known constant real matrices of appropriate dimensions that described the nominal system and $\Delta A(t)$, $\Delta B_2(t)$, $\Delta C_2(t)$ and $\Delta M_j(t)$ are real-valued matrix functions representing time-varying parameter uncertainties in the state and the input matrices. For technical simplification, we define that:

$$\tilde{B}_2 = B_2 + \{x(t)M\} \tag{6}$$

$$\Delta\tilde{B}_2 = \Delta B_2 + \{x(t)\Delta M\} \tag{7}$$

Therefore, the bilinear systems described in differential equation (1) can be defined as follows:

$$\dot{x}(t) = Ax(t) + B_1 w(t) + [\tilde{B}_2 + \Delta\tilde{B}_2(t)]u(t) \tag{8}$$

$$x(t_0) = x^0$$

For computational simplification, without loss of generality we shall make the following assumption.

Assumption 1

$$D_{12}^T [C_1 \quad D_{12}] = [0 \quad I]$$

$$\begin{bmatrix} B_1 \\ D_{21} \end{bmatrix} D_{21}^T = \begin{bmatrix} 0 \\ I \end{bmatrix} \qquad \Diamond$$

In this paper, the time-varying parameter uncertainties matrices $\Delta A(t)$, $\Delta\tilde{B}_2(t)$ and $\Delta C_2(t)$ considered here are of the form:

$$[\Delta A(t) \quad \Delta\tilde{B}_2(t) \quad \Delta C_2(t)] = QP(t)[E_1 \quad E_2 \quad E_3] \tag{9}$$

where Q, E_1, E_2 and E_3 are known constant real matrices and $P(t) \in R^{i \times j}$ is an unknown matrix function with Lebesgue measurable elements such that $P^T(t)P(t) \le I$.

2.1 Full information control

In this case, control law has access to both the stat x and exogenous disturbance w. Our aim is to design the full information control law such that the closed-loop system R_{zw} satisfies (10).

Definition 1 Suppose the constant $g > 0$ is given. The uncertain bilinear system (8), (2) is said to be stabilizable with an H_∞ norm bound g if there exists a state feedback control law, $u(t) = -Kx(t)$ such that for any admissible parameter uncertainty $P(t)$ the following conditions are satisfied:

(a) The closed-loop system is uniformly asymptotically stable.

(b) The closed-loop transfer function from disturbance to controlled output z, $R_{zw} = C_c(sI - A_c)^{-1}B_1$ satisfies the following H_∞ norm bound:

$$\|R_{zw}\|_\infty < g \tag{10}$$

where $A_c = A + QP(t)E_1 - (\tilde{B}_2 + QP(t)E_2)K$ and $C_c = C_1 - D_{12}K$ are state space model of the closed-loop system. $\Diamond$

In this paper we also use the concept of quadratic stabilization with an H_∞ norm bound, introduce in [8].

Definition 2 Suppose the constant $g > 0$ is given. The uncertain bilinear system (8), (2) is said to be quadratically stabilizable with an H_∞ norm bound g if there exists a state feedback control law, $u(t) = -Kx(t)$ and a real symmetric positive definite matrix X such that the inequality

$$A_c^T X + XA_c + g^{-2} XB_1B_1^T X + C_c^T C_c < 0 \tag{11}$$

holds for any admissible parameter uncertainties. $\Diamond$

In order to solve the robust H_∞ control problem involves solving a parameter- and a state vector x-dependent Riccati equation associated with an H_∞ norm bound $g > 0$ and the parameter uncertainties $\Delta A(t)$ and $\Delta B(t)$. Given the bilinear system (8), (2) and any desired g, we define the following x-dependent Riccati equation corresponding to the problem of quadratic stabilization with H_∞ norm bound $g > 0$.

$$\begin{aligned} &A^T X + XA + g^{-2} XB_1B_1^T X + e\,XQQ^T X \\ &-(X\tilde{B}_2 + \frac{1}{e}E_1^T E_2)R_c^{-1}(X\tilde{B}_2 + \frac{1}{e}E_1^T E_2)^T \\ &+\frac{1}{e}E_1^T E_1 + C_1^T C_1 + d\,I = 0 \end{aligned} \tag{12}$$

where

$$R_c = I + \frac{1}{e}E_2^T E_2 \tag{13}$$

In the quadratic stabilization x-dependent Riccati equation (12), $e > 0$ is design parameter and d is a sufficiently small positive constant.

Theorem 1 The uncertain bilinear system (8), (2) is quadratically stabilizable with an H_∞ norm bound $g > 0$ if and only if for all sufficiently small $d > 0$ there exists a constant e such that the quadratically stabilizable x-dependent Riccati equation (12) has a positive definite solution X. Moreover, a suitable feedback control law, $u(t) = -Kx(t)$, where K is given by

$$K = R_c^{-1}(\tilde{B}_2^T X + \frac{1}{e} E_2^T E_1) \tag{14}$$

Proof: It can be carried out using the similar argument as in [8]. ◊

Theorem 1 provides necessary and sufficient conditions for quadratic stabilization with an H_∞ norm bound g for the uncertain bilinear system (8), (2). The suitable stabilizing feedback control law can be constructed in term of solution to a x-dependent Riccati equation (12).

2.2 A H_∞ filter

We now consider the problem of measurement feedback control, in which the controller must generate the control signal according to $u(t) = Fy(t)$.

Definition 3 Suppose the constant $g > 0$ is given. In the uncertain bilinear system (8), (3), the state vector x is said to be estimatable with an H_∞ norm bound g if there exists the estimate, $\hat{x}(t) = Ly(t)$ such that for any admissible parameter uncertainty $P(t)$ the following conditions are satisfied:

(a) The filter L is uniformly stable.

(b) The system satisfies the following H_∞ norm bound:

$$\|R\|_\infty < g \tag{15}$$

where R is the transfer function from disturbance w to estimation error $(\hat{x} - x)$. ◊

Now, we also use the previous concept of quadratic stabilization with an H_∞ norm bound.

Definition 4 Suppose the constant $g > 0$ is given. In the uncertain bilinear system (8), (3), the state vector x is said to be quadratically estimatable with an H_∞ norm bound g if there exists the estimate, $\hat{x} = Ly(t)$ and a real symmetric positive definite matrix Y such that the inequality

$$A_o Y + Y A_o^T + g^{-2} Y C_1 C_1^T Y + B_1 B_1^T < 0 \tag{16}$$

holds for any admissible parameter uncertainties, where $A_o = A + QP(t)E_1 - L[C_2 + QP(t)E_3]$. ◊

In order to solve the robust H_∞ filter problem involves solving a parameter-dependent Riccati equation associated with an H_∞ norm bound $g > 0$ and the parameter uncertainties $\Delta A(t)$ and $\Delta B(t)$. Given the bilinear system (8), (3) and any desired g, we define the following algebraic Riccati equation corresponding to the problem of quadratic estimate with H_∞ norm bound $g > 0$.

$$\begin{aligned} &AY^T + YA^T + g^{-2} Y C_1^T C_1 Y + e YQQ^T Y \\ &-(YC_2^T + \frac{1}{e} E_1 E_3^T) R_f^{-1} (YC_2^T + \frac{1}{e} E_1 E_3^T)^T \\ &+\frac{1}{e} E_1^T E_1 + B_1 B_1^T + dI = 0 \end{aligned} \tag{17}$$

where

$$R_f = I + \frac{1}{e} E_3 E_3^T \tag{18}$$

In the quadratic filter algebraic Riccati equation (17), $e > 0$ is design parameter and d is a sufficiently small positive constant.

Theorem 2 Suppose the observation $y(t)$ is generated by the uncertain bilinear system (8), (3). There exists a quadratically stable filter L such that the system $R: w \mapsto (\hat{x} - x)$ is quadratically stable with an H_∞ norm bound $g > 0$ if and only if for all sufficiently small $d > 0$ there exists a constant e such that the quadratic filter algebraic Riccati equation (17) has a positive definite solution Y. Moreover, a suitable filter gain H_∞ is given by

$$H = R_f^{-1}(YC_2^T + \frac{1}{e} E_1 E_3^T)$$

Proof: It can be carried out using the dual argument of Theorem 1. ◊

Theorem 2 provides necessary and sufficient conditions for quadratic estimation with an H_∞ norm bound g for the uncertain bilinear system (8), (3). The suitable stable filter can be constructed in term of solution to an algebraic Riccati equation (17)

2.3 Measurement feedback control

In order to solve the problem of measurement feedback control, we make the following assumptions. Those are necessary and sufficient conditions for existence of the solutions of Riccati equations.

Assumption 2:

(a) $([A + \Delta A(t)], [\tilde{B}_2(T) + \Delta\tilde{B}_2(T)])$ is uniformly stabilizable and $([C_2 + \Delta C_2(t)], [A + \Delta A(t)])$ is uniformly detectable.

(b) $(C_1, [A + \Delta A(t)])$ has no unobservable mode on the imaginary axis and $([A + \Delta A(t), B_1)$ has no uncontrollable mode on the imaginary axis for any admissible parameter uncertainty $P(t)$. ◊

Under assumption 1 and 2, the quadratic stabilization x-dependent Riccati equation (12) and the quadratic filter algebraic Riccati equation (17) have stabilizing solutions.

We now consider the problem of real interest, in which our aim is to find a control law $u(t) = Fy(t)$ that satisfies $\|R_{zw}\|_\infty < g$. Generally, the signal $u(t) = -Kx(t)$ with $K = R_c^{-1}(\tilde{B}_2^T X + E_2^T E_1 / e)$ is the H_∞ full-information feedback or so-called state feedback control law. Moreover, the H_∞ measurement feedback or so-called output feedback control law is the robustly stable estimator of $u(t) = -Kx(t)$. It is given by

$$\dot{x}(t) = [A + \Delta A(t)]\hat{x}(t) + H(y - C_2\hat{x}(t)) + [\tilde{B}_2 + \Delta\tilde{B}_2(t)]u(t) \quad (20)$$

$$u(t) = -K\hat{x}(t) \quad (21)$$

in which $H = R_f^{-1}(YC_2^T + E_1E_3^T / e)$. Where X and Y are the solutions of Riccati equations (12) and (17), respectively.

The smooth conditions for the solvability of the robust H_∞ output feedback control design problem for bilinear systems are provided in Theorem 3.

Theorem 3 Suppose the assumption 1 and 2 hold. There exists an admissible control law such that the closed-loop system R_{zw} satisfies (10) for any admissible parameter uncertainty $P(t)$ if and only if:

(a) There is a solution to the quadratic stabilization x-dependent Riccati equation (12) such that $[A + \Delta A(t)] - ([\tilde{B}_2 + \Delta\tilde{B}_2][\tilde{B}_2 + \Delta\tilde{B}_2]^T - g^{-2}B_1B_1^T X$ is quadratically stable and $X \geq 0$.

(b) There is a solution to the quadratic filter algebraic Riccati equation (17) such that $[A + \Delta A(t)] - Y([C_2 + \Delta C_2)^T[C_2 + \Delta C_2] - g^{-2}C_1^T B_1)$ is quadratically stable and $X \geq 0$.

(c) $r(XY) < g^2$

Proof: The proof of this Theorem can be easily drawn from [6] ◊

3. QUADRATIC STABILIZATION x-DEPENDENT RICCATI EQUATION USING SUCCESSIVE APPROXIMATION

The solution of a quadratic stabilization x-dependent Riccati equation that is associated with bilinear systems can be hardly found, because it includes a state vector x. Thus we find the approximated solution using successive approximation.

Using iterative index i, the parameter uncertain bilinear system is defined as follows:

$$\dot{x}^{(i)}(t) = [A + \Delta A(t)]x^{(i)}(t) + B_1 w(t) + [\tilde{B}_2^{(i-1)} + \Delta\tilde{B}_2^{(i-1)}(t)]u^{(i)}(t) \quad (22)$$

$$x^{(0)}(t_0) = x_0$$

where,

$$\tilde{B}_2^{(i-1)}(t) = B_2(t) + \{x^{(i-1)}(t)M(t)\} \quad (23)$$

$$\Delta\tilde{B}_2^{(i-1)}(t) = \Delta B_2(t) + \{x^{(i-1)}(t)\Delta M(t)\} \quad (24)$$

In order to design the quadratically stabilizing output feedback control law, or find an approximated solution of x-dependent Riccati equation (12), we present a following algorithm.

Algorithm 1 A new algorithm for solving x-dependent Riccati equation using successive approximation

Initial step

(a) Set $i = 1$.

(b) Obtain $\tilde{B}_2^{(0)}$ and $\Delta\tilde{B}_2^{(0)}$ using an initial value of the state, $x^{(0)}(t_0)$. Also, define $E_2^{(0)}$ and $R_c^{(0)}$.

(c) Obtain the solution of the following algebraic Riccati equation, using the variables in (b).

$$A^T X + XA + g^{-2}XB_1B_1^T X + e\,XQQ^T X - (X\tilde{B}_2^{(0)} + \frac{1}{e}E_1^T E_2^{(0)})R_C^{(0)^{-1}}(\tilde{B}_2^{(0)^T} X + \frac{1}{e}E_2^{(0)^T} E_1) + \frac{1}{e}E_1^T E_1 + C_1^T C_1 + dI = 0$$

(d) Construct the following control law using the solution in (c).

$$u^{(1)}(t) = -R_c^{(0)^{-1}}(\tilde{B}_2^{(0)^T} X + \frac{1}{e}E_2^{(0)^T} E_1)x^{(1)}(t)$$

(e) Substitute the control law in (d) into the bilinear system (22) and obtain an initial value of the new state, $x^{(1)}(t_0)$, solving the differential equation.

Iterative step

(a) Set $i = i + 1$.

(b) Obtain $\tilde{B}_2^{(i-1)}$ and $\Delta\tilde{B}_2^{(i-1)}$ using an initial value of the state, $x^{(i-1)}(t_0)$. Also, define $E_2^{(i-1)}$ and $R_c^{(i-1)}$.

(c) Obtain the solution of the following algebraic Riccati equation, using the variables in (b).

$$A^T X + XA + g^{-2} XB_1 B_1^T X + e XQQ^T X$$
$$-(X\tilde{B}_2^{(i-1)} + \frac{1}{e} E_1^T E_2^{(i-1)}) R_C^{(i-1)^{-1}} (\tilde{B}_2^{(i-1)^T} X + \frac{1}{e} E_2^{(i-1)^T} E_1)$$
$$+\frac{1}{e} E_1^T E_1 + C_1^T C_1 + d I = 0$$

(d) Construct the following control law using the solution in (c).

$$u^{(i)}(t) = -R_c^{(i-1)^{-1}} (\tilde{B}_2^{(i-1)^T} X + \frac{1}{e} E_2^{(i-1)^T} E_1) x^{(i)}(t)$$

(e) Substitute the control law in (d) into the bilinear system (22) and obtain an initial value of the new state, $x^{(i)}(t_0)$, solving the differential equation.

Convergence determinative step

(a) Until the following $error$ is reduced sufficiently, execute the iterative step.

$$error = \left\| K^{(i)} - K^{(i-1)} \right\|$$

Where

$$K^{(i)} = R_c^{(i-1)^{-1}} (\tilde{B}_2^{(i-1)^T} X + E_2^{(i-1)^T} E_1 / e). \qquad \Diamond$$

In order to prove the proposed algorithm, the numerical example is simulated in next section.

4. NUMERICAL EXAMPLE

A bilinear system (1), (2), (3) with time-varying parameter uncertainties and exogenous disturbance is given as follows

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}$$
$$+\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + x_1 \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + x_2 \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$z = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}^T \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}$$

$$x(t_0) = \begin{bmatrix} 1.8 & -2.1 \end{bmatrix}^T$$

In this case, we assume that time-varying parameter uncertainty $P(t) = \cos(3.6p\,t) I$ and exogenous disturbance $w(t) = [1.7\sin(48p\,t) \quad -1.5\cos(36p\,t) \quad -2.1\cos(27p\,t) \quad 0.7\sin(41p\,t)]^T$. Also, we assume that $g = 0.7$, $d = 0.1$, $e = 0.5$, $E_1 = 0.3I$, $E_2 = 0.47I$, $E_3 = 0.2I$ and $Q = I$

The simulation results for proposed algorithm are given in figures 1-6. In figures 1, 2, (--) are the trajectories of state vector $x(t)$ and (-) are the trajectories of the state vector $\hat{x}(t)$ that is estimated from observation of output $y(t)$ using robust H_∞ filter. In figures 3-6, We see that the trajectories of the state vector and the control input vector converge into better ones according to increase the iteration. Where (⋯), (-·), (--), (-) lines are associated with iterative number i =1,2,3,4, respectively. Also, the H_∞ norm of closed-loop system is $0.4024 < \left\| R_{zw} \right\| < 0.4028$, and so the condition (b) in Definition 1 is satisfied.

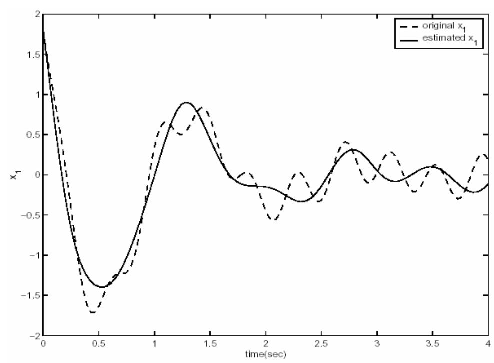

Figure 1: Trajectories of x_1 and $\hat{x}_1$

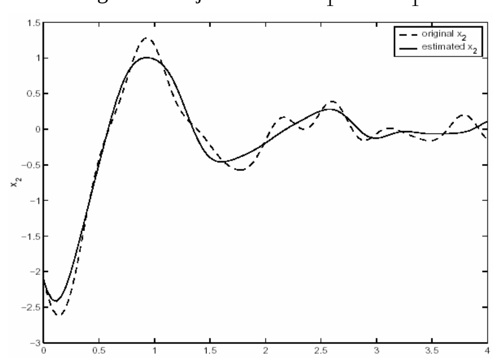

Figure 2: Trajectories of x_2 and $\hat{x}_2$

5. CONCLUSION

This paper has developed a H_∞ output feedback control for bilinear systems with time-varying parameter uncertainties and exogenous disturbance and has presented algorithm for the problem that is find an approximation solution of quadratic stabilization X-dependent Riccati equation using successive approximation. Using this solution, the control law that is quadratically stabilizable and guarantees robust H_∞ performance is designed. Also, we deal with a H_∞ filter and smooth conditions for existence of a quadratically stabilizable output feedback control law. We prove the efficiency of a proposed method by means of numerical example.

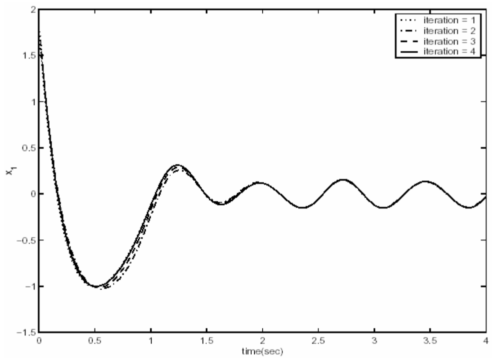

Figure 3: Trajectories of x_1

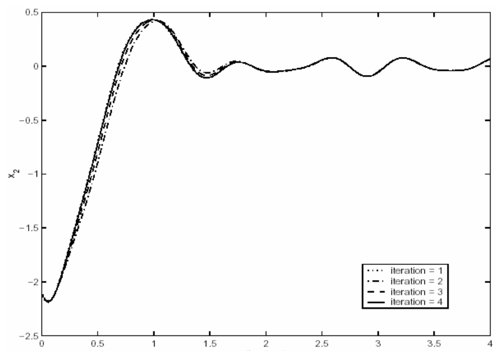

Figure 4: Trajectories of x_2

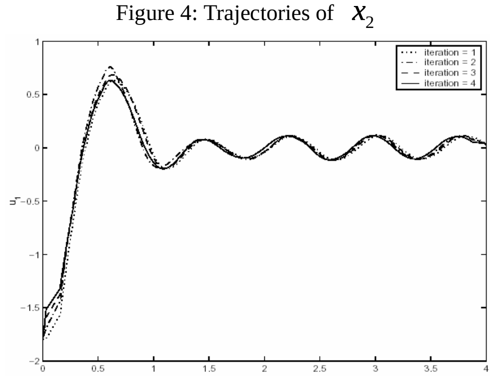

Figure 5: Trajectories of u_1

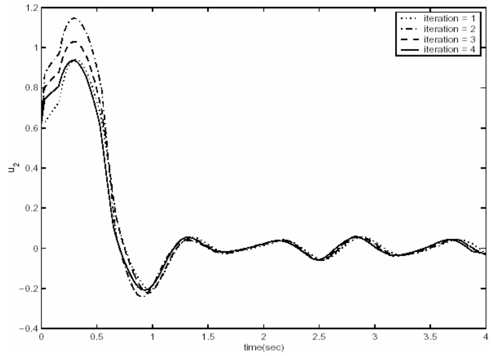

Figure 6: Trajectories of u_2

ACKNOWLEDGMENTS

This work was supported by the Korea Research Foundation Grant. (KRF-2002-041D00209)

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